Inflation and Unemployment in the Long Run Revisited*

Zachary Bethune
Rice University

Michael Choi
University of California, Irvine

Sébastien Lotz
Université Panthéon-Assas

Guillaume Rocheteau†
University of California, Irvine

June 17, 2026

Abstract

We construct a continuous-time, monetary model with frictional goods and labour markets to revisit the long-run relationship between inflation and unemployment. By endogenising the value of consumers' outside options and market power in accordance with standard consumer search theory, we generate novel predictions for the slope of the long-run Phillips curve, optimal monetary policy, and outcomes at the frictionless limit. The relationship between inflation and unemployment is non-monotone and, at low inflation rates, an increase in inflation reduces unemployment. The Friedman rule is suboptimal when firms' average bargaining power across markets is low. Markups and markdowns vanish as frictions disappear.

JEL Classification: D82, D83, E40, E50

Keywords: Unemployment, inflation, money, search, bargaining

*We thank the editor, Alberto Bisin, and three anonymous referees for their constructive comments. We also thank seminar participants at the Board of Governors of the Federal Reserve, Simon Fraser University, the University of Hawaii, the University of Melbourne, Monash University, University of California, Irvine, University of Wisconsin-Madison, Loyola Marymount University, Université Paris-Panthéon-Assas, Renmin University of China, the 2023 Dale Mortensen centre conference, the 2023 Search and Matching in Macro and Finance virtual seminar, the 2024 Theories and Methods in Macro conference in Amsterdam, the 2024 Bristol Macroeconomics Workshop, the Society for Economic Dynamics Annual Meeting 2024, the 2024 International Symposium on Money, Banking and Finance at Orléans, the 2024 Summer Workshop on Money, Banking, Payments, and Finance, and the Econometric Society World Congress in Seoul for their comments.

†Corresponding author: Guillaume Rocheteau, grochete@uci.edu

1 Introduction

According to the classical dichotomy, the long-run rate of inflation, perfectly correlated with money growth by virtue of the quantity theory, does not affect the equilibrium unemployment rate, resulting in a vertical long-run Phillips curve. While this view has been embraced by the canonical models of unemployment of Lucas and Prescott (1974) and Mortensen and Pissarides (1994), it has been challenged by Berentsen et al. (2011) – hereafter referred to as BMW. They show that, in theory, anticipated inflation acts as a distortionary tax that penalises monetary exchange and market activities. In their model, inflation reduces consumers' payment capacity, which has an adverse effect on firms' profits and their incentives to open vacancies in order to hire workers. As a result, the long-run Phillips curve slopes upward, in accordance with Friedman (1977). This logic has been formalised by integrating two workhorse models: the Mortensen and Pissarides (1994) model of unemployment – MP hereafter – and the Lagos and Wright (2005) model of monetary exchange.

While BMW offers an elegant description of goods and labour markets as two frictional markets that open sequentially, they introduce a subtle and overlooked asymmetry regarding the treatment of agents' outside options in each market. In the labour market, a worker who is negotiating her long-term employment contract with a firm can opt out and search for an alternative employer. Hence, by agreeing to an offer, the worker is giving up the opportunity to trade with a different firm, possibly at a different wage. In contrast, in the goods market, matched consumers face no opportunity cost from accepting a trade since their decision does not affect their future trading opportunities. Indeed, by assumption, in every period, consumers want to consume but are matched with at most one producer. Hence, they do not have the option to search for an alternative producer than the one they are matched with to satisfy their *current* desire for consumption. We will show that the lack of meaningful consumer outside options inhibits competitive pressures in the goods market, thereby undermining a core rationale for using search and bargaining models to formalise decentralised markets, namely, that they converge to a rent-free, perfect-competition outcome when trading frictions vanish. Moreover, it shuts down an intuitive market-power channel that operates through the user cost of money, and affects the relationship between inflation and unemployment. Finally, we show it is critical for the design of monetary policy and the optimality of the Friedman rule.

Our paper has two contributions: one is methodological and the other is theoretical. In terms of methodology, we construct a continuous-time monetary model of goods and labour markets with search frictions in both markets that provides a symmetric treatment of workers' and consumers' outside options and the determination of prices and wages. The model has MP and BMW as special cases. Specifically, we endogenise outside options by adding a single parameter in order to disentangle preference shocks from random matching shocks in the goods market. We will show that our economy differs fundamentally from the one in BMW in terms of its positive and normative implications.

Our theoretical contribution is threefold. The first contribution consists in proving the generic nonmonotonicity of the long-run Phillips curve. At low inflation rates, there is a negative relationship between inflation and unemployment whereas the relationship is positive at high inflation rates. This nonmonotonicity arises from two opposite effects of inflation on equilibrium unemployment. First, there is a negative effect on consumers' real balances, identified in BMW, that raises unemployment. Second, there is a new *market-power effect* according to which inflation reduces the value for consumers of opting out of a trade in order to search for an alternative producer. Indeed, search in monetary economies requires agents to hold real money balances at a cost that increases with inflation. This second effect raises firms' market power and incentivises them to open more vacancies, thereby reducing unemployment. The *market-power effect* of inflation dominates at low inflation rates whereas the *real-balance effect* dominates at high inflation rates.

Figure 1 illustrates this finding by plotting the theoretical long-run Phillips curve. There is an inflation rate above the Friedman rule, denoted π^*, that minimises unemployment. Below that inflation rate, the Phillips curve is downward-sloping, which generates a long-run trade-off for policymakers whose dual mandate is to keep inflation and unemployment low. Above π^*, the long-run Phillips curve is upward-sloping, in which case there is no trade-off between inflation and unemployment. These results rationalise the choice of an inflation target between the Friedman rule and π^*.[1]

In addition to showing that the Friedman rule fails to minimise the unemployment rate, the second theoretical contribution establishes that the Friedman rule fails to maximise social welfare when workers and consumers have high enough bargaining power across labour and goods markets. The logic goes as follows. The constrained-efficient allocations are implemented with the Friedman rule and a Hosios condition in each market. A deviation from the Friedman rule has a first-order, positive effect on firm entry and labour market tightness. There is also a negative effect on consumers' real money balances but the corresponding

[1] In our baseline model, consumer search is a threat that disciplines the demand of the firms during their negotiation with consumers, however, on the equilibrium path, consumers do not exercise their option to search once matched with a firm. In Online Appendix E, we consider horizontally differentiated products that induce search along the equilibrium path, and show that our main insight is robust, i.e., the long-run Phillips curve is downward sloping at low inflation rates.

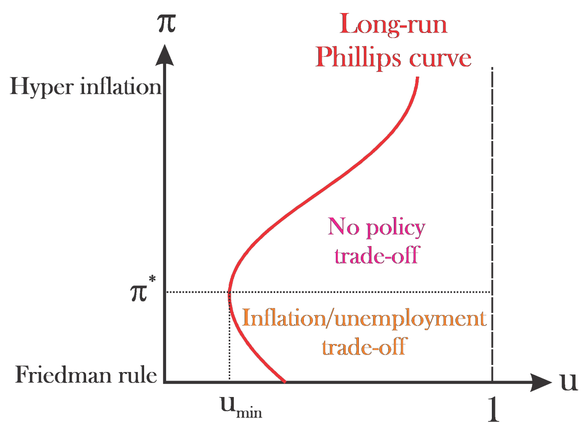

Figure 1: The long-run relation between inflation and unemployment

impact on entry is only second order. Hence, it is optimal to raise the money growth rate above the Friedman rule when entry is inefficiently low, e.g., due to high worker's and/or consumer's bargaining powers. Maybe surprisingly, this logic breaks down in the BMW model. Indeed, in the absence of consumers' outside options, a deviation from the Friedman rule only has a second-order effect on firm entry and social welfare and, under bargaining, the Friedman rule is always optimal.

Our third theoretical contribution is related to the limit of equilibrium outcomes as trading frictions vanish. A major result in the literature on decentralised markets is the convergence of equilibrium allocations to a Walrasian, or perfect-competition outcome as trading frictions vanish (Gale, 1986a,b, 1987). We show that Gale's result on frictionless limits holds in our two-market economies, i.e., markups and markdowns are driven to zero as the process of matching buyers and sellers becomes infinitely efficient. In the goods market, as search frictions vanish, the value of searching increases and rents vanish at the limit. In contrast, in the BMW model, positive rents expand when the search technology becomes more efficient because consumers hold larger real balances while there is no competitive pressure to reduce rents since consumers have no outside option. Table 1 provides an overview of the key differences between the predictions of our model and the ones of BMW.

	BMW	Our model
Slope of Phillips curve	positive	negative at low inflation rates
Friedman rule	optimal for welfare	suboptimal if low firms' bargaining power
Frictionless limit	rents & markups>0	rents & markups=0

Table 1: Comparison to BMW

In order to quantify the magnitude of the market power and real balance effects of inflation, we calibrate our model to the US using evidence from the Consumer Expenditure Survey to inform about the frequency of purchases. We show that the unemployment-minimising annual rate of inflation, π^*, is equal to 1.5%, i.e., the market power channel is dominant for annual inflation below 1.5%. The value of π^* relies on some calibration choices, including the frequency of preference shocks and bargaining powers in goods and labour markets. We show, however, that π^* remains strictly positive for various calibration strategies and can be as high as 6.5% when the market-power effect is counterfactually strong.

In the last section, we introduce two extensions. First, we generalise the relation between monetary policy and unemployment by incorporating a short-term nominal interest rate on liquid government bonds, and limited participation in the bond market. Monetary policy now has two instruments: the constant money growth rate, and the short-term nominal interest rate. When the inflation rate is sufficiently high, the relation between unemployment and the short-term interest rate is non-monotone. If the policymaker chooses both the short-term interest rate and the money growth rate to minimise the unemployment rate, it sets the former to zero – which corresponds to a liquidity trap – and the latter above the Friedman rule.

In our second extension, we study the short-run dynamics of our model. Perfect-foresight transitions to unanticipated money growth rate shocks can generate either a positively- or negatively-sloped short-run Phillips curve, depending on the state of the economy. For economies starting from low steady-state inflation, a positive money growth rate shock leads to a transitory increase in inflation that decreases unemployment since the market power channel dominates. However, the opposite is true for money growth rate shocks to economies with high steady-state inflation. In these cases, the short-run Phillips curve is upward sloping.

1.1 Literature review

Our model builds on Berentsen et al. (2011) that introduces a frictional labour market into the Lagos and Wright (2005) model.[2] Other models with frictional goods and labour markets include Lehmann and Van der Linden (2010), Petrosky-Nadeau and Wasmer (2015, 2017), Kaplan and Menzio (2016) and Michaillat and Saez (2015). In the first three, there

[2]Other models of unemployment and inflation based on the Mortensen-Pissarides framework include Shi (1998), Cooley and Quadrini (2004), and Lehmann (2012). A related approach is provided by Williamson (2015). Versions of the model with money and credit include Bethune et al. (2015), Branch et al. (2016), and Gu et al. (2023). A continuous-time version was constructed by Rocheteau and Rodriguez-Lopez (2014).

is no money, i.e., agents pay with transferable utility.[3] In the last, money is introduced in the utility function directly. In contrast, we explicitly formalise the role of money and the associated liquidity constraints that are critical for the formation of the terms of trade. Relative to BMW, we introduce competitive pressures by assuming that consumers have infrequent desires to consume so that their needs can be fulfilled by different producers over time. This follows the tradition of the consumer search literature where the opportunity cost of trading includes the forgone benefit of searching for other sellers, e.g., McCall (1970), Wolinsky (1986), and Anderson and Renault (1999). These models assume that products are horizontally differentiated in order to generate consumer search in equilibrium. We adopt a similar assumption in an extension of our model in Online Appendix E.

There are alternative approaches to model competition in monetary economies. Head and Kumar (2005); Wang (2016); Wang et al. (2020) introduce noisy search, as in Burdett and Judd (1983), into the Lagos-Wright model.[4] Rocheteau and Wright (2005) and Bethune et al. (2020) introduce directed and partially directed search, as exemplified by Moen (1997) and Lester (2011), into monetary economies. A crucial difference is that, in our model, switching across sellers to exercise consumers' market power takes time, i.e., the search process is sequential. Therefore, monetary policies affect the *intertemporal* outside option of searching for alternative sellers by raising the cost of holding money. In the noisy or directed search model above, search is simultaneous and thus the cost of holding money does not affect the outside option of switching to alternative sellers. This difference leads to different predictions regarding the normative and positive impact of monetary policies.

There are alternative explanations for why the long-run Phillips curve could be downward sloping at low inflation rates. In Rocheteau et al. (2007), where unemployment emerges due to indivisible labour, the long-run Phillips curve can be downward sloping if leisure and consumption are complements.[5] In Rocheteau and Rodriguez-Lopez (2014), the downward-sloping Phillips curve arises from a Tobin effect that induces agents to substitute real money balances for capital and financial assets as inflation increases, thereby promoting job creation and lowering unemployment. In Gu et al. (2023), inflation hurts the unemployed who do not have access to credit, which allows firms to negotiate lower wages. Relative to this literature, we claim that the non-monotone relation between inflation and unemployment is

[3]In particular, Kaplan and Menzio (2016) propose a theory of self-fulfilling unemployment fluctuations through the lens of market power. The crucial assumption is that a higher unemployment rate induces lower market power of firms because unemployed workers spend less and have more time searching for low prices.

[4]Julien et al. (2008), Galenianos and Kircher (2009), and Bajaj and Mangin (2023) also consider imperfect competition by introducing multilateral meetings into search models.

[5]Dong (2011) adopts the notion of competitive search equilibrium in this model to study the relationship between inflation and unemployment.

generic provided there is genuine consumer search in the goods market.

There is a literature on search and inflation under menu costs that shows that inflation erodes firms' market power, e.g., Benabou (1988) and Diamond (1993). In those models, the economy is cashless, i.e., money has no transaction role. Inflation reduces market power by preventing firms from maintaining their real price at a monopoly level as in Diamond (1971).[6] In our model, prices are perfectly flexible and, contrary to Benabou (1988) and Diamond (1993), inflation makes search more costly for consumers by raising the cost of carrying cash, which reduces the value of their outside option and raises firms' market power.

The extension in the last section of the paper with liquid bonds and limited participation in the bond market is related to Alvarez et al. (2001), Alvarez et al. (2002), and Williamson (2006). The existence of liquidity-trap equilibria is similar to Williamson (2012) and Rocheteau et al. (2018). Relative to these papers, we add a frictional labour market and endogenous outside options for consumers. Moreover, we provide conditions under which unemployment is minimum at the liquidity trap.

2 Environment

The benchmark environment, which builds on Choi and Rocheteau (2021), can be interpreted as a version of Lagos and Wright (2005), and Rocheteau and Wright (2005) where centralised and decentralised markets coexist in continuous time.

Time, agents, and commodities Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is composed of three types of infinitely-lived agents: a unit measure of workers, a measure ω of consumers, and an endogenous measure of firms, n.[7] There are two perishable goods, $y \in \mathbb{R}_+$ and $c \in \mathbb{R}$. Good c is taken as the numéraire, can be consumed and produced by all agents, and is traded competitively and continuously over time. Good y is produced by worker-firm pairs and valued by consumers only.

The flows of goods and payments between agents are summarized in Figure 2. The worker/firm pair produces y units of goods for the consumer (labelled B as for buyer) who

[6]Relatedly, New Keynesian models can feature a positive correlation between macroeconomic activity and inflation (see Walsh, 2017). For instance, in King and Wolman (1996), firms adjust prices infrequently as the inflation rate rises, resulting in a lower markup and a higher output. Devereux and Yetman (2002) show that, when the frequency of price adjustments is endogenised, the correlation between inflation and output becomes non-linear and non-monotone.

[7]The separation of workers and consumers provides tractability and is consistent with the description in BMW. One way to link the two is by allowing consumers' choice of real balances to be constrained by workers' wage as in, e.g., Rocheteau et al. (2021).

makes a payment p (expressed in the numéraire) to the firm, who compensates the worker with a wage w.

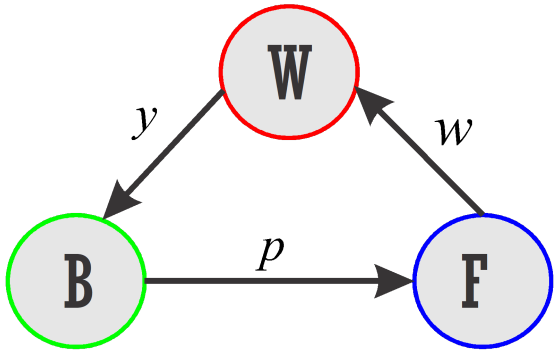

Figure 2: Agents: worker (W), firm (F), consumer (B)

Preferences Preferences of consumers, workers, and firms are represented by the following lifetime expected utility functions:

$$\mathcal{U}^b = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t) + \sum_{n=1}^{+\infty} e^{-\rho T_n} \varepsilon_{T_n} \upsilon\left[y(T_n)\right]\right], \tag{1}$$

$$\mathcal{U}^w = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right], \text{ and} \tag{2}$$

$$\mathcal{U}^f = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right], \tag{3}$$

where $C(t)$ measures the cumulative net consumption of the numéraire good.[8] Negative consumption of the numéraire good is interpreted as production. So preferences of all agents are linear in the numéraire.[9] The second term on the right side of (1) represents the consumption of the y good produced by firms for consumers. In (1) the idiosyncratic stochastic process $\{T_n\}$ indicates the times at which the consumer gets to consume good y. The utility function, $\upsilon(y)$, is such that $\upsilon(0) = 0$, $\upsilon' > 0$, and $\upsilon'' < 0$.

We introduce a meaningful outside option for consumers by assuming that the desire to consume is infrequent. At any point in time, consumers can be in one of two states, idle

[8]A similar cumulative consumption process is assumed in the continuous-time model of OTC trades of Duffie et al. (2005). If consumption (or production) of the numéraire happens in flows, then $C(t)$ admits a density, $dC(t) = c(t)dt$. If the buyer consumes or produces a discrete quantity of the numéraire good at some instant t, then $C(t^+) - C(t^-) \neq 0$.

[9]In the formulation of this environment, we separate agents according to their role (consumer, worker, firm) relative to the consumption or production of good y. It would be equivalent to consider a household composed of a unit measure of workers and a measure ω of buyers with a single integrated budget constraint that incorporates firms' profits, as in, e.g., Shi (1998).

or active, captured by $\varepsilon_t \in \{0, 1\}$.[10] An idle consumer has no desire to consume ($\varepsilon_t = 0$). The desire to consume arrives at Poisson rate $\lambda > 0$, in which case the consumer becomes active ($\varepsilon_t = 1$). This desire is fulfilled after the consumption of any quantity $y > 0$ (e.g., consumers are satiated), in which case the active consumer becomes idle. The measure of active consumers is denoted ω_1 and the measure of idle consumers is ω_0. The assumption that the consumer is satiated after having consumed any positive y is made for tractability. Alternatively, one could assume that the consumer wishes to purchase one unit of a good and negotiates with the firm about the quality, y, to be produced. In that case, satiation occurs after the consumption of one unit of any positive quality.

As an example, assume consumers wish to consume ice cream at some Poisson rate λ. Once they have consumed it, they no longer desire ice cream for a while. (In one calibrated version of the model, agents buy different goods, possibly characterised by different λ's.) The assumption $\lambda < +\infty$, according to which consumers remain temporarily idle (satiated) following consumption, $y_t > 0$, generates an opportunity cost from accepting a trade. This cost, together with bargaining shares, determines sellers' market power. In BMW, $\lambda = +\infty$, in which case consumers are always active.

Technology Each worker-firm pair produces both types of output: a constant flow, $x > 0$, of the numéraire good and endogenous quantities, y, of the decentralised-market good in bilateral meetings with consumers. The production of y units of the decentralised-market good requires $\varphi(y)$ units of numéraire.[11] It is such that $\varphi(0) = \varphi'(0) = 0$, $\varphi'(y) > 0$, and $\varphi''(y) > 0$. We denote y^* such that $\varphi'(y^*) = \upsilon'(y^*)$.

Frictions in the goods market We denote $q \equiv n/\omega_1$ as the market tightness in the decentralised goods market. It is defined as the ratio of the measure of firms to the measure of active consumers. The matching rate of a consumer is $\alpha \equiv \alpha(q)$ where $\alpha(0) = 0$, $\alpha' > 0$, $\alpha'(0) = +\infty$, and $\alpha'' < 0$. The matching rate of a firm is $\alpha^s \equiv \alpha(q)/q$.

Payments We distinguish meetings according to the method of payment. A fraction χ^d of meetings are such that the consumer can produce the numéraire to pay for her consumption. We interpret these meetings as credit meetings. With complementary probability, χ^m, the

[10] In Online Appendix E, we assume preferences are $\varepsilon\upsilon(y)$ and that ε is drawn from a continuous distribution with a positive density on an interval of $\mathbb{R}_+$.

[11] We also worked out a version of the model where the variable cost is in terms of workers' labour or disutility. It does not affect the allocations or results, except for the expression of the worker's compensation. See Online Appendix A.

consumer cannot produce the numéraire in the meeting and is not trusted to repay his debt in the future. In such meetings, the consumer pays with fiat money, an intrinsically useless object that is perfectly storable and durable. The quantity of money at time t is denoted M_t. The constant money growth rate is $\pi \equiv \dot{M}_t/M_t$ and new money is injected in the economy through lump-sum transfers (or taxes if $\pi < 0$) to consumers. The price of money in terms of the numéraire is denoted ϕ_t and the lump-sum transfer is denoted $\tau_t = \phi_t \dot{M}_t$.

In pairwise meetings in the goods market, the quantities produced and consumed, and payments, are determined according to the proportional solution of Kalai (1977), where the share of the surplus received by sellers is $\mu \in [0, 1]$. The reasons for using the Kalai solution are explained in Aruoba et al. (2007), and Hu and Rocheteau (2020) who also provide strategic foundations based on a variant of the Rubinstein game.[12]

Frictions in the labour market Labour market tightness is defined as the ratio of vacancies per unemployed worker, $\theta \equiv \nu/u$. The job finding rate of a worker is $f(\theta)$ with $f(0) = 0$, $f' > 0$, $f'(0) = +\infty$, $f'(+\infty) = 0$, and $f'' < 0$. The vacancy filling rate is $f(\theta)/\theta$. The flow cost of opening a vacancy is $k > 0$. We restrict our attention to contracts where workers are paid a constant wage, w, that is negotiated between the worker and the firm according to the Nash/Kalai solution. Unemployed workers receive a flow income b.

3 Equilibrium

We focus on steady-state equilibria where the distribution of agents across states and their value functions are constant.

3.1 Goods market

The value function of an active consumer with a units of real balances is $V^b(a)$ and the value function of an idle consumer is $W^b(a)$. Given the linearity of preferences with respect to the numéraire good, both value functions are linear, $V^b(a) = a + V^b$ and $W^b(a) = a + W^b$. Intuitively, agents can sell their units of money instantly at their competitive price and reoptimise their money holdings immediately after. (See Choi and Rocheteau, 2021, for details.) We denote $Z \equiv V^b - W^b$ as the loss in terms of lifetime expected utility from transitioning from being active to being idle. It represents the value of the consumer's outside option defined as the opportunity cost from accepting a trade.

[12] The monotonicity property of the Kalai solution guarantees that it implements efficient quantities at the Friedman rule (Aruoba et al., 2007). This result is instrumental for some of our proofs.

Pricing mechanism The goods market is described as a decentralised market where buyers and sellers trade bilaterally, as in Shi (1997) and Trejos and Wright (1995). A trade between a consumer and a firm is a pair, $(p, y) \in \mathbb{R}^2_+$, where p is the payment by the consumer expressed in the numéraire and y is the output produced by the firm. The surplus of the firm is the difference between the firm's revenue expressed in terms of the numéraire and the variable cost to produce y, $p - \varphi(y)$. The firm does not incur an opportunity cost since producing for its current consumer does not reduce its ability to serve its future consumers. The surplus of the consumer is the difference between the utility from consuming y net of the payment and the opportunity cost of accepting a trade, $\upsilon(y) - p + W^b - V^b = \upsilon(y) - p - Z$. The sum of the consumer's and firm's surpluses is $\upsilon(y) - \varphi(y) - Z$.

We adopt a simple rent-sharing rule, similar to the one we will use in the labour market. According to this rule, the profits of the firm are equal to a constant fraction, $\mu \in [0, 1]$, of the overall gains from trade,

$$\overbrace{p - \varphi(y)}^{\text{firm's surplus}} = \mu\overbrace{[\upsilon(y) - \varphi(y) - Z]}^{\text{gains from trade}}. \tag{4}$$

Moreover, we require trades to be pairwise Pareto-efficient.[13] As a result, the output, y, is the largest level, no greater than y^*, that is consistent with both (4) and, in the case of a monetary match, the liquidity constraint, $p \leq a$. In the following, we define

$$g(y) \equiv (1 - \mu)\varphi(y) + \mu\upsilon(y).$$

Lemma 1 ***(Pricing mechanism in the goods market.)*** *Consider a meeting between a consumer holding $a \in \mathbb{R}^+$ real balances and a firm.*

1. *In a credit match, there are gains from trade if $\upsilon(y^*) - \varphi(y^*) > Z$. The trade outcome is*

$$y = y^* \tag{5}$$

$$p = p^* \equiv g(y^*) - \mu Z. \tag{6}$$

2. *In a monetary match, there are gains from trade if*

$$\max_{y \geq 0} \{\upsilon(y) - \varphi(y) : \varphi(y) \leq a\} > Z. \tag{7}$$

[13]This pricing mechanism corresponds to the Kalai proportional solution. For the merits of this solution relative to the generalised Nash solution in environments with liquidity constraints, see Aruoba et al. (2011). For strategic foundations, see Hu and Rocheteau (2020).

The trade outcome is (5)-(6) if $a \geq p^*$, and, if $a < p^*$, then

$$y = g^{-1}(a + \mu Z) \tag{8}$$
$$p = a. \tag{9}$$

If consumers are not liquidity constrained, either because they have access to credit or they have abundant real balances, then y is at its first-best level, y^*, and p divides the gains from trade according to agents' bargaining shares. Inequality (7) gives a condition for a monetary trade to be incentive feasible. According to (8) and (9), if the consumer does not have enough real balances to buy y^*, then she spends all her money to buy the quantity y that is consistent with the proportional bargaining solution. We denote $y(a, Z)$ as the outcome of the negotiation given by (5) and (8). It is nondecreasing in a and Z.

The trade outcome is represented graphically in Figure 3 in the utility space (U^b, U^s), where $U^b = \upsilon(y) - p$ is the buyer's utility of consumption net of the payment and $U^s = p - \varphi(y)$ are the profits of the seller. The bargaining outcome is obtained at the intersection of the downward-sloping Pareto frontier and the upward-sloping proportional sharing rule.[14]

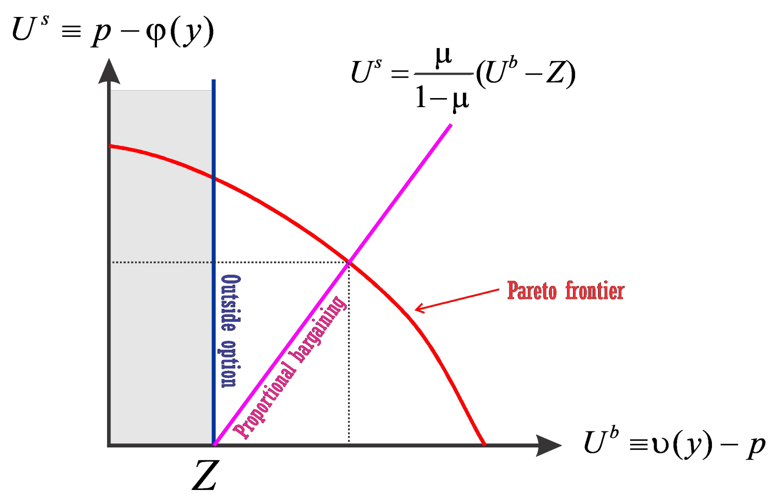

Figure 3: Bargaining in the goods market

[14]The Pareto frontier is linear when the liquidity constraint, $p \leq a$, does not bind and it is strictly concave otherwise.

Bellman equations The HJB equation for the value function of an active buyer, V^b, in a steady state (i.e., $\dot{V}^b = 0$) is

$$\rho V^b = \max_{a \geq 0} \left\{ -ia + \tau + \alpha \overbrace{(1-\mu)\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right]}^{\text{expected trade surplus}} \right\}, \tag{10}$$

where $i \equiv \rho + \pi$ is the nominal interest rate on an illiquid bond (i.e., a bond that cannot serve as means of payment in the decentralised goods market). An active buyer incurs the cost of holding real balances, ia, receives a lump-sum transfer, τ, enters a match with a firm at Poisson rate α, and receives $1-\mu$ share of the trade surplus. With probability χ^m, the match is monetary and the surplus is S^m given by

$$S^m(a,Z) \equiv \upsilon[y(a,Z)] - \varphi[y(a,Z)] - Z. \tag{11}$$

The surplus in monetary matches is increasing in a, and decreasing in Z. With complement probability, χ^d, the consumer can pay with the numéraire and the surplus is S^d given by

$$S^d(Z) \equiv \upsilon(y^*) - \varphi(y^*) - Z. \tag{12}$$

The surplus in credit matches is decreasing in Z.

The HJB equation for the value function of an idle buyer, W^b, is

$$\rho W^b = \tau + \lambda Z. \tag{13}$$

The idle buyer receives a preference shock with Poisson arrival rate λ, in which case she becomes active and enjoys a lifetime utility gain of $Z \equiv V^b - W^b$.

Money demand From (10), the optimal choice of real balances is given by

$$a \in \arg\max_{\hat{a} \geq 0} \left\{-i\hat{a} + \alpha(1-\mu)\chi^m S^m(\hat{a}, Z)\right\}. \tag{14}$$

It follows from (8) and the first-order condition associated with (14) that

$$a = (1-\mu)\varphi(y) + \mu\upsilon(y) - \mu Z \quad \text{where} \quad \frac{\alpha(1-\mu)\chi^m\left[\upsilon'(y) - \varphi'(y)\right]}{\mu\upsilon'(y) + (1-\mu)\varphi'(y)} = i, \tag{15}$$

if $-ia + \alpha(1-\mu)\chi^m S^m(a,Z) \geq 0$. Otherwise, $a = 0$.

Lemma 2 ***(The real balance effect.)*** *For given Z, the demand for real money balances is decreasing in i.*

For given Z, as inflation increases, consumers hold fewer real money balances. As a result, the output traded in pairwise meetings in the decentralised goods market falls.

Market power in the goods market Next, we determine the equilibrium value of the outside option, Z. Substituting (13) from (10), Z solves

$$(\rho + \lambda) Z = \max_{a \geq 0} \left\{ -ia + \alpha(1-\mu) \left[\chi^m S^m(a, Z) + \chi^d S^d(Z) \right] \right\}. \tag{16}$$

The left side of (16) can be interpreted as the opportunity cost of consumer search – the effective discount rate multiplied by the value of consumers' outside options – while the right side is the expected return from the search activity.

Lemma 3 ***(The market-power effect.)*** *Suppose $\chi^d > 0$. There is a unique solution of $Z \in (0, \upsilon(y^*) - \varphi(y^*))$ that solves (16). Moreover, $\partial Z / \partial i = -a/(\rho + \lambda)$.*

In Figure 4, Z is uniquely determined at the intersection of an upward-sloping curve—the left side of (16)—and a downward-sloping curve—the right side of (16). As i increases, the curve representing the right side of (16) moves downward and Z decreases. Intuitively, since the cost of holding real balances increases, searching for trading opportunities in the goods market becomes more costly, and thus the value of consumers' outside options decreases.

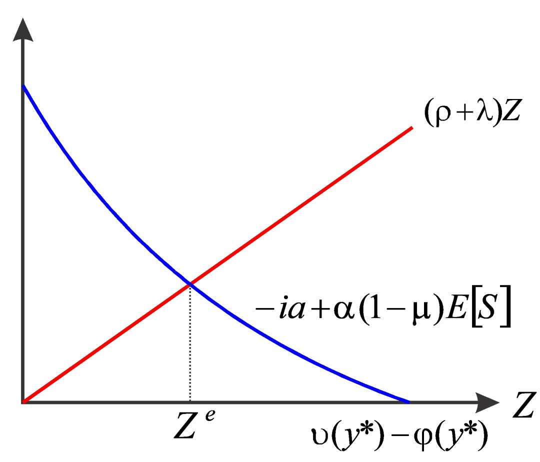

Figure 4: Determination of the value of consumers' outside options

In order to establish the connection between Z and firms' market power, we define the markup associated with a transaction as

$$MKUP \equiv \frac{p-\varphi(y)}{\varphi(y)} = \frac{\mu\left[\upsilon(y)-\varphi(y)-Z\right]}{\varphi(y)}. \tag{17}$$

The markup is the difference between the revenue from a sale and the variable production cost expressed in percentage terms of the latter.[15] For given y, the markup increases with μ but decreases with Z.

The distribution of consumers Finally, we compute the measure of idle and active buyers. The steady-state measure of active consumers solves $\alpha\omega_1 = \lambda(\omega - \omega_1)$, or rewritten,

$$\omega_1 = \frac{\lambda}{\alpha(q)+\lambda}\omega. \tag{18}$$

The measure of active consumers decreases with tightness in the goods market, q.

Takeaways From Lemmas 2 and 3, inflation operates through two distinct channels in the goods market. The real-balance effect implies that higher inflation lowers consumers' demand for real balances, thereby reducing output in pairwise meetings. The market-power effect implies that higher inflation decreases the value of consumers' outside option, which in turn lowers the quantity they can purchase with a given amount of real balances. As will become clear later, these two effects influence firms' profits in opposite directions.

3.2 Labour market

We now turn to the characterisation of the equilibrium of the labour market.

Bellman equations The value of a match composed of a worker and a firm is denoted J. It solves the following HJB equation,

$$(\rho+\delta)\,J = \alpha^s \overbrace{\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right]}^{\text{expected trade surplus}} + x - \rho U, \tag{19}$$

where δ is the job destruction rate, $\alpha^s \equiv \alpha(q)/q$ is the arrival rate of consumers, and U is the value function of an unemployed worker. The firm receives a share, μ, of the surplus

[15] Our measure of the markup is the average price of producing y goods, p/y, over average costs, $\varphi(y)/y$. This is related but different from defining the markup as price over marginal cost, $\varphi'(y)$, and more directly comparable to measures of gross sales margins that we use in the calibration.

generated by each trade. The second term on the right side is the flow of numéraire good produced by the worker-firm pair. The last term is the reservation wage of an unemployed worker. It solves

$$\rho U = b + f(\theta)\beta J, \tag{20}$$

where b is the income when unemployed, $\beta \in [0,1]$ is the worker's bargaining share, and $f(\theta)$ is the job finding rate. Here, we used that the wage is determined by a rent-sharing rule according to which the worker gets a constant share, β, of the match surplus, $E - U = \beta J$.

The HJB equation for an employed worker is

$$\rho E = w - \delta \beta J. \tag{21}$$

On the right side of (21), the worker receives a wage w, and, at rate δ, the match with the firm is destroyed, which generates a capital loss equal to βJ.

Free entry in the labour market Free entry of firms in the labour market implies

$$k = \frac{f(\theta)}{\theta}(1-\beta)J. \tag{22}$$

The flow cost of posting a vacancy, k, is equal to the vacancy filling rate multiplied by the value of a filled job, $(1-\beta)J$. Substituting $f(\theta)$ from (22) into (20), $\rho U = b + \beta k\theta/(1-\beta)$. We substitute this expression into (19) and replace J from (22) to obtain the equilibrium condition for market tightness at the steady state,

$$(\rho+\delta)\,\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s \mu \left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta. \tag{23}$$

Relative to the Mortensen-Pissarides model, the novelty is the term $\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right]$ that represents sales in the frictional goods market.

Lemma 4 ***(Labour market tightness and the effects of inflation.)*** *For given (α^s, a, Z), if $\beta < 1$ and*

$$\alpha^s \mu \left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x > b,$$

then there exists a unique $\theta > 0$ solution to (23).

1. *The real-balance effect. For all $a < p^* \equiv (1-\mu)\varphi(y^*) + \mu\upsilon(y^*) - \mu Z$, $\partial\theta/\partial a > 0$ with $\lim_{a \to p^*} \partial\theta/\partial a = 0$.*

2. *The market-power effect. For all $Z < \upsilon(y^*) - \varphi(y^*)$, $\partial\theta/\partial Z < 0$.*

From Lemma 4, conditional on the outcome of the goods market, there exists a unique solution $\theta > 0$ to (23). This solution increases with a and decreases with Z: firms are more profitable when consumers possess greater payment capacity, but less profitable when consumers enjoy more attractive outside options. These opposing forces represent the main channels through which inflation influences the labour market. Notably, the real-balance effect vanishes as a approaches p^*, which occurs as $i \to 0$ (or, equivalently, as $\pi \to -\rho$).

Wage determination and market power By using the rent-sharing rule, $E - U = \beta J$, and the Bellman equations above, we obtain the following expression for the wage.

Lemma 5 *The real wage is equal to*

$$w = \beta \left\{ \alpha^s \overbrace{\mu \left[\chi^m S^m(a, Z) + \chi^d S^d(Z) \right]}^{\text{firm's surplus in the goods market}} + x \right\} + (1 - \beta)b + \beta k\theta. \tag{24}$$

From (24), market power in the goods market, via μ and Z, affects the wage, and hence affects standard measures of market power in the labour market, such as the wage markdown. The latter is defined as

$$MKDOWN \equiv \frac{\hat{x} - w}{\hat{x}},$$

where $\hat{x}$ is the net expected revenue generated by a filled job, $\hat{x} = \mathbb{E}[p - \varphi(y)] + x$. It is also equal to

$$\hat{x} = \alpha^s \mu \left[\chi^m S^m(a, Z) + \chi^d S^d(Z) \right] + x. \tag{25}$$

By substituting w by its expression given by (24),

$$MKDOWN \equiv \frac{(1 - \beta) \left\{ \alpha^s \mu \left[\chi^m S^m(a, Z) + \chi^d S^d(Z) \right] + x - b \right\} - \beta k\theta}{\alpha^s \mu \left[\chi^m S^m(a, Z) + \chi^d S^d(Z) \right] + x}. \tag{26}$$

The markdown depends on bargaining powers in labour and goods markets as well as consumers' outside options in the goods market.

Distribution of workers The measures of employed and unemployed workers, at the steady state, are

$$n = \frac{f(\theta)}{\delta + f(\theta)} \quad \text{and} \quad u = \frac{\delta}{\delta + f(\theta)}, \tag{27}$$

respectively. Equation (27), which is analogous to the Beveridge curve, gives a positive, steady-state relationship between employment and market tightness.

Takeaways The key takeaway is that the outcome of the goods market shapes the labour market through its impact on firms' profitability and their incentives to create vacancies. The two main channels through which inflation affects the goods market—the real-balance channel and the market-power channel—also determine labour market tightness, and thus unemployment. By reducing real balances, inflation tends to lower labour market tightness and raise unemployment. By diminishing consumers' outside options, inflation instead raises labour market tightness and reduces unemployment. Finally, we showed that the real-balance effect disappears as $i \to 0$.

3.3 Definition of equilibrium

In order to simplify the definition of an equilibrium, we derive a relationship between the tightness of the goods market, q, and the tightness of the labour market, θ. From the definition $q\omega_1 \equiv n$, (18), and (27), the relationship between q and θ is given by

$$\frac{\lambda \omega q}{\lambda + \alpha(q)} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{28}$$

The implicit solution, $q = Q(\theta)$, from (28) is an increasing function of θ with $Q(0) = 0$ and $Q'(\theta) > 0$. Using $Q(\theta)$, we rewrite the matching rates for firms and consumers in the goods market as

$$\alpha^s(\theta) \equiv \frac{\alpha\left[Q(\theta)\right]}{Q(\theta)} \quad \text{and} \quad \alpha^b(\theta) \equiv \alpha\left[Q(\theta)\right],$$

respectively. The rate at which firms sell their output decreases with θ. Indeed, as θ increases, the number of active firms increases, which raises tightness in the goods market and reduces firms' matching rate with consumers.

Using the definitions of α^s and α^b above, from (14), (16), (23), and (24), an equilibrium is a 4-tuple, (θ, a, Z, w), solution to

$$\begin{aligned}
(\rho+\delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta, && (29)\\
a &\in \arg\max_{\hat{a}\geq 0}\left\{-i\hat{a} + \alpha^b(\theta)(1-\mu)\chi^m S^m(\hat{a},Z)\right\}, && (30)\\
(\rho+\lambda)Z &= -ia + \alpha^b(\theta)(1-\mu)\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right], \text{ and} && (31)\\
w &= \beta\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)b + \beta k\theta. && (32)
\end{aligned}$$

Proposition 1 *If $\chi^d > 0$, then there exists an active steady-state equilibrium.*

A sufficient condition for the existence of an active equilibrium is that a positive measure of transactions is conducted with credit, $\chi^d > 0$. Indeed, if θ becomes very small, the expected revenue of firms becomes very large because they can serve a large measure of consumers per firm. Hence, irrespective of b or x, there is always a sufficiently low θ so that firms' profits are positive and entry is profitable.

Nonmonetary equilibrium Nonmonetary equilibria are of interest because they make transparent the role of λ in shaping equilibrium outcomes, and they provide intuition for why high inflation rates lower labour market tightness and increase unemployment.

Proposition 2 ***(Nonmonetary equilibria.)*** *Suppose $\mu \in (0,1)$. There exists a unique nonmonetary equilibrium. Labour market tightness solves*

$$(\rho+\delta)\frac{k\theta}{f(\theta)}+\beta k\theta = (1-\beta)\left\{\alpha^s(\theta)\chi^d \overbrace{\frac{\mu(\rho+\lambda)}{\rho+\lambda+\alpha^b(\theta)\chi^d(1-\mu)}}^{\text{market power in the goods market}}\left[\upsilon(y^*)-\varphi(y^*)\right]+x-b\right\}. \tag{33}$$

1. *As λ increases, the value of consumers' outside options (Z) decreases, labour market tightness (θ) increases, wages (w) increase, and unemployment (u) decreases.*
2. *As χ^d increases, labour market tightness (θ) increases and unemployment (u) decreases.*

In (33), the endogenous market power of firms in the goods market is represented by the term

$$\hat{\mu} \equiv \frac{\mu(\rho+\lambda)}{\rho+\lambda+\alpha^b(\theta)\chi^d(1-\mu)}.$$

It increases with firms' bargaining power, μ, and it decreases with the rate at which consumers meet firms, $\alpha^b(\theta)$. If λ increases, i.e., consumers do not stay idle for long, then the opportunity cost of accepting a trade decreases, which makes the search for an alternative producer less profitable and raises producers' market power.

Finally, $\hat{\mu}$ decreases with the share of meetings where consumers have access to credit, χ^d. However, $\hat{\mu}\chi^d$ increases with χ^d so that labour market tightness rises as consumers have better access to credit. This result will be important later when comparing the economy under the Friedman rule with that under very high inflation. Under the Friedman rule, the economy is equivalent to a credit economy with $\chi^d = 1$. In this case, labour market tightness is higher than in the nonmonetary equilibrium, and unemployment is correspondingly lower.

The BMW economy The economy in BMW is a pure currency economy, $\chi^m = 1$, in which there is no opportunity cost for the consumer to complete a trade, $\lambda = +\infty$. From (18), all buyers are active at all points in time, $\omega_1 = \omega$. From (28), market tightness in the goods market is

$$q = \frac{f(\theta)}{\omega\left[\delta + f(\theta)\right]}. \tag{34}$$

From (31), $Z = 0$ and an equilibrium can be reduced to a pair (θ, y) that is solution to

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\mu\left[\upsilon(y)-\varphi(y)\right]+x-b\right\}-\beta k\theta \text{ and} \tag{35}$$

$$\frac{\upsilon'(y)-\varphi'(y)}{\mu\upsilon'(y)+(1-\mu)\varphi'(y)} = \frac{i}{(1-\mu)\alpha^b(\theta)}. \tag{36}$$

Given (θ, y), the wage is given by

$$w = \beta\left\{\alpha^s(\theta)\mu\left[\upsilon(y)-\varphi(y)\right]+x\right\}+(1-\beta)b+\beta k\theta. \tag{37}$$

Suppose $x < b$. Equation (35) gives a positive relationship between θ and y, with $\theta=0$ when $y = 0$ and $\theta = \bar{\theta} > 0$ when $y = y^*$. Equation (36) gives a positive relationship between y and θ, with $y = 0$ if $\theta < \underline{\theta}$ and $y \to y^*$ as $\theta \to +\infty$. There always exists a non-active equilibrium with $y = \theta = 0$ and, generically, if an active equilibrium exists, then the number of active equilibria is even.[16] In the following, we focus on the equilibrium with the highest θ.

Proposition 3 ***(Long-run Phillips curve in the absence of consumer search)*** *Assume $x < b$ and $\lambda = +\infty$ and focus on the equilibrium with the highest market tightness. An increase in i leads to a decrease in market tightness (θ), a decrease in wages (w), and an increase in unemployment (u).*

We provide a graphical proof (see Figure 5). At the high equilibrium, the MD curve representing (36) in the (θ, y) space intersects the JC curve representing (35) from above. As i increases, the MD curve (36) shifts downward. Hence, y and θ decrease. So the model predicts a positive relationship between unemployment and inflation in the long run, i.e., the long-run Phillips curve is upward sloping.

The logic is based on the following *real-balance effect* of inflation. As i increases, consumers reduce their real money holdings, which lowers the gains from trade that can be extracted in the goods market. Firms, whose profits fall, post fewer vacancies and the unemployment rate increases.

[16]These results mirror those of Rocheteau and Wright (2005) who describe a pure currency economy with free entry of sellers.

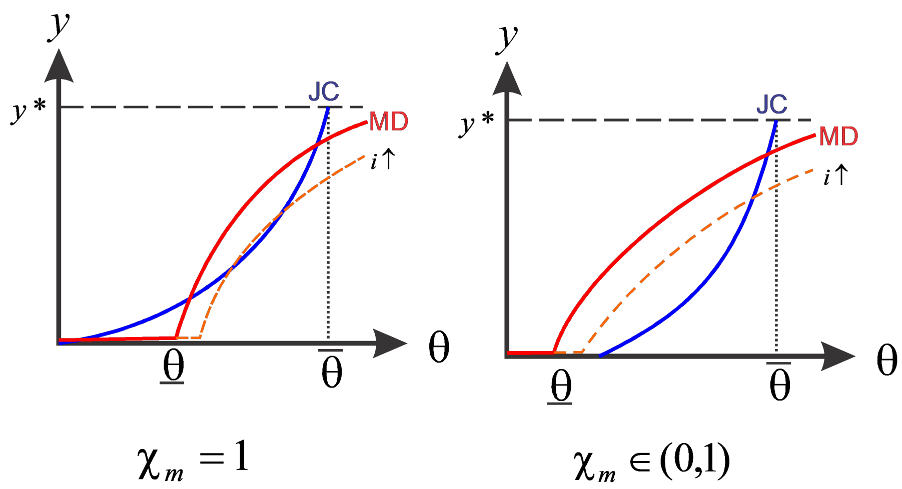

Figure 5: Left panel: Pure monetary economy. Right panel: Economy with money and credit.

The result in Proposition 3 is robust if we reintroduce some credit trades, $\chi^d \in (0, 1)$. What matters is that consumers have no outside option when $\lambda = +\infty$ so that inflation affects θ only through the real-balance effect.

4 Positive and normative implications

We now show that introducing consumers' endogenous outside options, $\lambda < +\infty$, generates new insights for the slope of the Phillips curve, the convergence to perfect competition as frictions vanish, and the optimal monetary policy.

4.1 The non-monotone long-run Phillips curve

To establish the nonmonotonicity of the long-run Phillips curve, we consider increases in the inflation rate of different magnitudes: (i) a small increase in the neighbourhood of the Friedman rule, $i = 0$, and (ii) a large increase from $i = 0$ that drives the value of money to zero.

Proposition 4 ***(The non-monotone long-run Phillips curve)*** *Suppose $\chi^m \in (0, 1)$.*

1. *A small increase in i, starting from $i = 0^+$, leads to: an increase in labour market tightness (θ); a decrease in the unemployment rate (u); and an increase in wages (w).*

2. *A large increase in i from $i = 0^+$ to $i = +\infty$ leads to: a decrease in labour market tightness (θ); an increase in the unemployment rate (u); and a decrease in wages (w).*

An increase in inflation affects the labour market through two channels. First, it reduces consumers' real balances, tightening liquidity constraints and limiting the amount of goods firms can sell. As shown in Proposition 3, this *real-balance effect* lowers market tightness, increases unemployment, and depresses wages. Second, higher inflation raises the cost for consumers of searching for an alternative producer, since they must hold real money balances until a new trading opportunity arises. When i is close to zero, output is close to y^*, and the real-balance effect is negligible, as demonstrated in Lemma 4.[17] Only the second channel, operating through consumers' outside options—the market-power effect—is of first-order importance. Inflation strengthens firms' market power by increasing the cost for consumers of exercising their outside option.[18]

When i is sufficiently large, the two effects described above are first order. When i is so high that the value of money is zero, i.e., all trades are conducted with credit, then firms are worse-off compared to the equilibrium at the Friedman rule, $i = 0$. The logic is the same as the one in Proposition 2: moving from $i = 0$ to $i = +\infty$ is equivalent to reducing the share of credit meetings, which reduces firms' profits.

Taken together, the two results above imply that the relation between θ and i is non-monotonic: it increases initially but eventually decreases once i becomes sufficiently large.

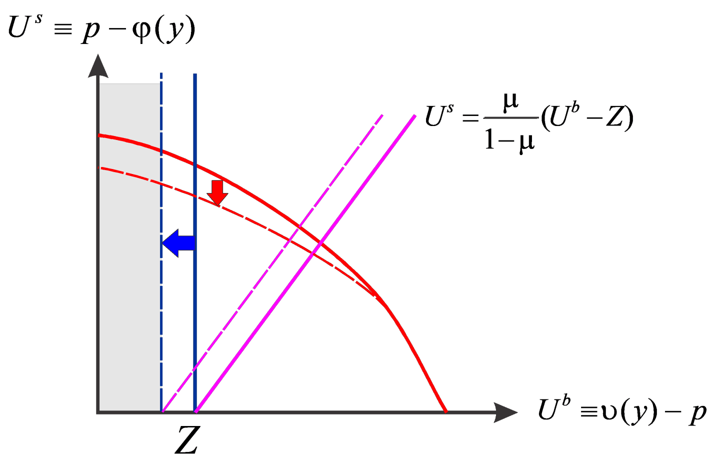

Figure 6: Effects of inflation on the bargaining outcome

[17] For this result, it is important that the bargaining solution is monotone and implements y^* when $i = 0$. Under Nash bargaining, $i = 0$ does not implement y^* and hence a deviation from the FR has a first-order effect on y. The two effects of inflation would still be present but it would not be guaranteed that the market power effect dominates at low inflation rates. However, the gradual Nash solution of Rocheteau et al. (2021) is monotone and would generate the same results.

[18] In Online Appendix E, we consider a version with horizontally differentiated products and show that as inflation increases, consumers become less picky and purchase varieties of the good that they value less – the standard "hot potato" effect of inflation arises.

We illustrate the two effects graphically in Figure 6 that plots the outcome of bargaining in pairwise meetings in the goods market. As inflation increases, a decreases, and the Pareto frontier of the bargaining problem shifts downward. It reduces the profits of the firm, U^s. But as inflation increases, Z decreases, thereby shifting the origin of the rent-sharing line to the left, which increases U^s. At the Friedman rule, $i = 0$, the Pareto frontier is linear when it intersects the rent sharing line, and this linear part does not shift as a decreases. Hence, only the shift of Z matters for the trade outcome.

Since real wages increase with firms' profits (we show in the proof of Proposition 4 that w and θ comove as i changes), the relationship between w and i is also non-monotone. A small, anticipated inflation rate pushes *real* wages up while a large inflation rate depresses *real* wages. Hence, the unemployment minimising level of i maximises real wages.

4.2 Frictionless limits

We now characterise equilibrium outcomes when search frictions in the goods market vanish. We write the matching function in the goods market as $\alpha(q) = A\bar{\alpha}(q)$, where $A > 0$, and we take the limit as $A \to +\infty$. We consider first as a reference point the BMW economy, in which consumers are always active ($\lambda = +\infty$).

Proposition 5 ***(Frictionless limits: The BMW model.)*** *Suppose $\lambda = +\infty$, $\chi^d > 0$, $\mu \in (0,1)$, and $\alpha(q) = A\bar{\alpha}(q)$. As $A \to +\infty$, $y \to y^*$, and $\theta \to +\infty$. Rents in pairwise meetings approach their maximum value, $\upsilon(y^*) - \varphi(y^*) > 0$. Markups remain bounded away from zero, $MKUP \to \mu\left[\upsilon(y^*)/\varphi(y^*) - 1\right] > 0$.*

If perfect competition is defined as the absence of rents (Makowski and Ostroy, 2001), the equilibrium outcome of BMW does not converge to perfect competition as frictions vanish. Indeed, rents attain their maximum value, $\upsilon(y^*) - \varphi(y^*)$, and markups remain strictly positive in the frictionless limit. In the left panel of Figure 7, the Pareto frontier lies farthest from the origin and the rent-sharing line intersects the Pareto frontier in its linear part, i.e., $y = y^*$, since the cost of holding money becomes irrelevant when consumers can locate sellers almost instantly.

We now contrast these results with those obtained in an economy where $\lambda < +\infty$, so that consumers retain meaningful outside options. We proceed in two steps. First, we examine the limit in which search frictions vanish in the goods market. Second, we consider the joint limit in which search frictions vanish in both the goods and labour markets.

Proposition 6 ***(Frictionless limits: The general case)*** *Suppose $\lambda \in (0, +\infty)$, $\alpha(q) = A\bar{\alpha}(q)$ and $f(\theta) \equiv B\overline{f}(\theta)$.*

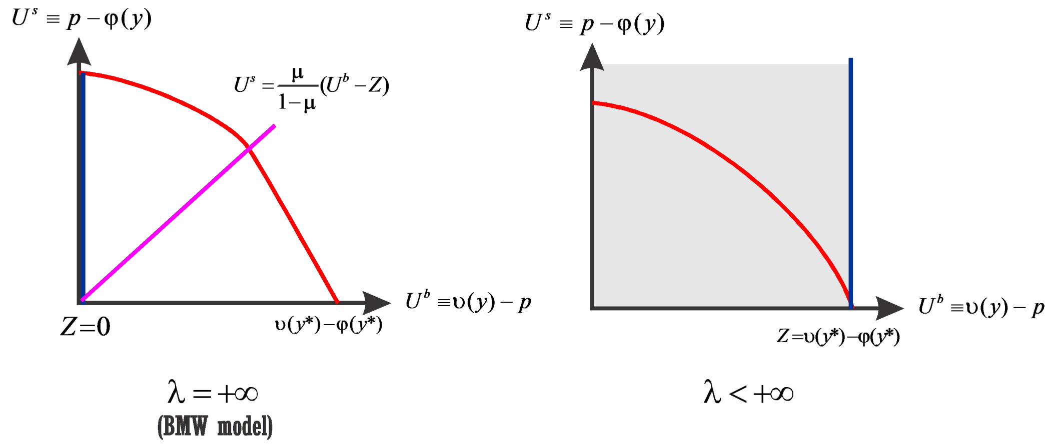

Figure 7: Outcome of bargaining when the goods market becomes frictionless ($A \to +\infty$). Left panel: BMW economy ($\lambda = +\infty$). Right panel: Economy with consumers' outside options ($\lambda < +\infty$).

1. ***Limit as the goods market becomes frictionless**. Consider the limit as $A \to +\infty$. Then, $Z \to \upsilon(y^*) - \varphi(y^*)$ and $MKUP \to 0$. If $x > b$, then $q \to +\infty$ and $\theta \to \theta^g_\infty$, where $\theta^g_\infty > 0$ is the unique solution to*

$$(\rho + \delta) \frac{k\theta^g_\infty}{f(\theta^g_\infty)} + \beta k \theta^g_\infty = (1 - \beta)(x - b). \tag{38}$$

   If $x \leq b$, then $\theta \to 0$.

2. ***Limit as both markets become frictionless**. Consider the limit as $A \to +\infty$ and $B \to +\infty$. Then, $Z \to \upsilon(y^*) - \varphi(y^*)$ and $w \to x$. If $x > b$, then $\theta \to \theta_\infty$, where*

$$\theta_\infty = \frac{(1 - \beta)(x - b)}{\beta k}. \tag{39}$$

   If $x \leq b$, then $\theta \to 0$.

As the frictions in the goods market vanish (Part 1 of Proposition 6), we obtain $Z \to \upsilon(y^*) - \varphi(y^*)$. Consequently, rents in pairwise meetings, $S^m(a, Z)$ and $S^d(Z)$, go to zero. This finding is illustrated in the right panel of Figure 7, which shows that the outcome of bargaining is pinned down by consumers' outside options. This corresponds to the definition of perfect competition in the goods market (e.g., Makowski and Ostroy, 2001). In the limit, quantities are efficient, $y \to y^*$, and the payment equals production cost, $p \to \varphi(y^*)$, implying that the average markup vanishes. Part 2 of Proposition 6 extends this result to the case in which frictions in both the goods and labour markets vanish. In that limit, a competitive outcome arises in both markets: in the labour market, the wage converges to worker's productivity, $w \to x$, so markdowns vanish.

4.3 Welfare

In the presence of search externalities in decentralised goods and labour markets, the monetary policy that minimises the unemployment rate is not necessarily the one that maximises social welfare.

4.3.1 Constrained-efficient allocations

We consider the problem of a planner maximising social welfare subject to search frictions in goods and labour markets. Welfare is the sum of the trade surpluses in pairwise meetings plus the production of the numeraire good by employed workers, x, and unemployed workers, b, net of the vacancy posting costs. (Here, b is not treated as a transfer). We denote $\epsilon_\alpha \equiv \alpha'(q)\, q/\alpha(q)$ and $\epsilon_f \equiv f'(\theta)\theta/f(\theta)$ the elasticities of the matching rates in goods and labour markets, respectively.

Proposition 7 ***(Constrained efficiency).*** *A decentralised equilibrium implements the constrained-efficient allocation if* $i_t = 0$,

$$\mu = \epsilon_\alpha\left(q^s\right), \tag{40}$$

$$1-\beta = \epsilon_f(\theta^s), \tag{41}$$

and the initial values n_0 *and* $\omega_{1,0}$ *equal the steady-state values* n^s *and* ω_1^s.

A decentralised equilibrium maximises social welfare when monetary policy implements the Friedman rule, $i_t = 0$, and the Hosios conditions in both the labour and goods markets hold, $\beta = 1 - \epsilon_f(\theta^{sp})$ and $\mu = \epsilon_\alpha\left[q(\theta^{sp})\right]$, where θ^{sp} is the constrained-efficient labour market tightness.[19] Imposing the Hosios condition in each market and the Friedman rule is sufficient to generate a constrained-efficient outcome but it is not necessary. Indeed, the planner can

[19] This result is a generalisation of Berentsen et al. (2007) to an economy where both goods and labour markets are frictional. The logic of this result requires that the Friedman rule implements y^* which is the case if the bargaining solution is monotone but would not be true for the Nash solution. See Aruoba et al. (2007) for a discussion. Relatedly, Mangin and Julien (2021) derive a "generalised Hosios condition"for a static version of BMW. Petrosky-Nadeau and Wasmer (2017) obtain a similar condition in a nonmonetary economy where credit, labour, and goods markets are frictional. In Online Appendix F, we show that if there is competitive search in both the goods and labour markets, as in Moen (1997), then the Hosios conditions are satisfied and the Friedman rule is optimal.

adjust two bargaining shares to target a single variable, θ; hence, multiple combinations of (β, μ) yield θ^{sp}.

4.3.2 Optimal monetary policy

We now study the optimality of the Friedman rule. In order to simplify the analysis, we follow Hosios (1990) and Pissarides (2000) and assume that agents are infinitely patient, i.e., $\rho \to 0$, which allows us to focus on steady-state welfare.

Proposition 8 ***(Optimal monetary policy)***

1. *If $\lambda = +\infty$, then the Friedman rule, $i = 0$, is optimal.*

2. *If $\lambda < +\infty$, then there exists a $\bar{\beta}(\mu) \in [0, 1)$ such that $i > 0$ is optimal if $\beta > \bar{\beta}(\mu)$. The threshold, $\bar{\beta}(\mu)$, rises in μ.*

When $\lambda = +\infty$, the Friedman rule is optimal regardless of whether labour market tightness is inefficiently high or low relative to the planner's allocation. If θ is excessively high because firms possess too much bargaining power, deviating from the Friedman rule may indeed reduce firm entry. However, such a deviation also lowers the quantity traded, thereby reducing welfare, and this adverse effect always dominates.[20]

When $\lambda < +\infty$, a small deviation from the Friedman rule, i.e., $i > 0$ (or, equivalently, $\pi > -\rho$), has a first-order impact on firm entry by reducing the value of consumers' outside option, Z. This deviation improves social welfare if the market tightness, θ, is inefficiently low at the steady-state equilibrium, which happens when firms have too little bargaining power.

In the left panel of Figure 8, we plot the welfare-maximising inflation rate as a function of β and μ. The choice of the parameter values, which are calibrated to the US economy, and functional forms will be discussed in greater detail in the next section. For now, it suffices to know that the matching functions, $f(\theta)$ and $\alpha(q)$, are Cobb-Douglas with elasticities equal to 0.6 in the labour market and 0.5 in the goods market. The function, $\bar{\beta}(\mu)$, in Proposition 8 is illustrated as an upward sloping black line. It contains all the combinations of (β, μ) that implement the constrained-efficient market tightness, including the Hosios condition in both markets, $\beta = 0.4$ and $\mu = 0.5$. If $\theta = \theta^{sp}$ is implemented for a pair, (β_0, μ_0), and if β increases, then $\theta = \theta^{sp}$ can be maintained by raising μ.

[20] A similar result is derived by Rocheteau and Wright (2005) in the context of a search-and-bargaining monetary model with free entry of sellers.

To the left of this black line, the average bargaining power of firms across goods and labour markets is too high. In that case, the optimal policy is to set $\pi = -\rho$ since an increase in the inflation rate above the Friedman rule would exacerbate the inefficiently high entry of firms. To the right of the line, the average bargaining power of firms—and thus entry—is too low. In this case, $i > 0$ is optimal, as higher inflation reduces consumers' outside options, thereby increasing firms' market power and incentivising entry. While higher inflation also raises the cost of holding money, this effect is of second-order importance in the neighbourhood of $y = y^*$. The welfare-maximising inflation rates are shown by the contour lines.

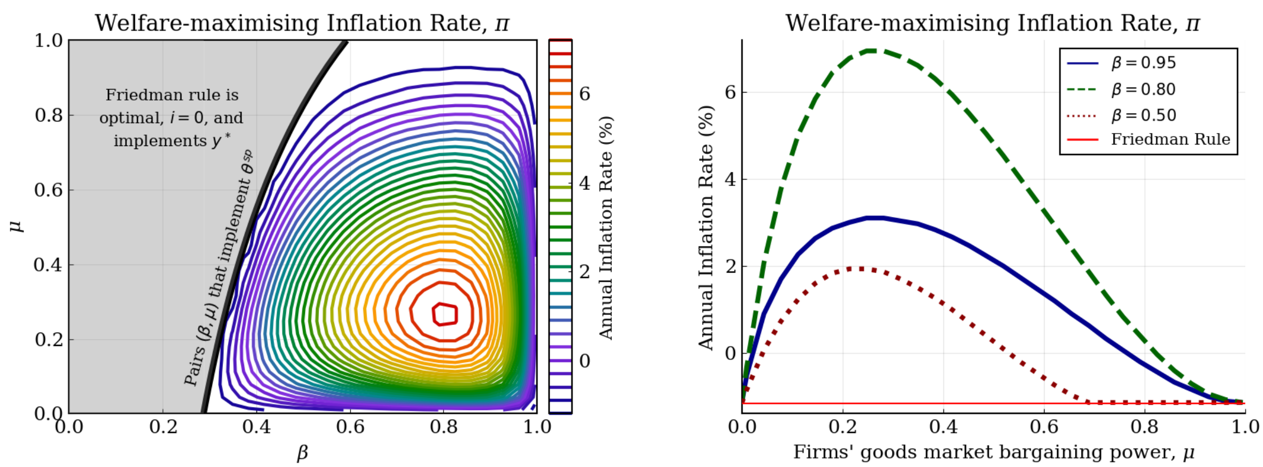

Figure 8: Welfare-maximising inflation rate, given μ and β.

In the right panel of Figure 8, the Friedman rule is represented by the horizontal red line, which is $\pi_{FR} \approx -0.014$ at an annual frequency in this parameterisation. For a given β, the welfare-maximising inflation rate is a non-monotone function of the bargaining power of firms in the goods market, μ. When $\mu = 0$, firms receive no surpluses from trade. Therefore, an increase in inflation reduces consumers' outside options but does not increase firm entry, i.e., the market power effect of inflation is not in operation. In this case, the welfare-maximising inflation rate is $\pi = \pi_{FR} \equiv -\rho$. When $\mu \in (0, 1)$, the market power effect of inflation exists and it becomes optimal to raise π above $-\rho$. When $\mu \to 1$, the optimal π returns to π_{FR} because consumers' outside options $Z \to 0$ and thus the market power effect again vanishes. This example illustrates that the market power effect is non-monotone in firms' bargaining power in general.

5 Quantitative Assessment

We now carry out a quantitative analysis by calibrating our model to the US. A unit of time corresponds to one month. We assume the matching functions in both labour and goods markets are Cobb-Douglas. Specifically, the job finding rate is given by $f(\theta) = \Xi\theta^{\eta}$ and the matching rate for consumers in the goods market is given by $\alpha(q) = \Psi q^{\psi}$. The cost of production in bilateral meetings is $\varphi(y) = Gy^{g}$ and utility is linear $\upsilon(y) = y$.

We calibrate λ using evidence about the average frequency of purchases across major expenditure categories in the 2021 Consumer Expenditure Survey (CEX; Bureau of Labor Statistics, 2023). We first estimate the average frequency of purchases within 15 expenditure categories, reported in Table 2, ranging from "shelter" and "food-at-home" to "personal care products and services" and "reading". The CEX reports the fraction of households who spent a positive amount in the sub-components of these categories (e.g., prepared flour mixes as a sub-component of food-at-home) in a given time period.[21] We interpret these fractions as the probability a representative household purchases a product in that category in a given time period. We then aggregate to the j-category level (e.g., food-at-home) by computing the average, expenditure-weighted probability for purchasing any product in that category, reported in the second column of Table 2. Some categories are purchased with high probability, like food at home ($Pr = 0.99$), food away from home ($Pr = 0.84$), and housekeeping supplies ($Pr = 0.92$). However, many categories are purchased with low probability, like household furnishings and equipment ($Pr = 0.07$), reading ($Pr = 0.09$), or vehicle purchases and expenses ($Pr = 0.12$). The largest expenditure category, shelter, is purchased relatively frequently ($Pr = 0.74$). Our targeted moment is the average, expenditure weighted, probability of a purchase across these categories of 0.492. In the model, the probability that a representative consumer makes at least one purchase over a unit of time is given by

$$\frac{\lambda}{\lambda+\alpha}(1-e^{-\alpha}) + \frac{\alpha}{\lambda+\alpha}(1-e^{-\lambda})(1-e^{-\alpha}).$$

The first term corresponds to the probability a consumer is active, $\lambda/(\lambda+\alpha)$, times the probability that this consumer leaves the active state in a unit of time (calibrated to one month), $1-e^{-\alpha}$. Likewise, the second term is the probability that a consumer is inactive, $\alpha/(\lambda+\alpha)$, times the joint probability of making two transitions from idle to active, then from active to idle, over a unit of time. This procedure leads to an estimate of $\lambda = 0.58$.

[21] In the CEX, the time period varies between weekly and quarterly across categories, depending on if the diary or interview survey was used.

	Data Moments	
Expenditure Category	Pr(Purchase) monthly	Expenditure Share (%)
Shelter	0.74	24.4
Vehicle purchases and expenses	0.12	16.2
Healthcare	0.30	10.0
Food at home	0.99	9.66
Utilities, fuels, and public services	0.71	7.76
Entertainment	0.12	6.56
Food away from home	0.84	5.57
Household furnishings and equipment	0.07	4.96
Gasoline and motor oil	0.53	3.95
Apparel and services	0.16	3.22
Household operations	0.19	3.01
Alcoholic beverages	0.24	1.64
Housekeeping supplies	0.92	1.48
Personal care products	0.31	1.42
Reading	0.09	0.21
Average (expenditure weighted)	0.49	-

Table 2: Purchase frequency in the Consumer Expenditure (CEX) Survey

The remaining parameters are fixed using an approach close to that in BMW. We set the discount rate $\rho = 0.001$ so that the real interest rate in the model matches the difference between the rate on 3-month Treasury bills and realised inflation, on average across a large sample period 1955-2005.[22] We use the vacancy posting cost k and job destruction rate δ to match the average unemployment rate and unemployment-to-employment (UE) transition rate.[23] The level parameter of the labour market matching function, Ξ, is normalised so that the vacancy rate is 1. The elasticity of the labour market matching function, η, targets the regression coefficient of labour market tightness θ on the UE rate of 0.6 as in BMW. Firms' bargaining power in the labour market, $1 - \beta$, is set to match the average wage markdown, $1 - w/\hat{x}$, where $\hat{x} = \alpha^s \mu \left(\chi^m S^m + \chi^d S^d\right) + x$ is average output per worker. Following evidence in Yeh et al. (2022) from the US manufacturing sector, we target an average wage markdown of 0.3. Unemployment benefits, b, represent both unemployment income and the value of non-work and is an important parameter to discipline the response of unemployment to labour productivity $\hat{x}$. We set b such that the elasticity of unemployment

[22]See St. Louis Federal Reserve (FRED; Federal Reserve Bank of St. Louis, 2025) series TB3MS and CPILFESL, respectively. We take the continuously compounded annual rate of change of the Consumer Price Index as our measure of inflation.

[23]The unemployment rate is given in St. Louis Federal Reserve (FRED; Federal Reserve Bank of St. Louis, 2025) series UNRATE, or equivalently from the Bureau of Labor Statistics (BLS; Bureau of Labor Statistics, 2024). We set the unemployment-to-employment transition rate to be 0.45 monthly to capture a quarterly rate of 0.83.

to labour productivity is 2/3 of its estimated value at business cycle frequencies.[24] This process implies that $b = 0.79$, or that $b/\hat{x} = 0.66$.

We set the level of the goods market matching function to $\Psi = 1$, and assume the matching rate of firms, α^s, and that of consumers, α^b, have the same elasticity, i.e., $\psi = 0.5$. We calibrate the bargaining power of firms in the goods market, μ, to match a markup target of 20%. We calibrate the level and curvature of the cost of production, (G, g), to match the relationship between money demand M/pY and i in the data between 1955-2005, using the adjusted M1 series in Lucas and Nicolini (2015) as our measure of money. Finally, we set $\chi^m = 0.8$ because in the Atlanta Fed data discussed by Foster et al. (2013), credit cards account for 23% of purchases in volume in 2010. In the Bank of Canada data discussed by Arango and Welte (2012), this number is 19% in 2009.

The targets and parameter values discussed above are summarised in Table 3. The calibration matches all the targeted moments exactly. In Online Appendix B we show our results are robust to alternative calibration strategies, including calibrating a multiple-goods version of our model and variation in our markup and markdown targets.

Parameter	Description	Targets	Value
ρ	Rate of time preference	Average real interest rate	0.001
k	Vacancy cost	Average unemployment rate	0.33
δ	Job destruction rate	Unemployment-to-employment rate	0.03
Ξ	Level of labour market matching	Average vacancies (normalization)	0.08
η	Elasticity of labour market matching	Elasticity of UE rate	0.60
Ψ	Level of goods market matching	—	1.00
ψ	Elasticity of goods market matching	Equal contribution to matching	0.50
β	Bargaining power of worker in labour market	Wage markdown	0.01
b	Unemployment benefits	relationship between u and $\hat{x}$	0.79
χ^m	Fraction of monetary meetings	Fraction of credit card transactions	0.80
μ	Bargaining power of firms in goods market	Markup	0.94
G	Level of production cost $\varphi(y)$	Level of money demand	0.68
g	Elasticity of production cost $\varphi(y)$	Elasticity of money demand	1.19

Table 3: Calibrated parameters

[24] We compute the elasticity of unemployment to labour productivity using the cyclical components of HP-filtered data (with a smoothing parameter of 1600) of quarterly unemployment and labour productivity from the Bureau of Labor Statistics.

5.1 Quantitative Results

In Figure 9, we illustrate how long-run inflation affects the aggregate unemployment rate (solid blue line in left panel), real money balances (middle panel) and consumers' outside options (right panel) across expenditure categories, under the baseline calibration. The Phillips curve is non-monotone in π, following the prediction in Proposition 4. Consumers' outside options, Z, decrease with inflation since money holdings become more costly – *the market power effect of inflation*. Even though lower money holdings constrain the payment that consumers can make to firms – *the real balance effect of inflation* – their lower outside options improve firms' market power.

In the left panel of Figure 9, we separately illustrate the real balance and market power effects of inflation by computing counter-factual Phillips curves as responses of unemployment to inflation due to changes only in i) real money balances (dash-dotted red line) or ii) consumers' outside options (dashed green line), respectively. Specifically, we recompute u using (27) and (29) keeping either Z or a constant. The outside option effect is quantitatively dominant for annual inflation rates between the Friedman rule and 1.5%, the latter representing the unemployment-minimising inflation rate. As inflation rises beyond 1.5%, the effect on liquidity constraints dominates and the Phillips curve turns upward-sloping.

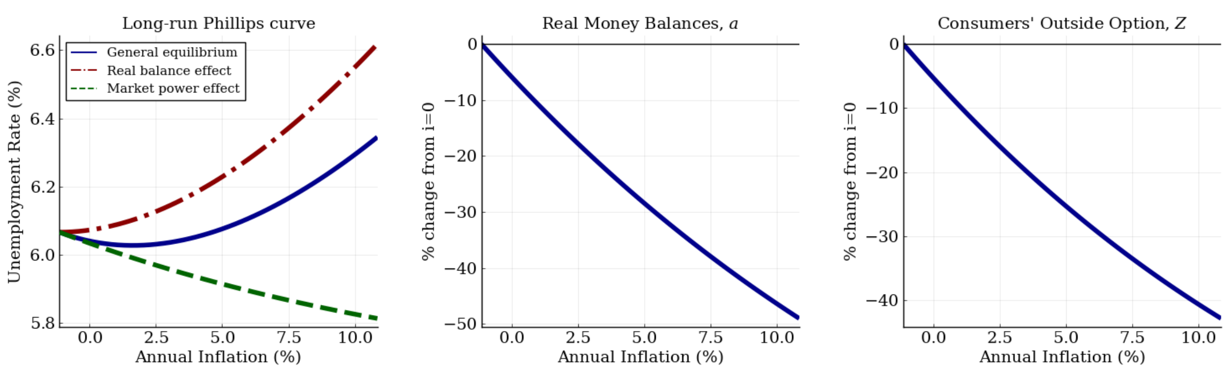

Figure 9: The effects of inflation on unemployment (left), real money balances (middle), consumers' outside options (right)

The strength of the market power effect depends on the rate at which the desire to consume arrives, λ. If it occurs relatively infrequently, then consumers' opportunity cost of consumption is relatively large since the value of returning to being idle is relatively low. If, however, consumers spend little time idle, the opportunity cost of consumption is low. In Figure 10, we illustrate the effects of changing λ on the unemployment-minimising level of inflation (left panel) and the overall shape of the long-run Phillips curve (right panel).

The left panel of Figure 10 shows that the unemployment-minimising inflation rate falls

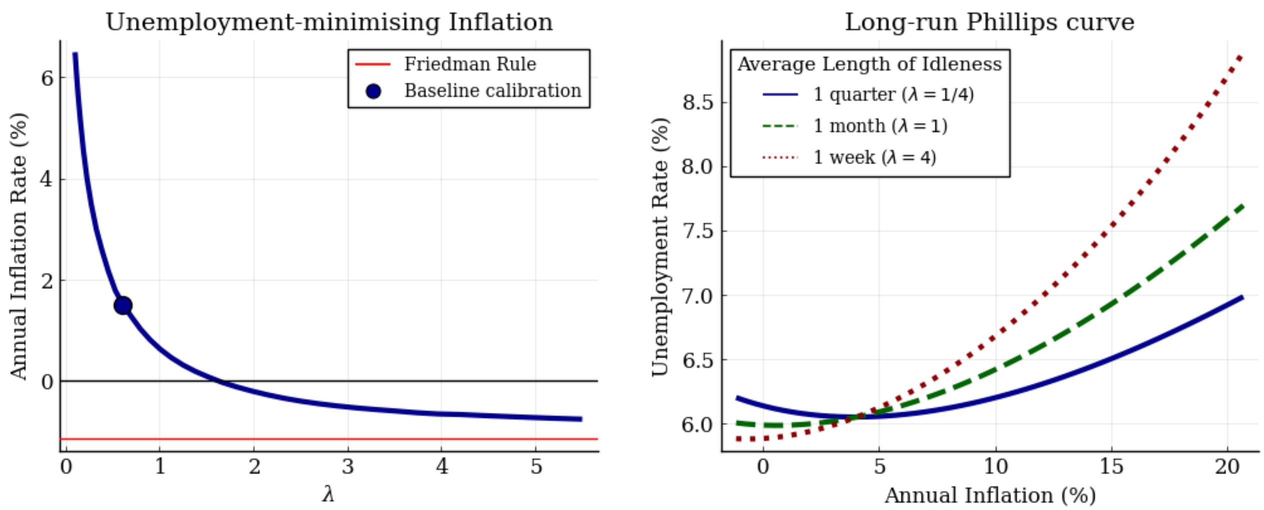

Figure 10: The unemployment-minimising inflation rate and λ (left); The long-run Phillips curve and λ (right);

as λ varies from close to zero to 5. As λ approaches zero, the unemployment-minimising annual inflation rate is just above 6%. Our baseline calibration is represented as a solid circle, in which unemployment-minimising inflation is 1.5%, close to the Federal Reserve's target of 2%. However, for any estimate of λ lower than 1.7 (or an average length of idleness greater than about 2 weeks), the unemployment-minimising inflation rate is positive.

We report the consumption-equivalent welfare cost of inflation in the left panel of Figure 11. For any inflation rate, π, we compute the percentage of consumption (across both goods) the representative household would give up to stay in the steady state with inflation at the Friedman rule. The welfare cost in the baseline calibration is shown in solid-blue while the welfare cost in a high-λ and low-λ (re-calibrated) steady state are shown in dotted-red and dashed-green, respectively. In the baseline, the welfare cost of 10% annual inflation is around 1% of consumption; considerably lower than the estimate in Lagos and Wright (2005) of around 5% but more in line with recent estimates in models that incorporate entry decisions, for instance, Bethune et al. (2020).[25] We find the welfare cost is lower when we re-calibrate to higher values of λ. This is largely driven by the fact that as λ increases, to match the average markup target our measure of firms' bargaining power in the goods market, μ, decreases.[26]

For all values of λ considered, the welfare-maximising inflation rate coincides with the

[25] See, e.g., Rocheteau and Wright (2009), for a discussion of the importance of the Hosios condition for the welfare cost of inflation in models with endogenous participation decisions.

[26] See Craig and Rocheteau (2008) for an explanation of why bargaining power is central to the welfare cost of inflation.

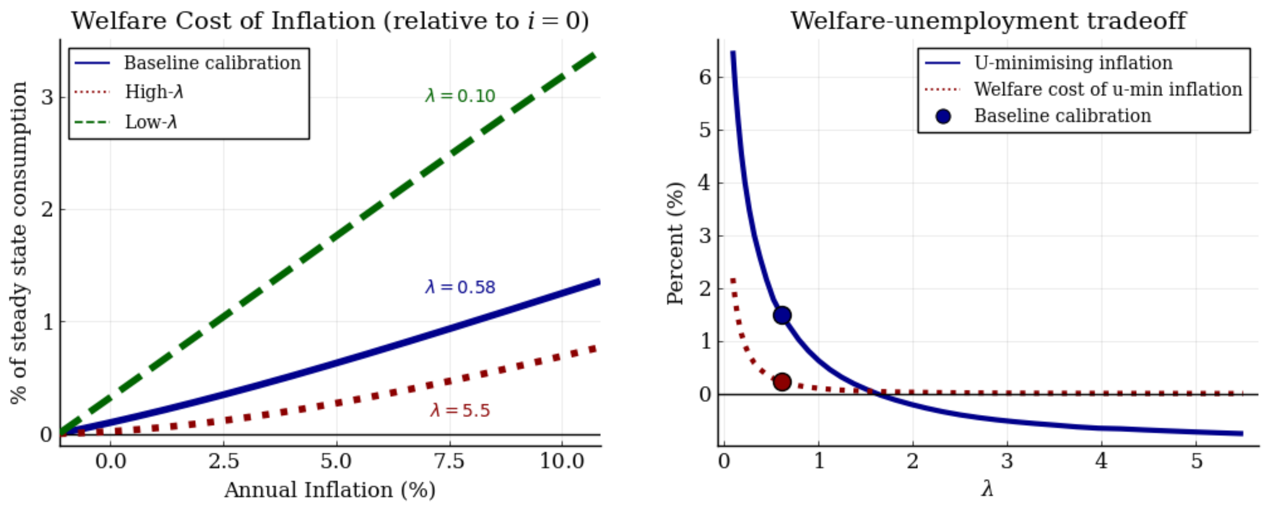

Figure 11: The welfare cost of inflation (left); The welfare-unemployment tradeoff (right);

Friedman rule. Consequently, mandating the monetary authority to implement the inflation rate that minimises unemployment entails a welfare loss relative to the constrained-efficient allocation. The right panel of Figure 11 quantifies this loss. For each λ, the solid blue line reports the unemployment-minimising inflation rate, while the dotted red line displays the associated welfare cost of setting inflation at that level rather than at the Friedman rule.

At the calibrated value, $\lambda = 0.58$, the welfare loss from targeting the unemployment-minimising inflation rate is negligible—approximately 0.12 percent of consumption. However, as λ approaches zero, the unemployment-minimising inflation rate rises, and the corresponding welfare loss increases monotonically. Thus, although the welfare loss from choosing an inflation target that minimises unemployment is modest at the calibrated parameterisation, it becomes economically significant in environments with a stronger outside-option channel.

6 Extensions

We explore two extensions of our model. First, we introduce short-term, liquid government bonds and examine the relationship between unemployment and the short-term nominal interest rate. Second, we analyse transitional dynamics and the short-run Phillips curve.

6.1 Nominal interest rates and unemployment

Thus far, monetary policy has taken the form of a constant money growth rate, π, which can be mapped one-to-one to the nominal interest rate of an illiquid bond, $i = \rho + \pi$. We can interpret this inflation rate as the target of the Central Bank. However, in current practice, central banks set the nominal interest rate on liquid, short-term bonds. In this section, we revisit the relationship between unemployment and nominal interest rates by distinguishing between illiquid and liquid bonds. We examine how changes in the short-term nominal interest rate affect firms' market power and unemployment, and we characterise the combination of two policy instruments – the money growth rate and the short-term nominal interest rate – that minimise unemployment. The main findings are presented below, with detailed derivations provided in Online Appendix C.

Liquid government bonds are of the pure discount (or zero coupon) variety. A bond pays one unit of numéraire when it matures at Poisson rate $\sigma > 0$. We consider the limit as the maturity of the bond approaches 0, i.e., $\sigma \to +\infty$. The nominal interest rate on liquid bonds is denoted $i_g = r_g + \pi$, where r_g is the real rate of return. The total supply of bonds is denoted A_g.

We assume limited participation in the bond market by dividing consumers into two types, $\kappa \in \{1, 2\}$, where types differ in terms of the assets – money or bonds – they can hold in their portfolios. A fraction ψ_1 of consumers, those of type $\kappa = 1$, can only use money to finance the consumption of good y. The remaining fraction, $\psi_2 = 1 - \psi_1$, corresponds to type-2 consumers who can use both bonds and money as means of payment.[27] One can think of type-1 consumers as unsophisticated investors who do not participate in the bond market while type-2 consumers are sophisticated investors who can hold financial assets.[28]

There are two types of equilibria. If A_g is low, there is a *liquidity trap* equilibrium where $i_g = 0$.[29] In such equilibria, type-2 consumers are indifferent between money and bonds. Both types of consumers have the same lifetime utility and the same outside options. Liquidity-trap equilibria are isomorphic to the monetary equilibria studied in Section 3. If A_g is above a threshold, then $i_g > 0$ and type-2 consumers only hold liquid bonds.

We call the relationship between u and the short-term nominal interest rate, i_g, a *modified* Phillips curve. In the neighbourhood of $i_g = i$, u falls as i_g decreases. Indeed, when $i_g = i$, the opportunity cost of holding liquid bonds is zero, and hence type-2 buyers hold enough bonds to purchase the first-best level of output, i.e., $y_2 = y^*$. Therefore, if i_g falls slightly below i, y_2 falls as well, but it only has a second-order effect on firms' profits. However, the increase in the spread, $s_g = i - i_g$, above 0 has a first-order effect on consumers' outside

[27]The assumption of limited participation in some asset markets has been used, e.g., by Alvarez et al. (2001), Alvarez et al. (2002), Williamson (2006), among others.

[28]One could endogenise participation in the bond market, and hence the measure ψ_κ, by introducing a distribution of participation costs across buyers. See, e.g., Rocheteau et al. (2018).

[29]Williamson (2012) provides another example of a New Monetarist model that delivers a liquidity trap equilibrium.

options, Z_2, which raises firms' market power. Consequently, it induces more firm entry and a lower unemployment rate. By the same logic as in the monetary economy studied earlier, the relationship between u and i_g is non-monotone when inflation, or i, is sufficiently large. We illustrate these findings in Figure 12 by plotting the modified Phillips curve for low and high inflation rates.

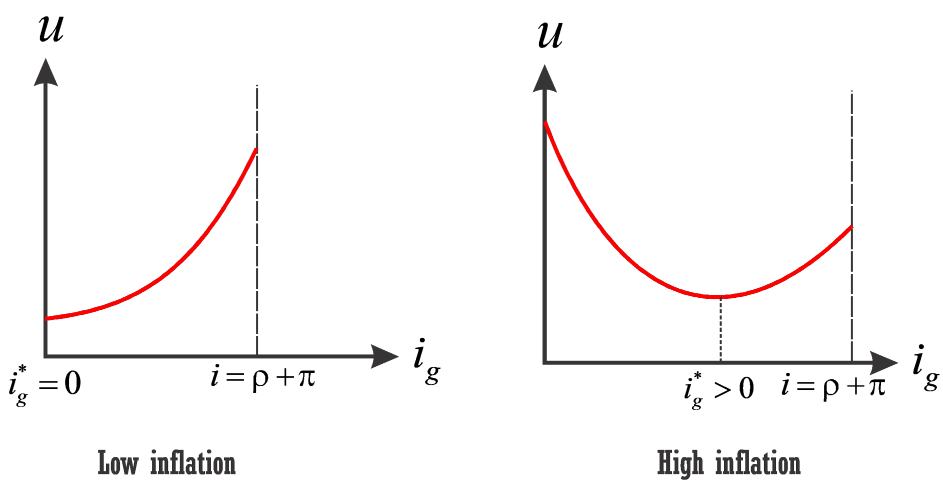

Figure 12: The modified Phillips curve: The relationship between u and i_g at low and high inflation rates

In terms of policy, we show that the unemployment rate is minimum when $i > 0$ and $i_g = 0$, i.e., the economy is in a liquidity trap but the inflation rate is above the Friedman rule. This result is obtained under the assumption that consumers of type 1 and 2 are identical in terms of their preferences and opportunities to consume. If type-1 and type-2 consumers receive preference shocks at different rates, then the unemployment minimising policy can feature $i > 0$ and $i_g > 0$.

6.2 Short-run dynamics of inflation and unemployment

Our model can be used to explore the relationship between inflation and unemployment in the short run. In this section, we study dynamics in a perfect-foresight equilibrium in the baseline environment in response to one-time, unanticipated money growth rate shocks.[30] We show that these shocks can generate a negatively-sloped short-run Phillips curve in the absence of nominal rigidities. The short-run Phillips curve can be either downward or upward sloping, depending on the initial state of the economy. Money growth rate shocks starting from lower long-run inflation rates are stimulative, increasing inflation and decreasing unemployment in

[30]In Online Appendix D, we show these results are robust to considering anticipated money growth shocks in a version of the model with a stochastic process for money growth.

the short-run. However, money growth rate shocks, starting from higher long-run inflation rates are contractionary, increasing both inflation and unemployment in the short-run.

Let $g_{M,t} = \dot{M}_t/M_t$ denote the money growth rate as a function of time. In Online Appendix D, we show that dynamic equilibria are time-paths, $(a_t, \omega_{1,t}, Z_t, \theta_t, n_t)$, that solve a system of ODEs given initial conditions, $\omega_{1,0}$ and n_0, and transversality conditions associated with a_t, Z_t, and θ_t. The economy is initially in a steady state with constant money growth rate, $\tilde{g}_M$. At time 0, there is a one-time, unanticipated increase in the money growth rate to $g_{M,0} > \tilde{g}_M$. The shock is persistent, $\dot{g}_{M,t} = \rho_{g_M}(\tilde{g}_M - g_{M,t})$ with $\rho_{g_M} > 0$.

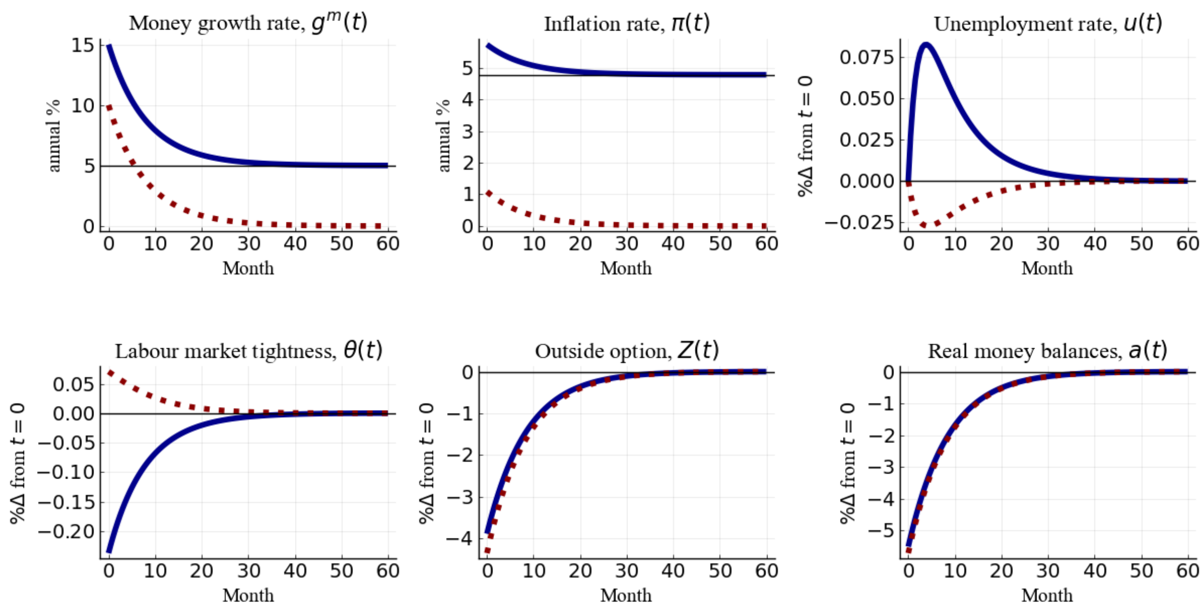

Figure 13: Responses to a one-time, unanticipated increase in money growth, $g_{M,t}$.

Figure 13 illustrates the impulse responses of the economy for a shock that increases the annualised money growth rate by 10 percentage points at onset in two economies with long-run inflation rates of 0% (dotted-red lines) and 5% (solid-blue lines). We set $\rho_{g_M} = 0.12$ so that the shock has a half-life of 6 months and use the calibration from our baseline economy for the other parameters.[31] The increase in the money growth rate leads to a temporary increase in inflation in both cases (top middle panel of Figure 13). The rise in inflation decreases consumers' outside option, Z_t, (middle panel of bottom row) and their real money holdings (bottom-right panel). As in the steady-state analysis, the market-power effect through Z and the real-balance effect work in opposite directions in influencing

[31] The half-life is given by the $T > 0$ such that $g_{M,T}/g_{M,0} = 1/2$. Given the process for $g_{M,t}$, it is equal to $T = ln(2)/\rho_{g_M}$.

labour market tightness. The market power effect is dominant at low inflation rates. The dotted-red lines illustrate that firms' expected revenue increases at onset, leading to higher labour market tightness and lower unemployment in the short-run. However, the real balance channel dominates for money growth shocks from steady states with larger inflation rates — the short-run Phillips curve can slope upward.

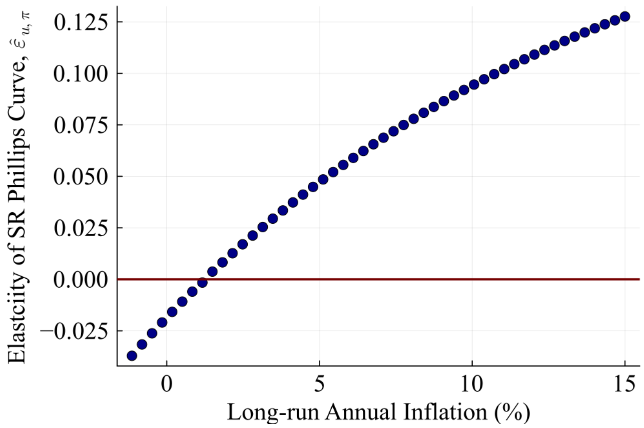

Figure 14: The slope of the short-run Phillips curve as a function of long-run inflation.

Figure 14 illustrates the non-monotonicity of the short-run Phillips curve more extensively by plotting a measure of the short-run elasticity of unemployment to inflation — the slope of the short-run Phillips curve — as a function of the long-run, steady-state inflation rate. We measure the elasticity, $\hat{\epsilon}_{u,\pi}$, as the percentage change in the unemployment rate from $t = 0$ to the maximum of $|u_t - u_0|$ along the impulse response, for a one percent change in annualised inflation at $t = 0$. Monetary expansion is stimulative for low-inflation economies, precisely those with long-run annual inflation rates of 1.5% or lower, and the short-run Phillips curve slopes downward. However, for economies with larger long-run inflation, monetary expansion becomes contractionary and the short-run Phillips curve slopes upward.[32] These exercises illustrate that our novel market power channel not only leads to a non-monotonicity of the long-run Phillips curve, but that these same economic forces lead to non-monotonicity in the short-run Phillips curve either in the case of unanticipated inflationary shocks, or anticipated.

7 Empirical evidence

We conclude the paper with a brief review of the empirical evidence on the long-run relationship between unemployment and inflation. In particular, we revisit BMW's empirical

[32]A similar result holds when money growth shocks are anticipated, as shown in Online Appendix D, that monetary expansion can either be stimulative or contractionary.

findings and provide a concise overview of the literature that employs more advanced econometric techniques.

Evidence with U.S. data To establish that the long-run Phillips curve is upward sloping, BMW applied the Hodrick-Prescott (HP) filter to U.S. data and showed a positive correlation between the HP-filtered unemployment and inflation trends from 1955 to 2005. Since our theory suggests that unemployment and inflation are negatively correlated when inflation is low, we estimate a regression-kink model with an unknown threshold. Let u_t denote the HP filtered U.S. unemployment trend and π_t is the inflation trend where the subscript t represents a quarter.[33] The reduced-form relationship is modeled as

$$u_t = \alpha + \beta_L \pi_t + \delta (\pi_t - \gamma) \mathbf{1}_{\{\pi_t > \gamma\}} + \varepsilon_t, \tag{42}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function; γ is the threshold level of inflation; β_L and $\beta_H = \beta_L + \delta$ are the low- and high-regime slopes; the intercept is α; ε_t is an i.i.d. disturbance with zero mean and constant variance. We estimate the model by minimising the mean squared error as described in Hansen (2017).[34]

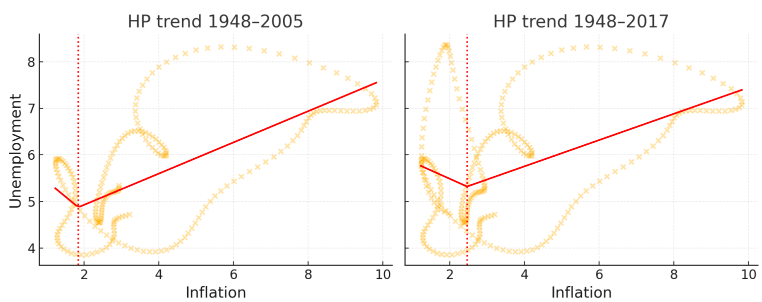

Figure 15: Estimated regression kink model with unknown threshold (red lines) based on HP filtered trend with smoothing parameter 1600 (yellow dots).

In Figure 15, we illustrate the estimation results with red lines for the time periods 1948–2005 (matching the end date used by BMW) and 1948–2017.[35] The slope of the red line

[33] Inflation is the quarter-on-quarter percent change of the CPI for All Urban Consumers (series CPI-AUCSL), annualised. The unemployment rate is the civilian unemployment rate (series UNRATE). Both series are from the Federal Reserve Bank of St. Louis (Federal Reserve Bank of St. Louis, 2025). We apply the HP filter with a smoothing parameter of 1600, which is standard for quarterly data.

[34] Hansen (2017) proposes a wild bootstrap algorithm to compute standard error for regression kink models with unknown threshold. We do not report these errors in order to avoid econometric issues regarding the data generating process of HP filtered trends.

[35] Instead of HP filtering, we provide the estimation results using Baxter-King (BK) and Christiano-Fitzgerald (CF) filter in Figure 8 in Online Appendix I. The results of using BK filtered data are similar to that of Figure 15. The estimated curve using CF filtered data is an upward sloping convex line, but still

changes from negative to positive at the threshold $\gamma = 1.8\%$ and 2.5% for the left and right panels, respectively. These results suggest that a V-shaped function provides a better fit for the long-run relationship than a linear or other piecewise linear functions.[36] While this reduced-form model does not provide definitive evidence, it is suggestive of the possibility that the long-run Phillips curve behaves differently when inflation is low versus when it is high.[37]

Evidence via identifying shocks Our regression above is agnostic about the source of variation in inflation and unemployment. A more rigorous approach involves estimating the slope of the long-run Phillips curve by identifying shocks that move the economy along the curve. A body of literature pursues this strategy (e.g., King and Watson (1994), King and Watson (1997), Benati (2015)), but its findings remain inconclusive and do not rule out a non-monotonic long-run Phillips curve. For instance, King and Watson (1994) find evidence of a negative long-run trade-off between inflation and unemployment in the US postwar. However, King and Watson (1997) show that these results depend on the choice of identifying assumptions. Benati (2015) adopts both classical and Bayesian structural VARs, and shows that U.S. data is compatible with both positively- and negatively-sloped long-run Phillips curves.

Ascari et al. (2022) introduce stochastic trends into a Bayesian VAR. They find that there is a threshold level of inflation below which potential output is independent of inflation, and above which potential output and inflation are negatively correlated. The threshold level of inflation is slightly below 4%. Bullard and Keating (1995) study a large sample of postwar economies using a structural VAR approach and find that inflation raised output in low-inflation countries, and either did not affect or reduced output in countries with a higher inflation rate.

Evidence on market power The evidence on inflation and firms' market power is also mixed. For instance, Chirinko and Fazzari (2000) and Neiss (2001) find a positive relationship between inflation and firm-level markups, while Banerjee and Russell (2001, 2005) find a negative relationship. Relatedly, Nekarda and Ramey (2020) find that monetary shocks lead markups and output to fall. The topic has gained renewed interest following the 2021-22

suggests that the long-run Phillips curve behaves differently at low and high rate. The raw U.S. data extends through 2024Q4, filters are applied to the full sample before subsetting, so endpoint trend estimates may reflect post-2017 observations.

[36] We also fit a polynomial of degree 4 to explain the relationship between u_t and π_t. The results are similar and suggest a similar slope change of the long-run Phillips curve (Figure 9 in Online Appendix I).

[37] In Online Appendix H we show that a similar slope change arises in several other countries.

inflation surge and discussion of "greedflation" as a source of rising prices (see, e.g., DePillis (2022)). Glover et al. (2023) document that firm-level markups in the US increased by 3.4 percentage points in 2021 while inflation increased by around 3 percentage points, and Hansen et al. (2023) find that increases in corporate profits contributed up to 45% of the Euro area inflation.

The evidence on inflation and market concentration is similarly mixed. Bräuning et al. (2023) show that concentration amplifies cost shock pass-through into prices, so higher concentration is likely to correlate with higher inflation. Kashyap and Tantri (2025) use a natural experiment in India to show that a rate hike increases industry concentration and markups. Gamber (2026) shows a contractionary monetary policy shock reduces entry and increases exit, which is consistent with a resulting increase in concentration. On the other hand, Horn and Fischer (2021) show that, in the US, aggregate mergers and acquisitions activity falls after a contractionary monetary policy shock.

8 Conclusion

In this paper we made a simple, but robust, observation regarding the long-run trade-off between unemployment and inflation. In the class of models pioneered by BMW, where goods and labour markets are frictional, and money plays an essential role to facilitate the exchange of goods and services, the relation between unemployment and inflation is non-monotone. As a result, the inflation rate that minimises unemployment is above the one prescribed by the Friedman rule. This result is robust in that it does not require parametric conditions to hold once one introduces consumer search in order to endogenise consumer outside options and firms' market power. We also show that consumers' outside options are crucial for the optimality of the Friedman rule and the convergence of equilibria to a competitive outcome, where rents, markups, and markdowns vanish at the frictionless limit.

References

Alvarez, F., A. Atkeson, and P. J. Kehoe (2002). Money, interest rates, and exchange rates with endogenously segmented markets. *Journal of Political Economy 110*(1), 73–112.

Alvarez, F., R. E. Lucas Jr, and W. E. Weber (2001). Interest rates and inflation. *American Economic Review 91*(2), 219–225.

Anderson, S. P. and R. Renault (1999). Pricing, product diversity, and search costs: A Bertrand-Chamberlin-Diamond model. *RAND Journal of Economics*, 719–735.

Arango, C. and A. Welte (2012). The Bank of Canada's 2009 methods-of-payment survey: Methodology and key results. *Bank of Canada Working Paper*.

Aruoba, S., G. Rocheteau, and C. Waller (2007). Bargaining and the value of money. *Journal of Monetary Economics 54*(8), 2636–2655.

Aruoba, S., C. Waller, and R. Wright (2011). Money and capital. *Journal of Monetary Economics 58*(2), 98–116.

Ascari, G., P. Bonomolo, and Q. Haque (2022). The long-run Phillips curve is... a curve. *Working paper*.

Bajaj, A. and S. Mangin (2023). Consumer choice and the cost of inflation. *Working paper*.

Banerjee, A. and B. Russell (2001, May). The relationship between the markup and inflation in the G7 economies and australia. *Review of Economics and Statistics 83*(2), 377–384.

Banerjee, A. and B. Russell (2005, June). Inflation and measures of the markup. *Journal of Macroeconomics 27*(2), 289–306.

Benabou, R. (1988). Search, price setting and inflation. *Review of Economic Studies 55*(3), 353–376.

Benati, L. (2015). The long-run phillips curve: A structural var investigation. *Journal of Monetary Economics 76*, 15–28.

Berentsen, A., G. Menzio, and R. Wright (2011). Inflation and unemployment in the long run. *American Economic Review 101*(1), 371–98.

Berentsen, A., G. Rocheteau, and S. Shi (2007). Friedman meets Hosios: Efficiency in search models of money. *Economic Journal 117*(516), 174–195.

Bethune, Z., M. Choi, and R. Wright (2020). Frictional goods markets: Theory and applications. *Review of Economic Studies 87*(2), 691–720.

Bethune, Z., G. Rocheteau, and P. Rupert (2015). Aggregate unemployment and household unsecured debt. *Review of Economic Dynamics 18*(1), 77–100.

Branch, W. A., N. Petrosky-Nadeau, and G. Rocheteau (2016). Financial frictions, the housing market, and unemployment. *Journal of Economic Theory 164*, 101–135.

Bräuning, F., J. L. Fillat Comenge, and G. Joaquim (2023). Cost-price relationships in a concentrated economy. Technical report, Working Papers.

Bullard, J. and J. W. Keating (1995). The long-run relationship between inflation and output in postwar economies. *Journal of Monetary Economics 36*(3), 477–496.

Burdett, K. and K. L. Judd (1983). Equilibrium price dispersion. *Econometrica*, 955–969.

Bureau of Labor Statistics (2023). Consumer expenditure surveys (CEX), table R-1. https://www.bls.gov/cex/tables.htm. 2021 survey data. Accessed December 2024.

Bureau of Labor Statistics (2024). Bureau of labor statistics (BLS), series lns14000000. https://www.bls.gov/. Accessed December 2024.

Chirinko, R. and S. Fazzari (2000, August). Market power and inflation. *Review of Economics and Statistics 82*(3), 509–513.

Choi, M. and G. Rocheteau (2021). New Monetarism in continuous time: Methods and applications. *Economic Journal 131*(634), 658–696.

Cooley, T. F. and V. Quadrini (2004). Optimal monetary policy in a Phillips-curve world. *Journal of Economic Theory 118*(2), 174–208.

Craig, B. and G. Rocheteau (2008). Inflation and welfare: A search approach. *Journal of Money, Credit and Banking 40*(1), 89–119.

DePillis, L. (2022). Is 'greedflation' rewriting economics, or do old rules still apply? *New York Times*.

Devereux, M. B. and J. Yetman (2002). Menu costs and the long-run output–inflation tradeoff. *Economics Letters 76*(1), 95–100.

Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Diamond, P. A. (1993). Search, sticky prices, and inflation. *Review of Economic Studies 60*(1), 53–68.

Dong, M. (2011). Inflation and unemployment in competitive search equilibrium. *Macroeconomic Dynamics 15*(S2), 252–268.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005). Over-the-counter markets. *Econometrica 73*(6), 1815–1847.

Federal Reserve Bank of St. Louis (2025). Federal reserve economic data (FRED). https://fred.stlouisfed.org/. CPI for All Urban Consumers (CPIAUCSL), 3-month Tbill Rate (TB3MS), Unemployment Rate (UNRATE). Accessed December 2024.

Foster, K., S. Schuh, and H. Zhang (2013). The 2010 survey of consumer payment choice. *Federal Reserve Bank of Boston Research Data Report*.

Friedman, M. (1977). Nobel lecture: Inflation and unemployment. *Journal of Political Economy 85*(3), 451–472.

Gale, D. (1986a). Bargaining and competition part I: Characterization. *Econometrica*, 785–806.

Gale, D. (1986b). Bargaining and competition part II: Existence. *Econometrica*, 807–818.

Gale, D. (1987). Limit theorems for markets with sequential bargaining. *Journal of Economic Theory 43*(1), 20–54.

Galenianos, M. and P. Kircher (2009). Directed search with multiple job applications. *Journal of Economic Theory 144*(2), 445–471.

Gamber, W. L. (2026). Firm dynamics, inflation, and the transmission of monetary policy.

Glover, A., J. M. del Rio, and A. von Ende-Becker (2023). How much have record corporate profits contributed to recent inflation. *Federal Reserve Bank of Kansas City Economic Review First Quarter*, 23–35.

Gu, C., J. H. Jiang, and L. Wang (2023). Credit condition, inflation and unemployment. *Working paper*.

Hansen, B. E. (2017). Regression kink with an unknown threshold. *Journal of Business & Economic Statistics 35*(2), 228–240.

Hansen, N.-J., F. Toscani, and J. Zhou (2023). Euro area inflation after the pandemic and energy shock: Import prices, profits, and wages. IMF Working Paper 23/131.

Head, A. and A. Kumar (2005). Price dispersion, inflation, and welfare. *International Economic Review 46*(2), 533–572.

Horn, C.-W. and J. J. Fischer (2021). Does monetary policy affect mergers and acquisitions?

Hosios, A. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies 57*(2), 279–298.

Hu, T.-W. and G. Rocheteau (2020). Bargaining under liquidity constraints: Unified strategic foundations of the Nash and Kalai solutions. *Journal of Economic Theory 189*.

Julien, B., J. Kennes, and I. King (2008). Bidding for money. *Journal of Economic Theory 142*(1), 196–217.

Kalai, E. (1977). Proportional solutions to bargaining situations: Interpersonal utility comparisons. *Econometrica 45*(7), 1623–1630.

Kaplan, G. and G. Menzio (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy 124*(3), 771–825.

Kashyap, N. and P. Tantri (2025). Firm market power channel of monetary policy transmission.

King, R. and A. L. Wolman (1996). Inflation targeting in a St. louis model of the 21st century. Federal Reserve Bank of St. Louis Review, 95.

King, R. G. and M. W. Watson (1994). The post-war US Phillips curve: a revisionist econometric history. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 41, pp. 157–219. Elsevier.

King, R. G. and M. W. Watson (1997). Testing long-run neutrality. *FRB Richmond Economic Quarterly 83*(3), 69–101.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lehmann, E. (2012). A search model of unemployment and inflation. *Scandinavian Journal of Economics 114*(1), 245–266.

Lehmann, E. and B. Van der Linden (2010). Search frictions on product and labor markets: Money in the matching function. *Macroeconomic Dynamics 14*(1), 56–92.

Lester, B. (2011). Information and prices with capacity constraints. *American Economic Review 101*(4), 1591–1600.

Lucas, R. E. and J. P. Nicolini (2015). On the stability of money demand. *Journal of Monetary Economics 73*, 48–65.

Lucas, R. E. and E. C. Prescott (1974). Equilibrium search and unemployment. *Journal of Economic Theory 7*(2), 188–209.

Makowski, L. and J. M. Ostroy (2001). Perfect competition and the creativity of the market. *Journal of Economic Literature 39*(2), 479–535.

Mangin, S. and B. Julien (2021). Efficiency in search and matching models: A generalized hosios condition. *Journal of Economic Theory 193*, 105208.

McCall, J. J. (1970). Economics of information and job search. *Quarterly Journal of Economics*, 113–126.

Michaillat, P. and E. Saez (2015). Aggregate demand, idle time, and unemployment. *Quarterly Journal of Economics 130*(2), 507–569.

Moen, E. (1997). Competitive search equilibrium. *Journal of Political Economy 105*(2), 385–411.

Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies 61*(3), 397–415.

Neiss, K. (2001). The markup and inflation: Evidence in OECD countries. *Canadian Journal of Economics 34*(2), 570–587.

Nekarda, C. J. and V. A. Ramey (2020). The cyclical behavior of the price-cost markup. *Journal of Money, Credit and Banking 52*(S2), 319–353.

Petrosky-Nadeau, N. and E. Wasmer (2015). Macroeconomic dynamics in a model of goods, labor, and credit market frictions. *Journal of Monetary Economics 72*, 97–113.

Petrosky-Nadeau, N. and E. Wasmer (2017). *Labor, Credit, and Goods Markets: The macroeconomics of search and unemployment.* MIT Press.

Pissarides, C. A. (2000). *Equilibrium unemployment theory.* MIT press.

Rocheteau, G., T.-W. Hu, L. Lebeau, and Y. In (2021). Gradual bargaining in decentralized asset markets. *Review of Economic Dynamics 42*, 72–109.

Rocheteau, G. and A. Rodriguez-Lopez (2014). Liquidity provision, interest rates, and unemployment. *Journal of Monetary Economics 65*, 80–101.

Rocheteau, G., P. Rupert, and R. Wright (2007). Inflation and unemployment in general equilibrium. *Scandinavian Journal of Economics 109*(4), 837–855.

Rocheteau, G., P.-O. Weill, and T.-N. Wong (2021). An heterogeneous-agent new-monetarist model with an application to unemployment. *Journal of Monetary Economics 117*, 64–90.

Rocheteau, G. and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica 73*(1), 175–202.

Rocheteau, G., R. Wright, and S. X. Xiao (2018). Open market operations. *Journal of Monetary Economics 98*, 114–128.

Shi, S. (1997). A Divisible Search Model of Fiat Money. *Econometrica 65*(1), 75–102.

Shi, S. (1998). Search for a monetary propagation mechanism. *Journal of Economic Theory 81*(2), 314–352.

Trejos, A. and R. Wright (1995). Search, bargaining, money, and prices. *Journal of political Economy 103*(1), 118–141.

Walsh, C. E. (2017). *Monetary theory and policy.* MIT press.

Wang, L. (2016). Endogenous search, price dispersion, and welfare. *Journal of Economic Dynamics and Control 73*, 94–117.

Wang, L., R. Wright, and L. Q. Liu (2020). Sticky prices and costly credit. *International Economic Review 61*(1), 37–70.

Williamson, S. D. (2006). Search, limited participation, and monetary policy. *International Economic Review 47*(1), 107–128.

Williamson, S. D. (2012). Liquidity, monetary policy, and the financial crisis: A New Monetarist approach. *American Economic Review 102*(6), 2570–2605.

Williamson, S. D. (2015). Keynesian inefficiency and optimal policy: A New Monetarist approach. *Journal of Money, Credit and Banking 47*(S2), 197–222.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *Quarterly Journal of Economics 101*(3), 493–511.

Yeh, C., C. Macaluso, and B. Hershbein (2022). Monopsony in the US labor market. *American Economic Review 112*(7), 2099–2138.

Proofs of propositions and lemmas

Proof of Lemma 1. In a monetary meeting, according to the proportional solution, (p, y), solves

$$\max_{p,y} \{p - \varphi(y)\} \quad \text{s.t. (4)} \quad \text{and} \quad p \leq a. \tag{43}$$

Equation (43) can be understood as the firm choosing (p, y) to maximise its profits subject to the conditions that: (i) the profits are a fraction $\mu \in [0, 1]$ of the whole gains from trade and (ii) the consumer's payment does not exceed her real balances. Since the firm's and the consumer's surpluses are proportional to each other, the problem can be reduced to the maximisation of the joint profits subject to a liquidity constraint:

$$\max_{p,y} \{\upsilon(y) - \varphi(y)\} \quad \text{s.t. } p = \varphi(y) + \mu [\upsilon(y) - \varphi(y) - Z] \leq a. \tag{44}$$

If the liquidity constraint does not bind, then $y = y^*$ and $p = (1 - \mu)\varphi(y^*) + \mu\upsilon(y^*) - \mu Z$, i.e., (5)-(6). Otherwise, $p = a$ and $a = (1 - \mu)\varphi(y) + \mu\upsilon(y) - \mu Z$, i.e., (8)-(9).

In a credit meeting, the solution is the same as that of the money meeting when $a = \infty$. Hence the solution is given by (5)-(6). ■

Proof of Lemma 2. Assuming $a > 0$, the first-order condition, (15), can be rewritten as

$$\frac{\varphi'(y)}{\upsilon'(y) - \varphi'(y)} = \frac{\alpha(1 - \mu)\chi^m - \mu i}{i}.$$

The left side is increasing in y. For all $i < \alpha(1 - \mu)\chi^m/\mu$, the right side is decreasing in i. Hence, y is decreasing in i. Given that $a = (1 - \mu)\varphi(y) + \mu\upsilon(y) - \mu Z$, the demand for real balances is decreasing in i for a given Z. ■

Proof of Lemma 3. The left side of (16) is linear and increasing in Z. The right side of (16) is decreasing in Z, is positive when $Z = 0$ provided that $\chi^d > 0$, and it approaches 0 as $Z \to \upsilon(y^*) - \varphi(y^*)$. Hence, as shown in Figure 4, there is a unique $Z \in (0, \upsilon(y^*) - \varphi(y^*))$ solution to (16). The right side is maximised with respect to a. Hence, by the envelope theorem, the derivative of the right side with respect to i is equal to $-a$. Thus, $\partial Z/\partial i = -a/(\rho + \lambda)$. ■

Proof of Lemma 4.

Equation (23) can be rewritten as

$$(\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k\theta = (1 - \beta) \left\{\alpha^s\mu \left[\chi^m S^m(a, Z) + \chi^d S^d(Z)\right] + x - b\right\}.$$

The left side is increasing in θ, it approaches 0 as $\theta \to 0$, and $+\infty$ as $\theta \to +\infty$. So, provided the right side is positive, there is a unique $\theta > 0$ solution to (23) taking (α^s, a, Z) as given.

Part 1: From the definition of S^m in (11),

$$\frac{\partial S^m(a,Z)}{\partial a} = \{\upsilon'[y(a,Z)] - \varphi'[y(a,Z)]\} \frac{\partial y(a,Z)}{\partial a},$$

where, from (8),

$$\frac{\partial y(a,Z)}{\partial a} = \frac{1}{g'(y)} > 0.$$

For all $a < p^*$, $y(a,Z) < y^*$, and hence $\partial S^m(a,Z)/\partial a > 0$. Thus, the solution θ to (23) increases with a. As $a \to p^*$, $y \to y^*$, and $\partial S^m(a,Z)/\partial a \to 0$. It follows that $\partial\theta/\partial a \to 0$.

Part 2: Again by (11),

$$\frac{\partial S^m(a,Z)}{\partial Z} \equiv \{\upsilon'[y(a,Z)] - \varphi'[y(a,Z)]\} \frac{\partial y(a,Z)}{\partial Z} - 1,$$

where, from (8),

$$\frac{\partial y(a,Z)}{\partial Z} = \frac{\mu}{(1-\mu)\varphi'(y) + \mu\upsilon'(y)}.$$

Using that

$$[\upsilon'(y) - \varphi'(y)] \frac{\partial y}{\partial Z} = \frac{\mu[\upsilon'(y) - \varphi'(y)]}{\varphi'(y) + \mu[\upsilon'(y) - \varphi'(y)]} \in [0,1),$$

it follows that $\partial S^m(a,Z)/\partial Z < 0$. From the definition of S^d in (12), $\partial S^d(Z)/\partial Z = -1$. Thus, the solution θ to (23) decreases with Z. ■

Proof of Lemma 5. We subtract ρU from both sides of (21) and use the rent-sharing rule, $E - U = \beta J$, to obtain

$$w = (\rho + \delta)\beta J + \rho U. \tag{45}$$

We substitute $(\rho + \delta)\beta J$ by its expression given by (19) to rewrite (45) as follows:

$$w = \beta\left\{\alpha^s \mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)\rho U. \tag{46}$$

We use $\rho U = b + \beta k\theta/(1-\beta)$ to rewrite the wage in (46) as (24). ■

Proof of Proposition 1.

Define $Z(\theta)$, the solution to (31), as a function of θ. To see that $Z(\theta)$ exists and is unique, note that the left hand side of (31) is linear, increasing in Z, while the right hand side (RHS) is decreasing for all $Z \in (0, \upsilon(y^*) - \varphi(y^*))$. If $Z = 0$, then $RHS \geq \alpha^b(\theta)(1-$

$\mu)\chi^d [\upsilon(y^*) - \varphi(y^*)] > 0$ if $\theta > 0$. If $Z = \upsilon(y^*) - \varphi(y^*)$, then $RHS = 0$. Hence, for all $\theta > 0$, there is a unique $Z \in (0, \upsilon(y^*) - \varphi(y^*))$ solving (31). Since $\alpha^b(\theta)$ is increasing in θ, it follows that RHS is increasing in θ, and hence $Z'(\theta) > 0$. Moreover, $\alpha^b(0) = 0$ implies $Z(0) = 0$.

We have seen that the set of solutions to (14) is either the corner solution 0, the interior solution given by (15), which we denote by $a_I^*(\theta)$, or both. Note that $a_I^*(\theta)$ is increasing continuously in θ, meaning the best response transitions from 0 to $a_I^*(\theta)$ as θ rises, and not the other way around. Hence, $S^m [a^*(\theta), Z(\theta)]$ can only jump upward as θ rises. We now define the following function:

$$\begin{aligned}\Gamma(\theta) \equiv \Big\{ &(1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m S^m [a^*(\theta), Z(\theta)] + \chi^d S^d [Z(\theta)]\right\} + x - b\right\} \\ &-\beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}\Big\}.\end{aligned} \tag{47}$$

When both 0 and $a_I^*(\theta)$ are optimal for consumers, we assume $a^*(\theta) = a_I^*(\theta)$, and hence $\Gamma(\theta)$ is right continuous. An equilibrium can be reduced to a θ solution to $\Gamma(\theta) = 0$. Since $S^m [a^*(\theta), Z(\theta)]$ only jumps upward, so does $\Gamma(\theta)$. As $\theta \to 0$, $a^*(\theta) \to 0$, so $\Gamma(\theta)$ converges to the value

$$(1-\beta)\left\{\alpha^s(0)\mu\chi^d [\upsilon(y^*) - \varphi(y^*)] + x - b\right\}.$$

Since $\alpha^s(0) = +\infty$, $\Gamma(0) = +\infty$. As $\theta \to +\infty$, $\alpha^s(\theta) \to 0$, $\theta/f(\theta) \to 1/f'(+\infty) = +\infty$ and hence $\Gamma(\theta) \to -\infty$. Since $\Gamma(\theta)$ can only jump upward, it must cut the x-axis and so a steady-state equilibrium exists. ■

Proof of Proposition 2. From (29)-(32), a nonmonetary equilibrium can then be reduced to a triple (θ, Z, w) that solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\mu\chi^d [\upsilon(y^*) - \varphi(y^*) - Z] + x - b\right\} - \beta k\theta, \tag{48}$$

$$(\rho+\lambda) Z = \alpha^b(\theta)(1-\mu)\chi^d [\upsilon(y^*) - \varphi(y^*) - Z], \text{ and} \tag{49}$$

$$w = \beta\left\{\alpha^s(\theta)\mu\chi^d [\upsilon(y^*) - \varphi(y^*) - Z] + x\right\} + (1-\beta)b + \beta k\theta. \tag{50}$$

From (49), we solve for Z in closed form and obtain

$$Z = \frac{\alpha^b(\theta)(1-\mu)\chi^d}{\rho+\lambda+\alpha^b(\theta)(1-\mu)\chi^d}[\upsilon(y^*) - \varphi(y^*)]. \tag{51}$$

We substitute the expression for Z into (48) to reduce an equilibrium to a single equation in θ, (33). The left side of (33) is increasing in θ from 0 to $+\infty$ while the right side is decreasing in θ from $+\infty$ to $(1-\beta)(x-b)$. Hence, a solution of θ exists and is unique.

Part 1: The right side of (33) is increasing in λ. Hence, θ increases with λ. It follows that the unemployment rate, $u = \delta/[\delta + f(\theta)]$, decreases with λ. From (48), θ is a decreasing function of Z, so Z decreases with λ. Finally, by (50), w increases in λ.

Part 2: In (33), the term

$$\frac{\alpha^s(\theta)\mu\chi^d(\rho+\lambda)}{\rho+\lambda+\alpha^b(\theta)\chi^d(1-\mu)}$$

increases in χ^d. Hence, market tightness increases and unemployment decreases in χ^d. ■

Proof of Proposition 4. Part 1: Labour market tightness is determined by $\Gamma(\theta;i)=0$ where

$$\Gamma(\theta;i) \equiv (1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m S^m\left[a^*(\theta;i), Z(\theta;i)\right] + \chi^d S^d\left[Z(\theta;i)\right]\right\} + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}.$$

When $i=0$,

$$\Gamma(\theta;0) \equiv (1-\beta)\left\{\frac{\alpha^s(\theta)\mu(\rho+\lambda)}{\rho+\lambda+\alpha^b(\theta)(1-\mu)}\left[\upsilon(y^*)-\varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}. \quad (52)$$

It is monotone decreasing in θ, so the equilibrium at the Friedman rule ($i=0$) is unique.

We now show that $\Gamma(\theta;i)$ is increasing in i in the neighbourhood of the Friedman rule. From (11),

$$\frac{\partial S^m(a,Z)}{\partial a} \equiv \left[\upsilon'(y)-\varphi'(y)\right]\frac{\partial y}{\partial a}.$$

In the neighbourhood of $i=0^+$, $y=y^*$ and $\upsilon'(y)-\varphi'(y)=0$. Hence, $\partial S^m(a,Z)/\partial a = 0$. The effect of a change in a^* on the match surplus, induced by an increase in i, is second order when i is close to 0 because the match surplus is maximum. However, from (11)-(12), when y is in the neighbourhood of y^*,

$$\frac{\partial S^m(a,Z)}{\partial Z} = \frac{\partial S^d(Z)}{\partial Z} = -1.$$

From (31),

$$\frac{\partial Z}{\partial i} = \frac{-a^*}{\rho+\lambda+\alpha^b(\theta)(1-\mu)},$$

where, at the Friedman rule, by (15) and (51)

$$a^* = \varphi(y^*) + \frac{(\rho+\lambda)\mu}{\rho+\lambda+\alpha^b(\theta)(1-\mu)}\left[\upsilon(y^*)-\varphi(y^*)\right] > 0.$$

Combining these results, $\chi^m S^m [a^*(\theta;i), Z(\theta;i)] + \chi^d S^d [Z(\theta;i)]$ is increasing in i, and hence $\Gamma(\theta;i)$ is also increasing in i. It follows that θ, such that $\Gamma(\theta;i)=0$, is increasing in i. The unemployment rate, $u = \delta/[\delta + f(\theta)]$, is decreasing in θ and hence decreasing in i. By the definition of $\Gamma(\theta;i)$ and (32), we can reexpress $\Gamma(\theta;i)$ as

$$\Gamma(\theta;i) = \frac{(1-\beta)}{\beta}(w-b) - k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}.$$

Since $\Gamma(\theta;i)=0$ in equilibrium, w and θ must comove as i changes. Therefore, w rises in i.

Part 2: We now consider the limiting case $i=+\infty$. Since agents do not carry money, the outcome is similar to a pure credit economy but with $\chi^d < 1$. From (30), $a^* = 0$. By the steps leading to (51),

$$Z = \frac{\alpha^b(\theta)\chi^d(1-\mu)}{\rho+\lambda+\alpha^b(\theta)\chi^d(1-\mu)}\left[v(y^*) - \varphi(y^*)\right]$$

and hence

$$\Gamma(\theta;+\infty) \equiv (1-\beta)\left\{\frac{\alpha^s(\theta)\mu\chi^d(\rho+\lambda)}{\rho+\lambda+\alpha^b(\theta)\chi^d(1-\mu)}\left[v(y^*)-\varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}. \tag{53}$$

From (52) and (53), $\Gamma(\theta;+\infty) < \Gamma(\theta;0)$ for all $\theta > 0$. So θ_0 solution to $\Gamma(\theta;0)=0$ is larger than $\theta_{+\infty}$ solution to $\Gamma(\theta;+\infty)=0$. Hence, the unemployment rate when $i=0$ is lower than the unemployment rate when $i=+\infty$. Since w and θ comove, w is lower when $i=+\infty$ than when $i=0$. Finally, note that there is a finite upper bound for i above which a monetary equilibrium does not exist. This upper bound is $i = \chi^m(1-\mu)\alpha^b(\theta)/\mu$ where θ is bounded above by the market tightness of a pure credit economy with $Z=0$. This upper bound of θ is finite and it solves

$$(1-\beta)\left\{\alpha^s(\theta)\mu\left[v(y^*)-\varphi(y^*)\right] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)} = 0.$$

■

Proof of Proposition 5 . From the assumption $\lambda = +\infty$, $Z=0$. Then from (29) and $\chi^d, \mu > 0$, $\theta \to +\infty$ as $A \to +\infty$. When $\lambda = +\infty$, buyers' money holding decision is given by (36) even when $\chi^d > 0$. Then from (36), for all $\theta > 0$, as $A \to +\infty$, $y \to y^*$. Hence surpluses in all pairwise meetings are equal to $v(y^*) - \varphi(y^*)$. We now turn to labour market tightness and matching rates. From (34),

$$\alpha^s(\theta) = A\frac{\alpha\left[q(\theta)\right]}{q(\theta)}, \text{ where } q(\theta) = \frac{f(\theta)}{\omega\left[\delta + f(\theta)\right]}.$$

Since $f(\theta) \to +\infty$ as $A \to +\infty$, tightness in the goods market, q, tends to $1/\omega$. Since $\alpha\left[q(\theta)\right]/q(\theta) \to \omega\alpha(\omega^{-1})$, from the expression above, $\alpha^s(\theta) \to +\infty$. The limit for the markup follows directly from (17). ■

Proof of Proposition 6. Part 1: From (28), as $A \to +\infty$

$$\frac{q}{\bar{\alpha}(q)} \to \frac{Af(\theta)}{\lambda\omega\left[\delta + f(\theta)\right]} = +\infty \text{ for all } \theta > 0.$$

So, if $\theta > 0$ at the limit, $q \to +\infty$ and the firm matching rate with a consumer tends to $A\bar{\alpha}(q)/q \to \lambda\omega\left[\delta + f(\theta)\right]/f(\theta)$. Since $A\bar{\alpha}(q) \to +\infty$, from (31), $Z \to \upsilon(y^*) - \varphi(y^*)$. Using that $S^m(a^*, Z) \to 0$ and $S^d(Z) \to 0$, from (29), θ solves

$$(\rho + \delta)\frac{k\theta}{f(\theta)} = (1-\beta)(x-b) - \beta k\theta.$$

It is consistent with $\theta > 0$ iff $x > b$. Hence, if $x \leq b$ then $\theta \to 0$ as $A \to +\infty$. Suppose $q \to q_\infty > 0$. Then, $\alpha^b = A\bar{\alpha}(q) \to +\infty$ and, from (31), $Z \to \upsilon(y^*) - \varphi(y^*)$. If $q_\infty = 0$, then $A\bar{\alpha}(q)/q \to +\infty$ and, from (29), $\theta = 0$ implies $Z \to \upsilon(y^*) - \varphi(y^*)$.

Part 2: We will show $w \to x$, other claims follow immediately from Part 1. If additionally $B \to +\infty$, then (38) becomes (39). By (32) and $S^m(a^*, Z), S^d(Z) \to 0$,

$$w = \beta x + (1-\beta)b + \beta k\theta.$$

By (39), the right side is equal to x as $B \to +\infty$. ■

Proof of Proposition 7. The planner's problem is

$$\max \int_0^{+\infty} e^{-\rho t}\varpi_t dt, \tag{54}$$

subject to

$$\begin{aligned} \dot{\omega}_{1,t} &= \lambda(\omega - \omega_{1,t}) - \left[\alpha\left(\frac{n_t}{\omega_{1,t}}\right)\right]\omega_{1,t}, \quad \text{and} && (55) \\ \dot{n}_t &= f(\theta_t)(1-n_t) - \delta n_t, && (56) \end{aligned}$$

where n_0 and $\omega_{1,0}$ are given and the instantaneous social welfare, ϖ_t, is given by

$$\begin{aligned}\varpi_t &= \omega_{1,t}\alpha\left(\frac{n_t}{\omega_{1,t}}\right)\left\{\chi^m\left[\upsilon(y_{m,t})-\varphi(y_{m,t})\right]+\chi^d\left[\upsilon(y_{d,t})-\varphi(y_{d,t})\right]\right\} \\ &\quad +n_t(x-b)+b-(1-n_t)\theta_t k.\end{aligned} \tag{57}$$

The state variables are $\omega_{1,t}$ and n_t. The control variables are $y_{m,t}$, $y_{d,t}$, and θ_t. We denote ξ_t as the current-value co-state variable associated with n_t, and ζ_t the current-value co-state variable associated with $\omega_{1,t}$. The current-value Hamiltonian is

$$\begin{aligned}\mathcal{H} &\equiv \omega_1\alpha\left(\frac{n}{\omega_1}\right)\left\{\chi^m\left[\upsilon(y_m)-\varphi(y_m)\right]+\chi^d\left[\upsilon(y_d)-\varphi(y_d)\right]\right\}+n(x-b)+b-(1-n)\theta k \\ &\quad +\xi\left\{f(\theta)(1-n)-\delta n\right\} \\ &\quad +\zeta\left\{\lambda(\omega-\omega_1)-\left[\alpha\left(\frac{n}{\omega_1}\right)\right]\omega_1\right\}.\end{aligned}$$

From Pontryagin's Maximum Principle,

$$(\theta_t, y_{m,t}, y_{d,t}) \in \arg\max \mathcal{H}(\theta_t, y_{m,t}, y_{d,t}, n_t, \omega_{1,t}) \quad \text{for all } t.$$

Hence, θ_t solves

$$k=\xi_t f'(\theta_t). \tag{58}$$

and $y_{m,t}=y_{d,t}=y^*$ for all t. The law of motion for the co-state variable, ξ_t, is given by $\rho\xi_t=\partial\mathcal{H}_t/\partial n_t+\dot{\xi}_t$, i.e.,

$$\rho\xi_t=\alpha'\left(q_t\right)\left[\upsilon(y^*)-\varphi(y^*)-\zeta_t\right]+x-b+\theta_t k-\left[f(\theta_t)+\delta\right]\xi_t+\dot{\xi}_t. \tag{59}$$

Similarly, the law of motion for ζ_t is given by $\rho\zeta_t=\partial\mathcal{H}_t/\partial\omega_{1,t}+\dot{\zeta}_t$, i.e.,

$$\left(\rho+\lambda\right)\zeta_t=\left[\alpha\left(q_t\right)-q_t\alpha'\left(q_t\right)\right]\left[\upsilon(y^*)-\varphi(y^*)-\zeta_t\right]+\dot{\zeta}_t. \tag{60}$$

Combining (58) and (60), the optimality condition for labour market tightness can be rewritten as

$$\begin{aligned}\left(\rho+\delta\right)\frac{k\theta_t}{f(\theta_t)} &= \epsilon_f(\theta_t)\left\{\frac{\alpha\left(q_t\right)}{q_t}\epsilon_\alpha(q_t)\left[\upsilon(y^*)-\varphi(y^*)-\zeta_t\right]+x-b\right\} \\ &\quad -\left[1-\epsilon_f(\theta_t)\right]k\theta_t-\frac{f''(\theta_t)}{f(\theta_t)f'(\theta_t)}k\theta_t\dot{\theta}_t.\end{aligned} \tag{61}$$

Using the definition of ϵ_α, and the law of motion for ζ_t, (60) can be rewritten as

$$(\rho+\lambda)\,\zeta_t = \alpha\,(q_t)\,[1-\epsilon_\alpha\,(q_t)]\,[\upsilon(y^*)-\varphi(y^*)-\zeta_t]+\dot{\zeta}_t. \tag{62}$$

From (61) and (62), a constant solution to the planner's problem is a pair (θ,ζ) that is a solution to

$$\begin{aligned}(\rho+\lambda)\,\zeta &= \alpha\,(q)\,[1-\epsilon_\alpha\,(q)]\,[\upsilon(y^*)-\varphi(y^*)-\zeta] \quad \text{and} && (63)\\ (\rho+\delta)\,\frac{k\theta}{f(\theta)} &= \epsilon_f(\theta)\left\{\frac{\alpha\,(q)}{q}\epsilon_\alpha(q)\,[\upsilon(y^*)-\varphi(y^*)-\zeta]+x-b\right\}-[1-\epsilon_f(\theta)]\,k\theta, && (64)\end{aligned}$$

provided that $n_0=n_s\equiv f(\theta)/[f(\theta)+\delta]$ and $\omega_{1,0}=\omega_1^s\equiv\lambda/[\alpha(\theta)+\lambda]$.

To show that the stationary solution is unique, we eliminate ζ from (64) by using (63). Then, θ solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)}=\epsilon_f(\theta)\left\{\frac{\alpha\,(q)}{q}\epsilon_\alpha(q)\frac{\rho+\lambda}{\rho+\lambda+\alpha(q)(1-\epsilon_\alpha)}\,[\upsilon(y^*)-\varphi(y^*)]+x-b\right\}-[1-\epsilon_f(\theta)]\,k\theta. \tag{65}$$

By (28), q increases in θ. Since $\alpha'', f''<0$, the elasticities ϵ_α and ϵ_f decrease in θ. Therefore, the right hand side of (65) falls and the left hand side rises in θ. Hence, the stationary solution is unique.

We now show that the constant solution is a solution to the planner's problem by invoking the Mangasarian sufficiency condition. First, the current-value Hamiltonian is jointly concave in (ι,n,ω_1), where $\iota\equiv(1-n)\theta$. To see this, note that $\omega_1\alpha\,(n/\omega_1)$ is jointly concave in (ω_1,n). Also, $f(\theta)(1-n)=f(\iota/u)u$ is jointly concave in (ι,u). Finally, the constant solution satisfies the following transversality conditions,

$$\begin{aligned}\lim_{t\to+\infty} e^{-\rho t}\xi_t n_t &= 0 \text{ and}\\ \lim_{t\to+\infty} e^{-\rho t}\zeta_t\omega_{1,t} &= 0.\end{aligned}$$

We now compare the optimality conditions with the equilibrium conditions. Since the planner requires $y_{m,t}=y^*$, monetary policy must implement the Friedman rule, i.e., $i_t=0$. From (16), Z at the Friedman rule solves

$$(\rho+\lambda)\,Z=\alpha(1-\mu)\,[\upsilon(y^*)-\varphi(y^*)-Z]\,. \tag{66}$$

From (23), θ at the Friedman rule solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)}=(1-\beta)\left\{\frac{\alpha(q)}{q}\mu\left[\upsilon(y^*)-\varphi(y^*)-Z\right]+x-b\right\}-\beta k\theta. \tag{67}$$

The comparison of the planner's optimality conditions, (63)-(64), and the equilibrium conditions (66)-(67), show that they coincide if the Hosios conditions in the goods and labour markets, (40)-(41), hold. ■

Proof of Proposition 8. _Part 1:_ When $\lambda=+\infty$, by (29)-(32), $\omega_{1,t}=\omega$, $Z=0$, and a steady state equilibrium can be reduced to a 3-tuple, (θ,a,w), that is a solution to:

$$\begin{aligned}(\rho+\delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]+x-b\right\}-\beta k\theta, \\ a &\in \arg\max_{a\geq 0}\left\{-ia+\alpha^b(\theta)(1-\mu)\chi^m S^m(a,0)\right\}, \text{ and} \\ w &= \beta\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]+x\right\}+(1-\beta)b+\beta k\theta.\end{aligned} \tag{68}$$

By (57), the instantaneous surplus becomes

$$\begin{aligned}\varpi &= \omega\alpha\left(\frac{n}{\omega}\right)\left\{\chi^m\left[\upsilon(y_m)-\varphi(y_m)\right]+\chi^d\left[\upsilon(y_d)-\varphi(y_d)\right]\right\} \\ &\quad +n(x-b)+b-(1-n)\theta k.\end{aligned} \tag{69}$$

The planner maximises ϖ by choosing the policy rate i, subject to the law of motion of workers (27) and the firms' free entry condition (68).

We first eliminate y^m in ϖ with the free-entry condition. By (68),

$$\alpha^s(\theta)\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]=\frac{1}{\mu(1-\beta)}\left[(\rho+\delta)\frac{k\theta}{f(\theta)}+\beta k\theta\right]-\frac{x-b}{\mu}. \tag{70}$$

Since $S^m(a,0)=\upsilon(y_m)-\varphi(y_m)$ and $S^d(0)=\upsilon(y_d)-\varphi(y_d)$, we can eliminate the trade surplus in the planner's objective function in (69) by (70). Also, we can eliminate n by using $n=f(\theta)/[\delta+f(\theta)]$. Therefore, we can rewrite the planner's problem as $\max_\theta \varpi(\theta)$, where

$$\varpi(\theta)\equiv\frac{f(\theta)}{\delta+f(\theta)}\left[\frac{k\theta}{\mu(1-\beta)}\left(\frac{\rho+\delta[1-\mu(1-\beta)]}{f(\theta)}+\beta\right)-\left(\frac{1}{\mu}-1\right)(x-b)\right]+b.$$

The expression in the square bracket rises in θ. When the square bracket is positive, the right side strictly rises in θ. Therefore, the maximiser of $\varpi(\theta)$, that we call $\bar{\theta}$, is the largest element in the set of feasible market tightness, provided that $\varpi(\bar{\theta})>b$. Since the free-entry condition (68) defines a positive relationship between θ and y_m, the maximum tightness, $\bar{\theta}$, is achieved if $y_m = y_d = y^*$, which corresponds to the Friedman rule.

Now we argue $\varpi(\bar{\theta}) > b$. Let θ_0 be the value of θ such that the right side of (70) is 0. At $\theta = \theta_0$, by (69),

$$\varpi(\theta_0) = \frac{\theta k}{\delta + f(\theta)} \left(\frac{\rho + \beta[\delta + f(\theta)]}{1 - \beta} \right) + b.$$

Since $\varpi(\theta_0) > b$, $\varpi(\theta)$ rises in θ for all $\theta \geq \theta_0$. Since $\bar{\theta} > \theta_0$, $\varpi(\bar{\theta}) > b$.

<u>*Part 2:*</u> When $\lambda < +\infty$, we can rewrite (57) by replacing $\omega_1 = \lambda/[\alpha(\theta) + \lambda]$ and $n = f(\theta)/[\delta + f(\theta)]$:

$$\begin{aligned} \varpi(\theta, y_m, y_d) \quad = \quad & \frac{\lambda \alpha(\theta)}{\alpha(\theta) + \lambda} \left\{ \chi^m \left[\upsilon(y_m) - \varphi(y_m) \right] + \chi^d \left[\upsilon(y_d) - \varphi(y_d) \right] \right\} \\ & + \frac{f(\theta)}{f(\theta) + \delta}(x - b) + b - \frac{\delta \theta}{\delta + f(\theta)} k. \end{aligned}$$

At the Friedman rule, an increase in i reduces y_m and Z, but the change in y^m only has a second-order effect on firms' profits and market tightness. The only first-order effect is due to the drop in Z. As Z falls, firms get more profits from trade and thus θ rises. Hence, a local derivation from the Friedman rule exists when the derivative of $\varpi(\theta, y_m, y_d)$ with respect to θ is positive at the Friedman rule, fixing $y_m = y_d = y^*$, i.e.,

$$\frac{\partial \varpi(\theta, y^*, y^*)}{\partial \theta} \propto \epsilon_f(\theta) \left[\frac{\lambda \alpha^s(\theta) S(y^*, 0)}{\lambda + \alpha(\theta)(1 - \epsilon_\alpha)} \epsilon_\alpha(\theta) + (x - b) \right] - \left(\frac{1}{n} - \epsilon_f(\theta) \right) k\theta, \qquad (71)$$

is strictly positive, and $\propto$ means the left and right side have the same sign. The right side of (71) is equivalent to that of (65) when $\rho = 0$. Hence the right side of (71) falls in θ. Since we have assumed $f'(0) = +\infty$, the right side explodes as $\theta \to 0$. Thus the Friedman rule is suboptimal if and only if θ is sufficiently small.

According to (33), the labour tightness, θ, at the Friedman rule solves

$$(\rho + \delta) \frac{k\theta}{f(\theta)} = (1 - \beta) \left\{ \alpha^s(\theta) \frac{\mu (\rho + \lambda)}{\rho + \lambda + \alpha^b(\theta)(1 - \mu)} \left[\upsilon(y^*) - \varphi(y^*) \right] + x - b \right\} - \beta k\theta. \quad (72)$$

There is a unique solution of θ because the right side of (72) falls and the left side rises in θ. It is easy to check that the solution of θ rises in μ and falls in β. Given μ, if $\beta \to 1$, then $\theta \to 0$. Hence, there exists a $\bar{\beta}(\mu) < 1$ such that the Friedman rule is locally suboptimal if and only if $\beta > \bar{\beta}(\mu)$. Moreover, $\bar{\beta}(\mu)$ rises in μ because the market tightness θ at the FR increases in μ by (72). ■