Frictional Goods Markets: Theory and Applications*

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Abstract

We analyze dynamic general equilibrium models with more-or-less directed search by informed buyers and random search by uninformed buyers. This nests existing specifications and generates new insights. A quantitative application concerns the welfare cost of inflation, which is known to be quite high with pure random search and low with pure directed search. Our calibration implies the impact of inflation is fairly low, in part because, in addition to the usual costs, it provides benefits by more heavily taxing high-price sellers that inefficiently profit from exploiting the uninformed. Other applications analyze analytically and numerically changes in credit conditions and information.

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> [Some consumers] receive no information on the price in this market. (It is natural to think of them as tourists, having no local information.) A second type of consumer (resident) receives some information... It would be interesting to develop models with both types of consumers and, I suspect, would result in a different structure of equilibrium. Diamond (1971)

1 Introduction

A classic issue in economics concerns the impact of inflation on welfare, price dispersion, markups, and other endogenous variables. To study such phenomena, perhaps especially welfare, we think it is important to use a microfounded theory where money ameliorates explicit trading frictions, since inflation is primarily a tax on monetary exchange. This project develops a framework where money is a medium of exchange, and uses it to study several issues, both theoretically and quantitatively. A central feature is that consumers can be more or less informed about sellers' terms of trade – e.g., some may perfectly informed and direct their search to the most attractive sellers, while others are completely uninformed and hence pick sellers at random. Another feature is that, in addition to money, agents can use credit at a cost, which is important for several reasons explained below.

As motivation, first note that at some point there emerged a consensus that the impact of inflation on welfare was small, as can be seen in many papers using Walrasian theory augmented with reduced-form devices like money-in-utility or cash-in-advance assumptions, e.g., Cooley and Hansen (1989) or Lucas (2000). See Rocheteau and Nosal (2017) and Diercks (2017) for more discussion and references, but as a rough average over studies, eliminating a 10% annual inflation was found to be worth around 0.5% of consumption. That was challenged by economists working with search-and-bargaining theory, e.g., Lagos and Wright (2005), where the same policy is worth closer to 5.0% of consumption – literally an order of magnitude higher. However, in otherwise similar environments using competitive search equilibrium, which means directed search and price posting rather than random search and bargaining, e.g., Rocheteau and Wright (2005, 2009), the result goes back down to around 1%.

We pursue this by integrating models with different types of search. In one version, with partially directed search as in Lester (2011), a fraction λ of buyers are informed about the terms offered by all sellers, while others are not and search randomly. In another version, with noisy search as in Burdett and Judd (1983), a fraction λ observe the terms offered by $h > 1$ sellers, but not all, while the rest see $h = 1$. Both generate price dispersion, but its nature differs in the two versions. Also, both have ex ante homogeneous sellers behaving differently in equilibrium: some post lower prices p and higher quantity or quality q to attract more-well-informed buyers; others cater to the less-well-informed at less-attractive terms. This is similar to Lester and Burdett-Judd, although a difference is that we always give buyers the option to bargain.[1]

Moreover, those papers use basically static, partial equilibrium, nonmonetary models, while the substantive issues here require dynamic, general equilibrium, monetary theory. Hence we build on the New Monetarist literature.[2] For our purposes this is natural for several reasons. First it provides a tractable integration of search and general equilibrium theory. Second, it captures in a simple way the asynchronization of expenditures and receipts at the heart of any analysis of money or credit. Third, it easily accommodates random, directed or noisy search, as well as bargaining or posting, and this project is all about how these details of market microstructure matter.

As regards combining money and credit, our transaction cost approach is not meant to be deep, but it has much precedent (see Rocheteau and Nosal (2017) for citations). In any case, credit is a key element of the framework for several reasons. First, as explained below (see fn. 9), pure-currency economies with price posting display a nuisance indeterminacy of steady states that is eliminated by introducing costly credit. Second, it adds discipline by generating statistics, like the ratio of credit to money purchases, for which we can compare models and data. Third, a serious analysis of inflation should

[1] Related work includes Craig and Rocheteau (2008), Gill and Thanassoulis (2009), Delacroix and Shi (2013), Stacey (2019), Moen et al. (2017) and Shi and Delacroix (2018).

[2] This literature is surveyed in Lagos et al. (2017). In particular, we use an extenion of Lagos and Wright (2005), although in principle, related models like Shi (1997), Molico (2006) or Menzio et al. (2013a) can be used. An advantage of the last two is that they generate endogenous distributions of liquidity, but that makes them much less tractable. Still, as suggested by a referee, we briefly discuss below one version with an endogenous distribution, with the details in a Supplementary Appendix.

give consumers the opportunity to substitute out of cash, and not just into autarky, which is the only option in pure-currency economies. Fourth, we derive some results on credit conditions different from those in nonmonetary models. For these reasons, plus realism, we want both, although pure-credit or pure-currency economies are special cases.[3]

We present analytic and quantitative results. As an example of the former, note that in very many models the optimal monetary policy is the Friedman rule, which corresponds to a nominal interest rate of $i = 0$. Here $i > 0$ can be desirable due to market-composition effects, and although of course others have discussed cases where $i > 0$ is desirable (again see Rocheteau and Nosal (2017) for citations), the economic mechanism here seems both novel and compelling. Intuitively, in equilibrium there are low- and high-price sellers, and the latter inefficiently survive by exploiting uninformed buyers. Inflation is effectively a tax impinging more heavily on these inefficient sellers, because they are more expensive, and that counters its usual negative effects. On net $i > 0$ may be optimal, and must be optimal in some versions of the model, as explained below in terms of information externalities and second-best theory.[4]

Quantitatively, we find that eliminating 10% inflation is worth around 1% of consumption, or less, depending on details. Several reasons for the low numbers are discussed below, but one is that while inflation has costs, it also has benefits because it discourages inefficient high-price sellers. Moreover, the results do not imply currency is unimportant, as eliminating it has a welfare cost between 3.5 and 5.5%, but taxing it some through inflation can be beneficial. On net, the optimal inflation rate is always

[3] As to what our transaction cost of credit represents, we are agnostic. A narrow interpretation pushed by Gomis-Porqueras et al. (2014) is that using credit makes avoiding taxation harder; Wallace (2013) emphasizes monitoring; other relevant considerations are record keeping and enforcement. See Gu et al. (2016) and references therein for more on money and credit. As in that paper, we emphasize that credit here is a payment instrument, like cash, not a way to smooth over the life cycle, like mortgages or student loans. According to *The Economist* (Oct 15, 2016), US merchants paid over $40 billion to process charge card transactions in 2015, and despite policy reforms aimed at reducing this, the cost keeps going up. So payment systems are not costless, and hence seem worth studying.

[4] Along similar lines we explain how our setup affects empirical observations deemed interesting in the literature. For one, we can match the finding in Benabou (1992b) of a negative relation between markups and inflation through a channel different from Benabou (1992a) or Head and Kumar (2005). See also Shi (1998). We can also match in a novel way the positive empirical relation between price dispersion and inflation in Parsley (1996) or Debelle and Lamont (1997), although note that other papers get different empirical results, including Reinsdorf (1994) and Caglayan et al. (2008).

above the Friedman rule, and can be as high as 2.7% , not far from the target of real-world central banks. Since the framework features endogenous credit, we also ask how that matters. It turns out that changes in credit conditions have a very different impact in models with and without money – indeed, in monetary economies tighter credit can actually increase welfare, for second-best reasons, as explained below. Finally, we study the effects of changes in information. As one example, eliminating that friction by making information free is worth between 1.2 and 2.7% of consumption.

The rest of the paper involves making the assumptions explicit, proving our claims, and performing quantitative experiments. Section 2 contains the basic theory, Section 3 presents the calibration, and Section 4 concludes. [5]

2 Theory

First, we embed Lester (2011) in a dynamic monetary environment, with exogenous or with endogenous information. Second, we do the same for Burdett and Judd (1983).

2.1 Basic Assumptions

Each period in discrete time has two subperiods: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. There is a unit measure of infinite-lived buyers and a large measure of sellers. The above-mentioned asynchronization of expenditures and receipts is this: a buyer may want a good q from a seller in the DM, but his income accrues in the CM; so he must either bring cash from the previous CM, or use debt to be paid in the next CM. While in the CM, everyone trades a different good x and labor ℓ, pays taxes, settles debts and adjusts money balances. Period utility is $U(x) + u(q) - \ell$ for buyers and $U(x) - q - \ell$

[5] An alternative motivation for the project is to understand retail markets better. In addition to having money and credit, plus directed and random search, the framework is built to also capture the following features of retail: price dispersion; quality dispersion; high and variable markups; and mainly posted prices but also some bargaining. Some of these features are self evident; others require documentation. On price and quality dispersion, see Ellison and Ellison (2005, 2014), and Jaimovich et al. (2019). Data on markups and the use of money and credit are discussed below. For related work on retail see, e.g., Faig and Jerez (2005), Gourio and Rudanko (2014), Paciello et al. (2019) and Liu et al. (2019).

for sellers, with the usual properties. Agents discount between the CM and DM at $\beta \in (0, 1)$, but not between the DM and the next CM, without loss of generality.

Buyers and sellers meet bilaterally in the DM and trade (p, q), where p is the payment in the next CM's numeraire. If q represents quantity then $P = p/q$ is the unit price; if q represents unobserved quality then we would say p is the price (considering two wines at $\$10$ and $\$100$ a bottle, someone not knowing the latter is better would say it has a higher price, but it may actually be a better deal). We call $P = p/q$ the markup. As is standard in competitive search theory, as surveyed in Wright et al. (2019), the DM partitions sellers into submarkets with the same (p, q), and in each submarket agents match randomly with arrival rates depending on that submarkets's tightness, or buyer-seller ratio, n. Thus a submarket is identified by $\Gamma = (p, q, n)$.

Each period, with probability λ a buyer is *informed*, meaning that he sees $h \in \{2, \infty\}$ draws of Γ, and with probability $1 - \lambda$ he *uninformed*, meaning he only knows the distribution across submarkets, where as mentioned we first take λ as fixed, then endogenize it. We first consider the case $h = \infty$, so that buyers see Γ in every submarket, then consider $h = 2$ so that buyers only see two draws. Similar to Diamond's remarks in the epigraph, we call uninformed buyers *tourists* and informed buyers *locals*, although we do not take that literally.[6] As a benchmark, buyers choosing money balances in the CM do not know if they will be informed in the next DM; later we discuss what happens under alternative specifications.

Anticipating some results, equilibrium in this model has two submarkets, one with local shops catering to the informed, and one with tourist shops serving only the uninformed. Local shops offer favorable terms to attract informed customers, and since their terms are attractive, buyers at local shops could bargain, but prefer to accept the posted (p, q). Tourist shops only get uninformed customers, and we can interpret them as bargaining. However, as usual, agents are never observed negotiating on the equilibrium

[6] Suppose Mr. A and Ms. B live in the same town, but Mr. A knows which shops have good deals on apples and not bananas, while Ms. B knows the opposite. On days when Mr. A and Ms. B both need apples, with leaving town, he acts like our locals and she acts like our tourists, and vice versa when they need bananas. This is equivalent to having agents randomly transiting between locations, where they know more about some than others.

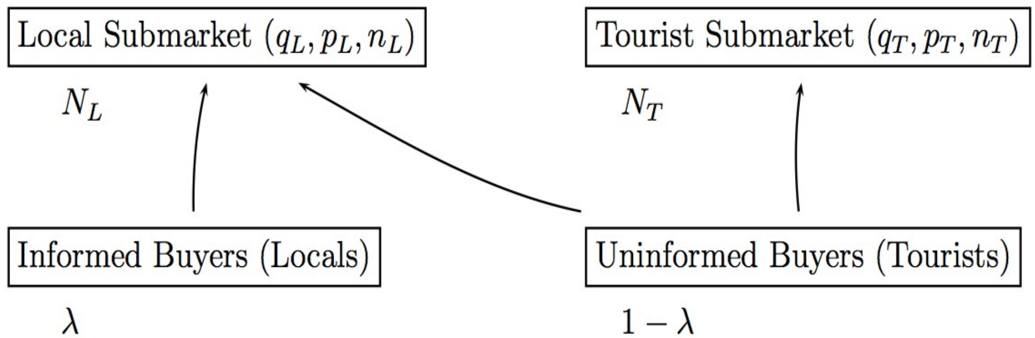


Figure 1: Decentralized market structure

path: a buyer always accepts a seller's first offer, but that is disciplined the threat of rejecting and making a counteroffer. Hence, whether tourist shops involve bargaining or posting is a matter of interpretation, but a key implication is that sellers cannot generally extract all the surplus from buyers.

Fig. 1 depicts the DM structure, where N_L and N_T are the measures of local and tourist shops, and $N = N_L + N_T$. If ω_j denotes the ex ante probability a buyer goes to submarket j, before knowing if he will be informed, then

$$\omega_T = \frac{(1-\lambda)N_T}{N_L + N_T} \quad \text{and} \quad \omega_L = \frac{N_L + \lambda N_T}{N_L + N_T}. \tag{1}$$

Thus, ω_T is the probability of being uninformed times the probability of finding a tourist shop. As there is a unit measure of buyers, ω_j is also the measure of buyers in submarket j. Market tightness in each submarket is

$$n_T = \frac{1-\lambda}{N_T + N_L} \quad \text{and} \quad n_L = n_T + \frac{\lambda}{N_L}. \tag{2}$$

Within a submarket agents meet according to a CRS matching technology: for a seller, the probability of meeting a buyer is $\alpha(n)$; for a buyer the probability of meeting a seller is $\alpha(n)/n$. As usual, $\alpha(n)$ is strictly increasing if $\alpha(n) < 1$, $\alpha(n)/n$ is strictly decreasing if $\alpha(n) < n$, $\alpha(0) = 0$, and $\alpha(\hat{n}) = 1$ for some $\hat{n} \in (0, \infty]$. Also, $\alpha''(n) < 0$ for $n < \hat{n}$.[7]

[7] Notice in the case that $h = \infty$ tourist shops only get tourists, while local shops get locals and tourists that luck out, making $n_L \geq n_T$, with $n_L > n_T$ if $\lambda > 0$. Hence, local submarkets are tighter. A similar result holds when $h = 2$, although in this case informed agents may also visit tourist shops since they only see two draws.

Consider a buyer in the CM (a seller is similar). His state is his net worth, $A = \phi m + \tau - d - \gamma(d)$, where m is money brought from the previous DM, ϕ is the price of m in terms of numeraire, τ is a lump sum transfer used to inject currency, d is debt from the previous DM, and $\gamma(d)$ is the transaction cost from using debt, where $\gamma'(d), \gamma''(d) > 0$ $\forall d > 0$. We assume $\gamma(0) = \gamma'(0) = 0$ to guarantee $d > 0$, since as usual interior solutions simplify the presentation. Also, while the transaction cost here is borne by buyers, the results are similar if it is instead borne by sellers, with one caveat: if the cost is borne by sellers they may in principle want to charge more for using credit, a practice that is sometimes but not always banned by platform rules or state regulations. In any case, we abstract from that complication in this paper.

2.2 Model 1 with Exogenous Information

When λ is fixed, a buyer's CM problem is

$$W(A) = \max_{x,\ell,\hat{m}} \left\{ U(x) - \ell + \beta V(\phi_+ \hat{m}) \right\} \text{ st } x = w\ell - \phi\hat{m} + A,$$

where w is the wage, $\hat{m}$ is money taken out of the CM, and V is the DM value function, depending on real balances at tomorrow's prices, $z \equiv \phi_+ \hat{m}$. We focus on stationary equilibrium, where W and V are independent of time, and to ease notation adopt a CM technology $x = \ell$ since that means $w = 1$. Then, after eliminating ℓ, we get

$$W(A) = A + \max_x \{U(x) - x\} + \max_z \{-(1+\pi) z + \beta V(z)\},$$

where $1 + \pi = \phi/\phi_{+1}$ is inflation, equal to the growth of the money supply in stationary equilibrium. For buyers the FOC for $z > 0$ is $1 + \pi = \beta V'(z)$, and for sellers $z = 0$ because they do not need liquidity in the DM. For both, z is independent of A and $W'(A) = 1$, as usual in these models.

This means buyers' DM trading surplus is $S = u(q) - p - \gamma(d)$. Similarly, sellers' DM surplus is net revenue $R = p - q$, and expected profit is $\Pi = \alpha(n) R - k$, where k is a fixed cost for sellers entering the DM. Let q^* be the efficient quantity defined by $u'(q^*) = 1$. Then, to guarantee some sellers enter, impose

$$k < (1 - \underline{\eta})[u(q^*) - q^*], \tag{3}$$

where $\underline{\eta} = \min_n\{\eta(n)\}$ and $\eta(n) = n\alpha'(n)/\alpha(n)$ the elasticity of matching. As mentioned, equilibrium has at most 2 types of sellers: local shops that set terms to attract the informed buyers; and tourist shops that try to exploit the uninformed. Then by CRS we can assume there is one representative submarket of each type. Also, while (3) implies $N_L > 0$, it is possible to have $N_T > 0$ or $N_T = 0$, as discussed below.

Consider first submarket L. As standard, the outcome can be found by maximizing local buyer's expected surplus subject to free entry by sellers:

$$\max_{p,q,n}\left\{\frac{\alpha(n)}{n}\left[u(q) - p - \gamma(p-z)\right]\right\} \text{ st } \alpha(n)(p-q) = k. \tag{4}$$

Then $\Gamma_L = (p_L, q_L, n_L)$ solves $k = \alpha(n_L)(p_L - q_L)$ plus the FOC's wrt q and n,

$$\begin{aligned} u'(q_L) &= 1 + \gamma'(p_L - z) & (5)\\ p_L - q_L &= \frac{(1-\eta_L)\left[u(q_L) - q_L - \gamma(p_L - z)\right]}{\eta_L u'(q_L) + 1 - \eta_L}. & (6)\end{aligned}$$

By (6), the seller gets a fraction $(1-\eta_L)/[\eta_L u'(q) + 1 - \eta_L]$ of the DM trade surplus.

Consider next submarket T, where sellers bargain, or equivalently post the bargaining outcome. We use Kalai (1977) bargaining, which has several advantages over Nash bargaining in models with liquidity considerations (Aruoba et al. (2007)), and hence is more common in monetary theory. Kalai's solution is found by maximizing the buyer's surplus subject to him getting a share θ of the total surplus:

$$\max_{p,q}\{u(q) - p - \gamma(p-z)\} \text{ st } p - q = (1-\theta)[u(q) - q - \gamma(p-z)]. \tag{7}$$

Now $\Gamma_T = (p_T, q_T, n_T)$ solves $\alpha(n_T)(p_T - q_T) = k$ plus the FOC's

$$\begin{aligned} u'(q_T) &= 1 + \gamma'(p_T - z) & (8)\\ p_T - q_T &= (1-\theta)[u(q_T) - q_T - \gamma(p_T - z)]. & (9)\end{aligned}$$

Conveniently, the conditions for Γ_T are the same as those for Γ_L, except $1-\theta$ replaces $(1-\eta_L)/[\eta_L u'(q) + 1 - \eta_L]$. We assume $\theta \leq \eta(n)\ \forall n$ to guarantee that while buyers can bargain at local shops, they prefer to accept (p_L, q_L).[8]

[8] After finding equilibrium we check (p_L, q_L) is such that buyers do not want to bargain on the equilibrium path or off – i.e., they do not want to bring a different z and bargain.

Buyers' DM payoff is the CM continuation value plus the expected surplus,

$$\begin{aligned} V(z) &= W(z+\tau) + \omega_L \frac{\alpha(n_L)}{n_L} \left[u(q_L) - p_L - \gamma(p_L - z) \right] \\ &\quad + \omega_T \frac{\alpha(n_T)}{n_T} \left[u(q_T) - p_T - \gamma(p_T - z) \right]. \end{aligned} \tag{10}$$

At tourist shops p and q depend on z, but not local shops. Thus,

$$V'(z) = 1 + \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} \left[u'(q_T) q_T' - p_T' - \gamma'(d_T)(p_T' - 1) \right], \tag{11}$$

where q_T' and p_T' are derivatives wrt to z. Inserting (11) into $(1+\pi) = \beta V'(z)$, we get

$$i = \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} \left[u'(q_T) q_T' - p_T' - \gamma'(d_T)(p_T' - 1) \right], \tag{12}$$

where i is a nominal interest rate defined by the Fisher equation $1 + i = (1+\pi)/\beta$. As usual, this means i is the return agents require in the next CM to give up a dollar in this CM, and we can price such trades whether or not they occur in equilibrium.

The Fisher equation makes it equivalent to use i or π as our policy variable. We adopt the usual restriction $i > 0$, or $\pi > \beta - 1$, but consider the limit $i \to 0$, or $\pi \to \beta - 1$, called the Friedman rule. Given all this we have:

Definition 1 *A stationary equilibrium with fixed λ is a nonnegative list $\langle \Gamma_L, \Gamma_T, z \rangle$ such that Γ_j solves the relevant conditions in each submarket given z, and z solves the money demand problem given (Γ_L, Γ_T). It is a monetary equilibrium if $z > 0$.*

While the interest here is in monetary equilibrium, let us mention a few results for pure-credit economies (all proofs are in the Appendix):

Lemma 1 *With fixed λ equilibrium with $z = 0$ exists and is unique. If $N_L, N_T > 0$, so local and tourist shops coexist, then $n_L > n_T$, $p_L < p_T$, $P_L < P_T$, $R_L < R_T$, and $q_L > q_T$.*

When local and tourist shops coexist, the former have lower revenue per unit, $R_L < R_T$, and since $\Pi_L = \Pi_T$ they must make it up on the volume, which means $n_L > n_T$. In fact very similar results hold in monetary equilibrium. To show that, the following is useful:

Lemma 2 *In monetary equilibrium (i) $z < \hat{p} = \max\{p_L, p_T\}$; (ii) $V''(z) < 0 \; \forall z < \hat{p}$.*

From (i), buyers must cash out (spend all they have) in some shops. Since we show below $p_T > p_L$, they for sure cash out in tourist shops, but they may or may not in local shops. From (ii), (12) has unique solution. This is due to costly credit, which is one important reason to include it.[9] Given this we can show:

Proposition 1 *With fixed λ, equilibrium with $z > 0$ exists and is unique if i is not too big. If local and tourist shops coexist, the comparison in Lemma 1 holds.*

In terms of qualitative properties of monetary equilibrium, when it is unique, higher i lowers z, p_j, q_j, raises n_j and d_j, and can raise or lower P_j. This is proved in the Appendix, where Lemma 4 shows how Γ_j varies with z and Lemma 5 shows how z varies with i. These sharp results may be surprising, as it is usually not clear in competitive search models if the solution is monotone or even continuous in parameters; we make progress using the methods of monotone comparative statics (see Menzio et al. (2013b) and Choi (2015) for related applications).

Also, since buyers must cash out in some trades, $N_T = 0$ implies $q_L < q^*$, and $N_T > 0$ implies $q_T < q^*$ but we can have $q_L < q^*$ or $q_L = q^*$ depending on parameters. Fig. 2 shows the outcomes in (λ, i) space. Area $\mathcal{A}_1$ has $N_L, N_T > 0$, with credit used in tourist but not local shops, as i is sufficiently low that buyers hold enough cash to pay p_L. Area $\mathcal{A}_2$ has $N_L, N_T > 0$, with credit used in all shops, as i is higher and buyers hold less cash. Area $\mathcal{A}_3$ has $N_T = 0$, as λ is high enough to eliminate tourist shops. Also, for a given λ there is a bound for i above which monetary equilibrium breaks down.[10]

[9] In particular, $V(z)$ is twice differentiable with $V''(z) < 0$ due to a smooth cost of credit $\gamma(d)$, which avoids an indeterminacy of monetary steady states analyzed in a series of papers following Green and Zhou (1998). See Jean et al. (2010) for details, but to understand the idea, heuristically, consider indivisible DM goods. If sellers think all buyers bring $m = X$ to market, they all post $p = X$ as long X is not too small; and if they all post $p = X$, buyers all bring $m = X$ as long as X is not too big. So $p = m = X$ is an equilibrium for any X in some range. A similar indeterminacy arises with divisible goods. These problems are eliminated, however, with costly credit. Intuitively, for the case with indivisible goods, even if all sellers charge $p = X$, a buyer can bring $m < X$ and put the difference on his credit card. The demand for money is then driven by the desire to reduce the costly use of credit.

[10] The calibration below puts us in $\mathcal{A}_2$, but even before that an empirical point can be made about the finding in Benabou (1992b) of a negative relation between markups and inflation. While the impact of i

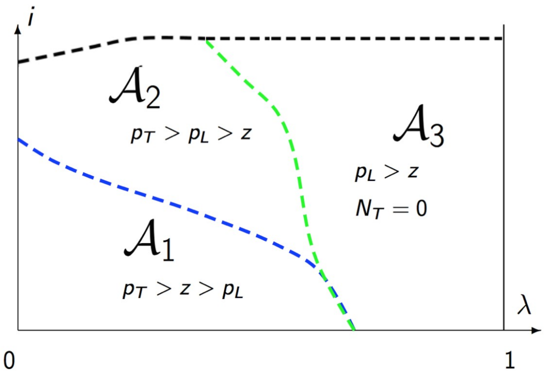


Figure 2: Monetary equilibrium regions in (λ, i) space.

Now consider efficiency. Since CM payoffs are constant with respect to the interventions considered here, welfare is well measured by the sum of DM payoffs net of entry, production and transaction costs:

$$\Omega = \sum\nolimits_j \omega_j \left\{ \frac{\alpha(n_j)}{n_j} \left[u(q_j) - q_j - \gamma\left(p_j - z\right) \right] - \frac{k}{n_j} \right\}. \tag{13}$$

For two reasons $N_T > 0$ is undesirable. First $q_T < q_L$, so tourist shops are inefficient compared to local shops on the intensive margin. Second $n_T < n_L$, so with $\alpha\left(n\right)$ concave the number of DM meetings is not maximized, which is inefficient on the extensive margin. Hence, it would be good to regulate or tax tourist shops away. However, we do not take this too seriously, as it may be hard to identify them or dictate their terms of trade.

As regards monetary policy, note that $q_L, q_T \rightarrow q^*$ as $i \rightarrow 0$, but we also have to consider the effect on market composition, the mix of N_L and N_T. For $\lambda = 1$ or $\lambda = 0$ the optimal policy is $i = 1$, but that may not be true if $\lambda \in (0,1)$. To see this, consider area $\mathcal{A}_1$ in Fig. 2, where higher i has two opposing effects: (1) z and hence the surplus in submarket T fall; and (2) N_T falls, making buyers less likely to end up at tourist shops. There is an area $\mathcal{A}_1^* \neq \varnothing$ in the lower right part of $\mathcal{A}_1$ where the net effect is positive, since N_T is small and hence the loss from reducing the surplus is dominated by the gain from downsizing submarket T.

on P_j is ambiguous, even if $\partial P_T/\partial i > 0$ and $\partial P_L/\partial i > 0$, average P in the DM can fall with i since N_T goes down and N_L up. Also, average P across the DM and CM can fall since the former shrinks. We show later how this pans out numerically, but the point is that it matters *which* markup we consider.

While many models imply $i = 0$ is optimal, others have shown $i > 0$ is optimal in different environments for different reasons, but we think the idea here is novel and somewhat compelling. To pick on a straw man, our result is less trite than saying, e.g., people smoke too much, and since cigarettes are often cash goods, inflation is beneficial for health reasons. That is trite without explaining why cigarettes cannot be taxed directly or purchased on credit. In our environment, it is natural to think it is difficult to identify and hence tax tourist shops directly, and it is an equilibrium outcome, not an assumption, that they use more cash.

Similar second-best logic to these results on monetary policy can be applied to credit conditions. From an individual's perspective, both money and credit are costly, but from a social perspective the former is not. This is because the revenue from inflation can be rebated to tax payers, while the resources required for credit constitute deadweight loss, so individuals tend to use too much credit and too little cash, and a higher cost of credit mitigates that. In fact, other general equilibrium effects can make welfare go up even in the case where the cost of credit is rebated to agents. To make this precise, define a ranking by saying $\gamma_2(d)$ is more costly than $\gamma_1(d)$ when $\gamma_2^{-1}(\xi)$ is weakly flatter than $\gamma_1^{-1}(\xi)$ $\forall \xi > 0$. Then we can get several comparative results on the cost of credit, some of which are now summarized along with the effects of monetary policy:

Proposition 2 *Money: for fixed λ, if it is near 0 or 1 then $i = 0$ is optimal, but if $(\lambda, i) \in \mathcal{A}_1^*$ then $i > 0$ is optimal. Credit: if γ becomes more costly, Ω rises for λ near 0 or 1 but falls for $(\lambda, i) \in \mathcal{A}_1^*$.*

To mention one more result before endogenizing λ, note that several well-known papers predict that prices and/or price dispersion fall with increases in buyer information (Salop and Stiglitz (1977); Varian (1980); Burdett and Judd (1983); Stahl (1989)). Yet as Ellison and Ellison (2005) say "evidence from the Internet... challenged the existing search models, because we did not see the tremendous decrease in prices and price dispersion that many had predicted." Similarly, Baye et al. (2006) say "Reductions in information costs over the past century have neither reduced nor eliminated the levels of price dispersion observed." While our framework is not designed to apply specifically to Internet shopping, it speaks to the issues.[11] If $z = 0$ one can easily check that higher λ lowers prices, consistent with nonmonetary models. In monetary equilibrium, however, this can be overturned because money demand changes with λ.

To see this, first, it is easy to show Γ_j depends on z, but not directly on λ or other endogenous variables, and $\partial z/\partial\lambda \geq 0$ iff $\partial p_j/\partial\lambda \geq 0$. Examples show z can be increasing or decreasing in λ. So more information can raise or lower prices measured by p_j, which again makes sense if q is unobserved. Alternatively, measuring prices by $P_j = p_j/q_j$, the results are less clear since q_j and p_j co-move with z, but examples show it can also rise or fall with λ. The reason is simple: for some parameters (e.g., low θ), higher λ implies buyers bring more money to the DM, and when they have more money sellers charge higher prices. So higher λ need not reduce prices. It is even easier to see that higher λ need not reduce price dispersion: $\lambda = 0$ and $\lambda = 1$ both imply no dispersion; $\lambda \in (0,1)$ implies dispersion; so dispersion is nonmonotone in λ. The general message is that it one needs to be careful saying what search theory predicts about information and prices or price dispersion.

2.3 Model 1 with Endogenous Information

Although points can be made with λ fixed, the above presentation is also a stepping stone to a framework with endogenous information. While there are different ways to proceed, here buyers choose λ at cost $s(\lambda) \geq 0$, with $s(0) = 0$, $s' > 0$ and $s'' \geq 0$. Thus, investing in information (talking to more people, reading more newspapers, etc.) in the CM makes them more likely to know where to get good deals on any goods they may want or need when they go to the DM. Notice the information externality: the uninformed benefit when there are more informed buyers because that reduces the number of tourist shops. Also note that agents' DM information status is still random, as they are still informed with probability λ, but now that is a choice variable. Further,

[11] One might say online shopping does not use much cash, but for us monetary transactions include check, debit and paypal. All these payment methods are monetary in the sense that one must work to get purchasing power before spending, and purchasing power held as demand deposits, like cash, bears approximately zero interest. This is distinct from credit, where one first spends, then works to pay it off.

as a benchmark, let us assume the realization occurs after buyers leave the CM, so as long as they all choose the same λ they again choose the same z.[12]

The terms of trade are determined exactly as before, but now each buyer chooses (z, λ) to maximize the expected DM payoff

$$\begin{aligned} V(z) &= W(z+\tau) + \omega_L \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z)] \\ &\quad + \omega_T \frac{\alpha(n_T)}{n_T}[u(q_T) - p_T - \gamma(p_T - z)] - s(\lambda). \end{aligned}$$

The choice of z is still determined by (12). For the choice of λ, first notice from (1) that the marginal impact on the probability of entering submarket L is $\partial\omega_L/\partial\lambda = \hat{\omega}_T \equiv N_T/\left(N_T + N_L\right)$. Hence, it solves the FOC

$$s'(\lambda) = \left\{ \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z)] - \frac{\alpha(n_T)}{n_T}[u(q_T) - p_T - \gamma(p_T - z)] \right\} \hat{\omega}_T. \tag{14}$$

Definition 2 *A stationary monetary equilibrium with endogenous information is a non-negative list $\langle \Gamma_L, \Gamma_T, z, \lambda \rangle$ solving the conditions in Definition 1 plus (14).*

There is always an equilibrium where $\lambda = N_L = 0$, but the interest here is in equilibrium with $\lambda > 0$. Notice $s' > 0$ implies $\lambda < 1$ for standard reasons: if $\lambda = 1$ then there are only local shops, so $\lambda = 1$ is not a best response. The next result establishes existence with $\lambda \in (0, 1)$ and gives conditions for uniqueness.[13]

Proposition 3 *Monetary equilibrium with endogenous $\lambda \in (0, 1)$ exists and it is unique if i and $s'(0)$ are not too big. If local and tourist shops coexist, the comparison in Lemma 1 holds.*

[12] We also solved the case where buyers are informed for sure if they pay a fixed cost. Typically equilibrium has both informed and uninformed buyers, and they carry different z. In that version $i > 0$ is optimal in any equilibrium with $\lambda > 0$; in the benchmark model $i > 0$ can be optimal, too, but it depends on parameters. Also, a referee suggested it may be interesting to have persistent heterogeneity in z. One way to get that is to add a one-time cost to becoming informed; that does not affect the results much. Another is to have multiple rounds of DM trade before the CM convenes; which affects some quantitative results, as discussed briefly below, with details in the Supplementary Appendix.

[13] In general one might expect multiplicity. If λ is higher there will be fewer uninformed buyers, that leads to fewer tourist shops, and that makes agents carry less cash. But then tourist shop profit falls, so n_T rises, and hence being uniformed is worse. Hence buyers want to acquire more information. Moreover, if there are multple equilibria, they can be ranked by $(z, \hat{\omega}_T)$ (see the proof of Proposition 3). While other information-based theories display multiplicity, this is different, and again relies on interaction between information and money; still, we prefer to focus on other issues and not dwell on it .

In terms of qualitative properties, when monetary equilibrium is unique, higher i lowers z, p_j, and q_j, raises n_j and d_j, and can raise or lower P_j. These sharp results, verified in Lemma 6 in the Appendix, are the same as Section 2.2, with fixed λ, but now there are additional findings. For one, higher i lowers $\hat{\omega}_T$, the probability a tourist ends up at a tourist shop (the analogous effect is ambiguous with fixed λ). For another, we can show the impact on all these variables of a lower marginal cost of information is the same as the effect of higher i.

The next result shows that $i > 0$ is optimal when the cost of information acquisition is low, where the economic intuition is similar to that behind Proposition 2.

Proposition 4 *Money: With endogenous λ, $i > 0$ is optimal when $s'(\lambda)$ is small $\forall\lambda$. Credit: If γ becomes more costly, Ω falls when i is small and $s'(\lambda)$ is small $\forall\lambda$.*

2.4 Model 2 with Exogenous Information

Until now informed buyers see every posted (p, q). Suppose instead they see exactly two random draws, the simplest version of what Burdett and Judd (1983) call noisy search. Interestingly, this version yields a very different type of price dispersion: a continuous distribution of posted terms.[14]

While sellers can in principle post any (p, q), without loss of generality we can assume they simply post their surplus, $R = p - q$. To see this, define the bilateral frontier by letting $\Sigma(R)$ be a buyer's maximal surplus given a seller's surplus is R:

$$\Sigma(R) \equiv \max_{p,q} \{u(q) - p - \gamma(p - z)\} \quad \text{st} \quad p - q = R. \tag{15}$$

Sellers only post (p, q) on this frontier, along which each point corresponds to a unique R; then, given R, we can find the implied $p = p(R)$ and $q = q(R)$. For $z < R + q^*$, the solution is given by $p = R + q$ and the FOC

$$u'(q) = 1 + \gamma'(R + q - z), \tag{16}$$

[14] Acemoglu and Shimer (2000) study a similar setup but without the details we need (e.g., they do not have divisible goods, money or costly credit). Still, some results are similar.

which indicates that $q(R)$ and $\Sigma(R)$ decrease while $p(R)$ increases with R. For $z \geq R + q^*$, the solution is $q = q^*$ and $p = R + q^*$.

Sellers posting higher R have a lower probability of trade. If $n(R)$ is market tightness for sellers that post R, the free entry condition

$$\alpha\,[n(R)]\,R = k \tag{17}$$

holds for each R posted in equilibrium. Buyers visit the seller that offers the highest expected payoff $\alpha\,[n(R)]\,\Sigma(R)/n(R)$. Let $R_L = p_L - q_L$ be a seller's trade surplus with directed search, the same as local shops' surplus in Section 2. Lemma 7 in the Appendix shows buyers' expected payoff is strictly quasi-concave and maximized at R_L. As a result, no seller posts any $R < R_L$, because R_L attracts more buyers and yields more surplus per trade. Therefore, only $R \geq R_L$ will be posted.

Let $F\,(R)$ be the CDF of posted R. When λ is large, competition creates a mass point at R_L. Thus, there exists λ^* such that $\lambda \geq \lambda^*$ implies all sellers post R_L, and $F\,(R_L) = 1$. For $\lambda < \lambda^*$, some sellers post $R > R_L$ to take advantage of the uninformed buyers, and $F(R)$ has an atomless part in an interval $[\underline{R}, \bar{R}]$, plus potentially a mass point at $R_L \leq \underline{R}$. In equilibrium $\bar{R} = R_T$, where $R_T = p_T - q_T$ is a seller's surplus under bargaining, as at a tourist shop in Section 2 because: if a seller posts $\bar{R} > R_T$ buyers opt to bargain; if he posts $\bar{R} < R_T$ he can profitably deviate to R_T. Since sellers that post R_T only trade with uninformed buyers, the buyer-seller ratio is $n_T = (1-\lambda)/N$. By free entry, $\alpha(n_T)R_T = k$, the measure of sellers is $N = (1-\lambda)/\alpha^{-1}(k/R_T)$.

The size of the mass point at R_L can be derived from free entry, $\alpha(n_L)R_L = k$. For a seller that posts R_L, tightness is

$$n_L = \frac{1-\lambda}{N} + \frac{2\lambda}{N}[1 - F(R_L)] + \frac{2\lambda}{N}\frac{F(R_L)}{2}. \tag{18}$$

The first term on the RHS is the measure of uninformed buyers that sample this seller; the second is the measure of informed buyers who sample him plus one other seller, where the other one posts $R > R_L$; and the third is the measure of informed buyers who sample him plus one other seller, where the other one posts R_L. It follows that the size of the mass point μ is

$$\mu \equiv F(R_L) = \max\left\{2 - \frac{(1-\lambda)}{\lambda}\left[\frac{\alpha^{-1}(k/R_L)}{\alpha^{-1}(k/R_T)} - 1\right], 0\right\}, \tag{19}$$

where the max operator takes care of the case when F is atomless.

As is standard, one can show $F(R)$ is continuous and atomless for $R \in [\underline{R}, \bar{R}]$.[15] Then a seller posting $R \in [\underline{R}, \bar{R}]$ can be interpreted as facing a buyer-seller ratio

$$n(R) = \frac{1-\lambda}{N} + \frac{2\lambda}{N}[1 - F(R)]. \tag{20}$$

By (17), (20) and $F(R_T) = 1$, we have

$$F(R) = 1 - \frac{1-\lambda}{2\lambda}\left(\frac{\alpha^{-1}(k/R)}{\alpha^{-1}(k/R_T)} - 1\right). \tag{21}$$

The lower support of the atomless part $\underline{R}$ solves $F(\underline{R}) = \mu$, or

$$\underline{R} = \frac{k}{\alpha(\alpha^{-1}(k/R_T)[\frac{2\lambda(1-\mu)}{1-\lambda} + 1])}. \tag{22}$$

We summarize as follows:

Lemma 3 *For $\lambda \geq \lambda^* \equiv 1 - n_T/n_L$, all sellers post R_L. For $\lambda < \lambda^*$, a fraction μ of sellers post R_L where μ is given by* (19). *Other sellers post $R \in [\underline{R}, \bar{R}]$ where $\bar{R} = R_T$ and $\underline{R} \geq R_L$ solves* (22). *For $R \in [\underline{R}, \bar{R}]$, F is given by* (21).

To derive money demand, first, let $\tilde{F}$ be the distribution function of the lowest of two draws from F, $\tilde{F}(R) = (1-\lambda)F(R) + \lambda\left\{1 - [1 - F(R)]^2\right\}$. Then

$$V(z) = W(z+\tau) + \int \frac{\alpha[n(R)]}{n(R)}\{u[q(R)] - p(R) - \gamma[p(R) - z]\}\, d\tilde{F}(R).$$

The FOC wrt z is given by

$$i = \int \frac{\alpha[n(R)]}{n(R)}\gamma'(p(R) - z)d\tilde{F}(R). \tag{23}$$

Definition 3 *With noisy search, a stationary monetary equilibrium is a list $\langle F, z\rangle$ where F is characterized by Lemma 3 and $z > 0$ solves* (23).

[15] If F has a mass point at $R' \in [\underline{R}, \bar{R}]$, a seller posting R' has a profitable deviation to $R' - \varepsilon$ for $\varepsilon > 0$. If F has a flat spot between R' and $R'' > R'$, a seller posting R' has a profitable deviation to R''.

Proposition 5 *Stationary monetary equilibrium with noisy search exists and is unique if i is not too big. It implies $\partial p(R)/\partial R > 0$, $\partial q(R)/\partial R \leq 0$ and $\partial n(R)/\partial R < 0$ $\forall R \in [R_L, R_T]$.*

As regards welfare, similar to (13), we have

$$\Omega = N \int_{\underline{R}}^{\bar{R}} \alpha\left[n\left(R\right)\right] \Sigma(R) dF(R) = \int_{\underline{R}}^{\bar{R}} \frac{\alpha\left[n\left(R\right)\right]}{n(R)} \Sigma(R) d\tilde{F}(R). \tag{24}$$

Now in any equilibrium with price dispersion $i = 0$ is suboptimal:

Proposition 6 *Assume $\lambda < \lambda^*$. In stationary monetary equilibrium with noisy search, near $i = 0$, as λ or i rise, z falls, the p distribution falls in the sense of first-order stochastic dominance, and Ω rises.*

As λ rises, price competition among sellers gets stronger and prices fall, so buyers are better off. As i rises, buyers carry less z and that has two effects: buyers use more credit, but the cost of that is small near $i = 0$; and sellers post lower prices, which improves welfare. Altogether Ω is maximized at $i > 0$ whenever price dispersion arises because F is continuous around R_T. In Section 2, raising i from 0 hurts the entire tourist submarket, but now it only impacts sellers posting R_T. The measure of such sellers is 0, so this loss is small when the R distribution is continuous.

2.5 Model 2 with Endogenous Information

As in Section 2.3 assume buyers can choose λ by paying $s(\lambda)$. Given (λ, z) Lemma 3 still holds, and λ solves the FOC $s'(\lambda) = \partial\Omega/\partial\lambda$, with Ω given by (24). When a buyer chooses a higher λ, then the marginal impact on the distribution $\tilde{F}$ is

$$\frac{\partial \tilde{F}(R)}{\partial \lambda} = F(R)[1 - F(R)].$$

Given this, the FOC for λ can be written

$$s'(\lambda) = \int \frac{\alpha\left[n(R)\right]}{n(R)} \left\{u\left[q(R)\right] - p(R) - \gamma\left[p(R) - z\right]\right\} [1 - 2F(R)] dF(R). \tag{25}$$

Definition 4 *With noisy search, a stationary monetary equilibrium with endogenous information is a list* $\langle F, z, \lambda \rangle$ *solving the conditions in Definition 3 plus (25).*

This model has a new channel for possible multiplicity. In Model 1, individuals always have less incentive to become informed when others choose higher λ, because that reduces the fraction of tourist shops; here the marginal benefit of λ is not necessarily decreasing in aggregate λ. Intuitively, buyers have more incentive to acquire information when there is more price distribution, and with noisy search dispersion is nonmonotone in aggregate λ, suggesting that it is possible to have multiple equilibria even holding z fixed. Hence a stronger condition is needed for uniqueness.

Proposition 7 *Stationary monetary equilibrium with noisy search exists and is unique if* i *is not too big and* $s'(\lambda)$ *is sufficiently small* $\forall \lambda \in [0, 1]$. *The second claim in Proposition 5 remains true.*

When equilibrium is unique, around $i = 0$, both z and λ fall with i. Buyers choose lower λ because as z falls all shops post lower prices, which leads to a drop in the aggregate price dispersion, and so buyers have less incentive to acquire information. Letting $\eta(n) \equiv \alpha'(n)n/\alpha(n)$ be the elasticity of the matching function, we have:

Proposition 8 *In stationary monetary equilibrium with noisy search, near* $i = 0$*, as* i *rises,* z *and* λ *fall provided that* $\eta(n)$ *falls in* n.

Lower z and lower λ have opposing effects on prices and welfare, and since z and λ both fall with i here, the overall impact is ambiguous.

3 Quantitative Results

We now quantify the effects of inflation, information, and credit conditions, three key facets of the above analysis. Both Models 1 and 2 are used, with λ endogenous, although other specifications are discussed. All experiments use the same calibration strategy and targets unless otherwise noted.

3.1 Calibration

The period length is set to a year, although this does not matter much.[16] Then we set $\beta = 1/(1+r)$ with $r = 0.03$. The CM and DM utility functions are $U(x) = \log(x)$ and $u(q) = Bq^{1-b}/(1-b)$, with (b, B) set to match aggregate money demand, i.e. the empirical relationship between nominal interest rates and a measure of money scaled by output, M/PY. With $U(x) = \log(x)$, real CM output is $x^* = 1$ (a normalization), while DM output in the same units from submarket j is $\alpha(n_j)N_j[p_j - \gamma(p_j - z)] - N_j k$. Aggregate output sums these, while M/P is given by the equilibrium value of z, and both depend on i, as analyzed above.

Our measure of money is $M1$. The rationale is that checks and debit cards are about as liquid as currency, and they are backed by deposits that must be accumulated before expenditures, while credit means buying now and paying later. The best $M1$ data is the $M1J$ series from Lucas and Nicolini (2015), which augments the usual series with money market accounts after regulatory amendments in 1980 made them about as liquid as checking accounts. They provide annual observations from 1919 to 2008 and show the empirical money demand relationship is stable over the sample using the nominal rate on T-bills. To fit the data with $u(q) = Bq^{1-b}/(1-b)$, intuitively, changing B shifts the curve up or down and is set to match a mean M/PY of 0.27, while b captures the elasticity and is set to minimize the sum of squared residuals between model and data.

We assume $\gamma(d) = Cd^c$, where willingness to substitute between money and credit is captured by c, and the share of credit purchases by C. Letting D be credit averaged across all purchases, and emulating the procedure for money demand, (c, C) is set to match the empirical relation between D/Y and i. For data we use consumer credit

[16]To see why, consider the simplest job search model where V_0 and $V_1(w)$ are the values of unemployment and employment at wage w, α is the arrival rate of jobs, and κ is a search cost. Then $rV_0 = \alpha[V_1(w) - V_0] - \kappa$. To change from, e.g., a weekly to a monthly model, we can simply multiply r, α, κ and w by 4 without changing payoffs or observables like the unemployment rate and hourly wages – the only caveat is we must respect $\alpha \leq 1$, which is not a problem when moving to higher frequencies. The same idea applies here. In particular, with shorter periods agents get to rebalance z more often, but the lower arrival rates imply they hold cash for just as long on average. Now one might take issue with an annual model because it means households make at most one DM purchase per year, but if that is problematic, simply interpret a household as a collection of many buyers as in Shi (1997) (and similar to macro-labor, where a firm is interpreted as a collection of many vacancies).

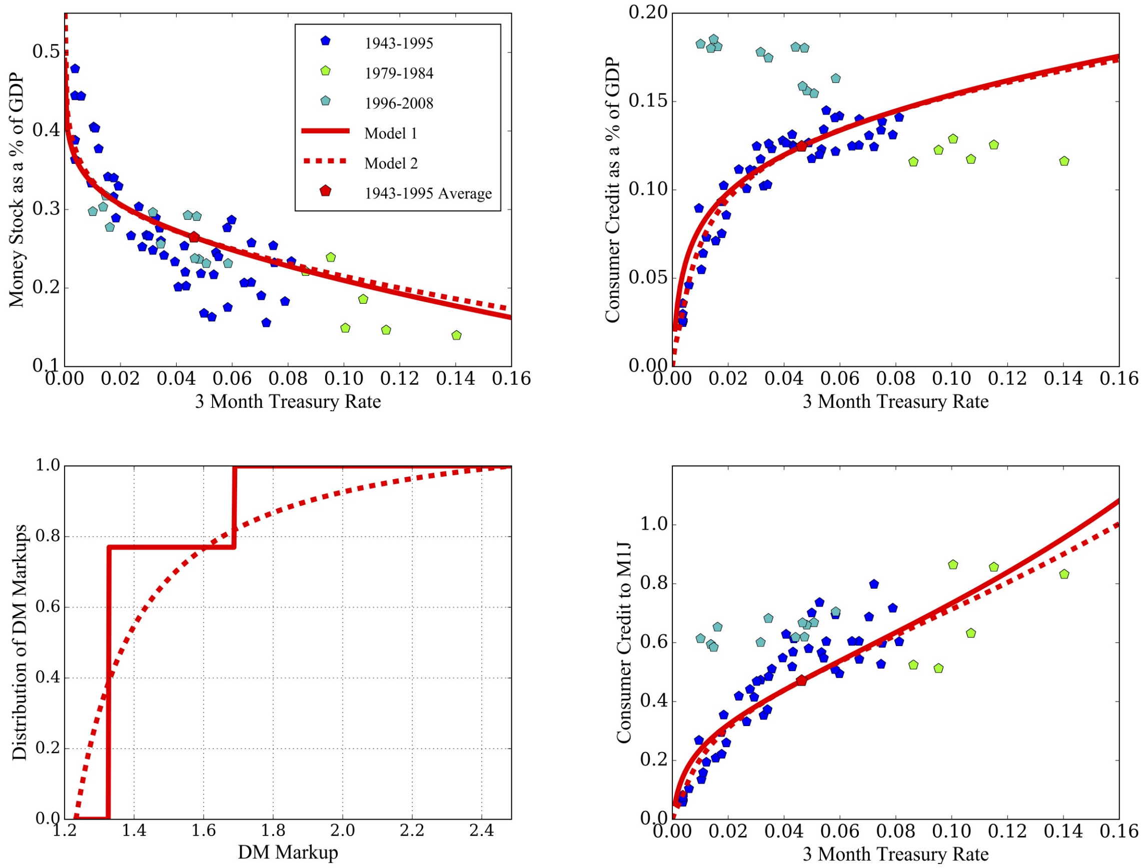


Figure 3: Estimated money and credit demand curves.

on household, family and other personal expenditures exclusive of loans secured by real estate, which is appropriate as this largely supports retail trade, and provides a reasonably long time series (see FRB's G.19 consumer credit release, FRED Series TOTALSL). Note we do not calibrate to micro payment data, but it turns out we match this well: in equilibrium about 30% of DM transactions by value are made with credit, in the middle of the range of numbers in Boston Fed and Bank of Canada survey and diary data (see Liu et al. (2019) and references therein to primary sources).

Estimated money and credit demand curves are shown in Fig. 3. Note that the demand for credit is increasing in i because this is the nominal, not the real, interest rate, and that is effectively the cost of using cash. For money demand (top-left panel) the relationship looks stable and the fit is good, consistent with Lucas and Nicolini (2015). For credit demand (top-right) the result is reasonable given there is apparently a structural break in the 1990s. Understanding that break is beyond the scope of this paper, but it is important to say that including or excluding observations post 1996 does not change the results much (see the Supplementary Appendix). Given this, we omit data post 1996, but again that matters little for the main conclusions.

The crucial factor for us is substitutability between money and credit, depending on i, and that is captured well, as shown by the relationship between D/z and i (bottom-right). This relationship is reasonably stable and displays clear movement out of cash and into credit as i rises. While we slightly over predict credit demand for high nominal rates in the early 1980s, the fit is otherwise quite good. We also mention that the aggregate cost of credit is small, with $\gamma/Y = 0.3\%$ and $\gamma/D = 2.0\%$, roughly in line with interchange fees on credit cards. Based on all this, we think the model does well at accounting empirically for the use of money and credit.

The cost of acquiring information is assumed quadratic, $s(\lambda) = E\lambda^2/2$, so marginal cost is linear with slope E. We set E, which in turn disciplines λ, to match the average retail markup. In retail survey data (see https://www.census.gov/retail) the average ratio of gross margins to sales from 1992-2008 is 0.28, implying an average markup of $1 + 0.28/(1 - 0.28) = 1.39$. This delivers a value of $\lambda = 0.39$ in Model 1 and $\lambda = 0.63$ in Model 2, and in both cases the implied resources spent on information in equilibrium is around 4.2% of output. Note that the fraction of informed buyers is larger in Model 2 because there the informed see only two, not all, prices, and so matching the average markup requires a larger λ.

We use a common DM matching technology, $\alpha(n) = n/(1+n)$. Buyers' bargaining power in tourist shops is θ, and their effective bargaining power in local shops, where they accept the terms of trade without negotiating, is $\hat{\theta} = u'(q)/\left[u'(q) + n\right]$. Price dispersion is disciplined by the difference, $\theta - \hat{\theta}$, which depends on the measure of informed buyers λ. We set θ to match the relative standard deviation in retail prices of 15.5% found in Kaplan et al. (2019) (using the alternative of 19% in Kaplan and Menzio (2015) gives very similar results). In Model 1, this implies $\theta = 0.72$ and $\hat{\theta} = 0.92$, which means even at tourist shops the uninformed get a considerable surplus. In Model

2, $\theta = 0.31$ and $\hat{\theta} = 0.85$, which means high-priced tourist shops are very costly, although since λ is larger buyers run into these shops less often. Fig. 3 (bottom-left) plots the distribution of DM markups, which in Model 1 are 1.3 and 1.7 in submarkets L and T, respectively, and in Model 2 range from 1.2 to 2.4.[17]

Finally, entry cost k is set to get an aggregate markup across the CM and DM of 1.1, as standard in macro going back to Basu and Fernald (1997). Since our DM and CM markups are 1.39 and 1.0, k gets the size of the DM to match the average trade-weighted markup. To be clear, we do not calibrate the size of the CM and DM – like the size of the DM submarkets L and T, they are implied by observable targets. It turns out that the DM contributes about $1/4$ of Y, with around $1/4$ of that coming from tourist shops and the rest from local shops.

Description	Model 1	Model 2	Source/Target
DM utility curvature, b	0.67	0.58	$(z/Y, i)$ relationship
DM utility level, B	0.56	0.59	avg. z/Y
Credit cost curvature, c	5.29	6.31	$(D/Y, i)$ relationship
Credit cost level, C	12.28	82.12	avg. D/Y
Cost of information level, E	0.052	0.046	retail markup 40%
Sellers' entry cost, k	0.020	0.014	agg. markup 10%
Bargaining power, θ	0.72	0.31	avg. price dispersion

Table 1: Calibrated parameter values

Table 1 shows the calibrated parameters. Section 3.5 and a Supplementary Appendix check robustness – e.g., cutting the price dispersion and markup targets in half does not affect the too much, suggesting they are not overly sensitive to measurement issues along these lines. It is also worth emphasizing there are not many parameters, considering the theory has a lot of ingredients, and they are all tied down by reasonable targets.

[17] Note that our range for markups is actually quite close to the data, where at the low end are Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, while at the high end are Specialty Foods, Clothing, Footwear and Furniture. Although we do not push this, it would not be a big stretch to think these low-markup (high-markup) stores in a loose sense capture our local (tourist) shops – e.g., at least some of us buy gas all the time and have a good feel for the prices at different vendors, but buy footwear much less often and hence search randomly.

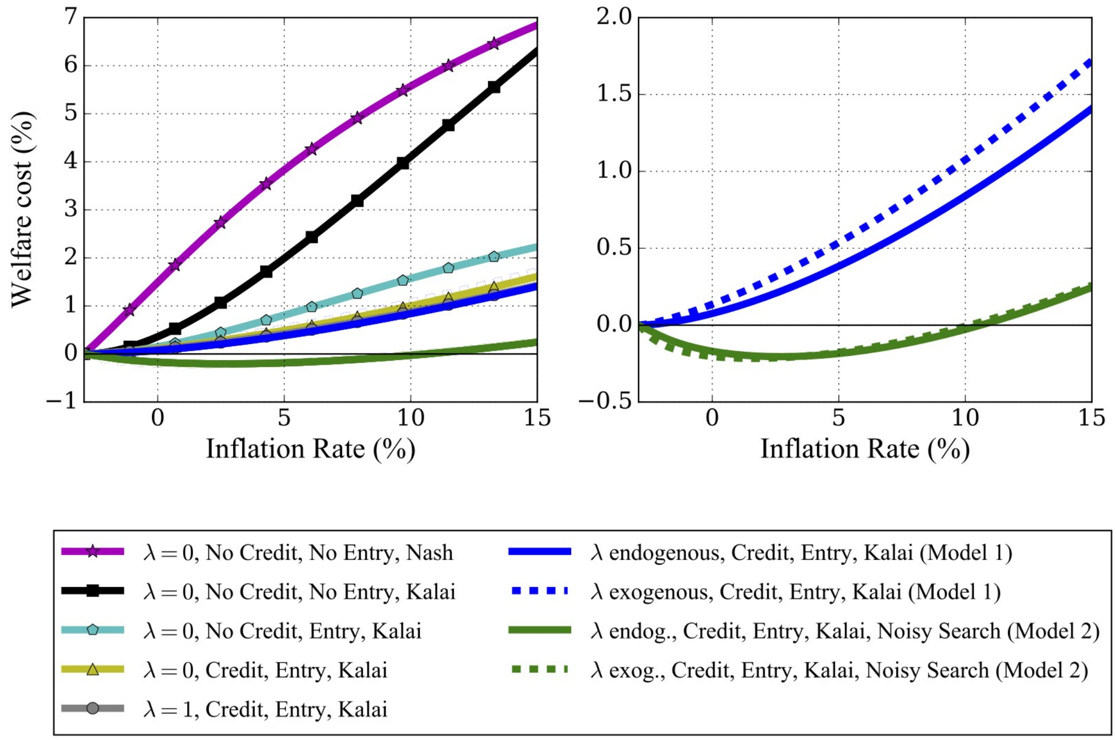


Figure 4: Cost of inflation: Model comparisons.

3.2 Inflation

As is standard, we compute the equilibrium payoff Ω at a given inflation rate $\tilde{\pi}$, typically 10%, then compute the percentage reduction in total consumption agents would accept to reduce $\tilde{\pi}$ to the value π^0 consistent with $i = 0$, which is $\pi^0 = -0.029$ in our calibration. Fig. 4 shows this for $\tilde{\pi}$ varying from π^0 to 15%. However, while this may be natural when π^0 is optimal, as it is in many models, that may not be the case here. Hence, we also consider other measures (e.g. the cost of having the average $\hat{\pi}$ in the data rather than π^*), but begin with the usual cost of 10% inflation rather than π^0. The result 0.8% of consumption in Model 1, and approximately 0 in Model 2.

These numbers are much lower than what one gets with only random search, and one might guess this comes from a calibrated λ close to 1, but that is not the case: it is 0.38 in Model 1 and 0.63 in Model 2. To understand this, Fig. 4 shows the welfare cost for our baseline models with endogenous λ, plus versions with $\lambda = 0$ and $\lambda = 1$ (calibrated using a similar procedure, except we have to drop some targets). Even at $\lambda = 0$ our results are lower than previous papers with pure random search, such as Lagos and Wright (2005), for several reasons: we use Kalai rather than Nash bargaining; we have entry; we allow credit; and our calibration targets differ. Eliminating the first three differences, the results get closer: at $\lambda = 0$, with no credit or entry, and using Nash bargaining, we get 5.6%, compared to 6.9% for a similar specification in Lagos and Wright (2005), with the residual due to calibration targets.

To decompose this further, using Kalai instead of Nash bargaining reduces the cost from 5.6% to 4.0%, adding entry reduces it further to 1.6%, and allowing credit brings it to 1.0%, in either Model 1 or 2; then endogenizing λ brings it to 0.8% and 0 in Models 1 and 2, respectively. This helps us understand how each component of the model contributes to the results. Also shown are results with λ fixed as π varies, which are slightly higher (intuitively, because higher π increases the benefit of information, and hence λ, when it is endogenous). Thus, the difference in the cost of 10% inflation with λ endogenous and exogenous is 0.3% in Model 1 and near 0 in Model 2.

In both versions optimal inflation is $\pi^* > \pi^0$, but for Model 1 this is hard to see in the chart, as π^* is very close to π^0. It is clear in Model 2, however, where $\pi^* = 2.7\%$ is well above the Friedman rule and – interestingly enough – not far from inflation targets at many central banks. This difference across models is due to the way calibration determines bargaining power, as discussed in Section 3.1. In Model 1 buyers get decent deals at tourist shops because $\theta = 0.72$, so the distortion is not so bad, even though $\lambda = 0.39$ implies they run into tourist shops frequently. In Model 2, calibrating to the same targets yields a much lower value of $\theta = 0.31$, which means buyers get very bad deals at tourist shops, and that matters a lot even if $\lambda = 0.63$ implies they run into them less frequently. In particular, there is a big difference across models in the highest markups observed in equilibrium, which are 1.7 in Model 1 and 2.4 in Model 2. Given these high markups at some tourist shops in Model 2, the gain from taxing them by inflation is large and hence π^* is high. One message is that the micro details of market

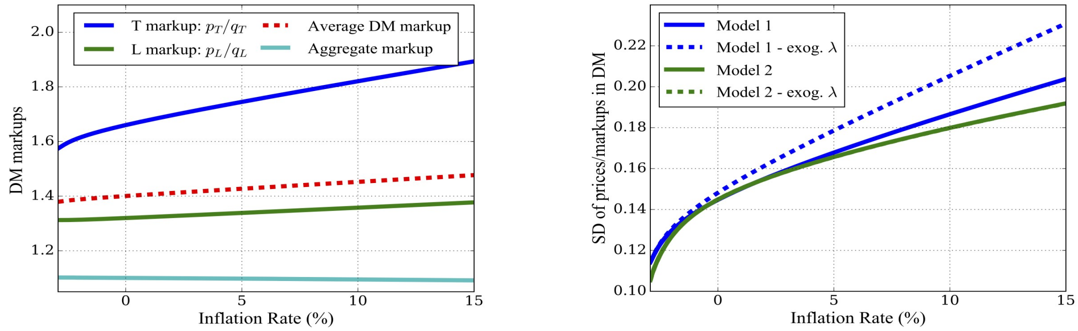


Figure 5: Effects of inflation on markups (in Model 1) and price/markup dispersion.

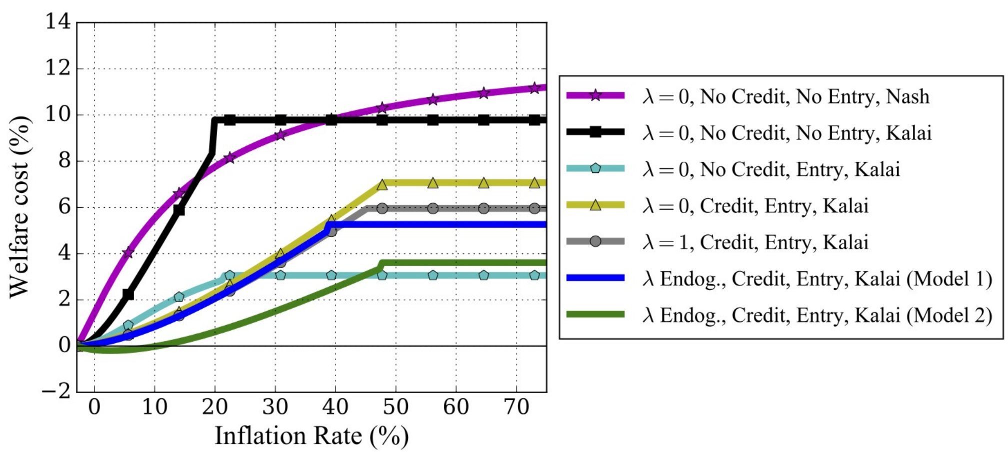
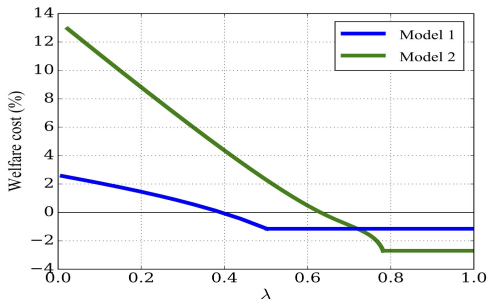


Figure 6: Cost of hyper-inflation: Model comparisons.

structure can make a major difference in quantitative results.

The left panel of Fig. 5 shows the effects of π on markups and price dispersion. Notice DM markups rise with π, counter to some empirical findings. However, since the DM shrinks with π, the average markup falls, although the effect is not big. The right panel shows DM price dispersion increases with π with the calibrated parameters. For completeness, Fig. 6 shows the welfare cost as inflation gets much higher. Notice there is a critical $\pi < \infty$, which varies with the specification, at which we drive out currency. That does not mean the DM necessarily shuts down, however, since there is still credit



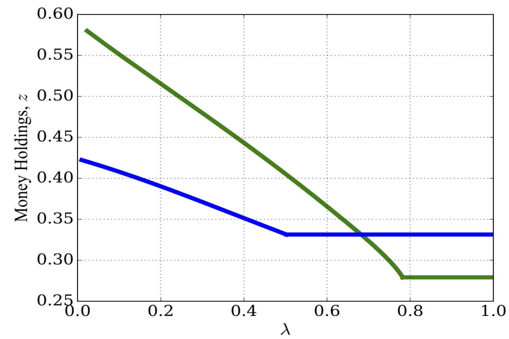


Figure 7: Effects of changes in information costs on Ω and z.

in some specifications, but the figure shows eliminating currency implies a welfare cost of 5.5% in Model 1 and 3.5% in Model 2. Hence, currency is not unimportant, although taxing it some through inflation is beneficial.

3.3 Information

The left panel of Fig. 7 shows the cost, or benefit if negative, of changes in λ, induced by a change in the cost of information, $s(\lambda)$ (i.e., the parameter E). There are two forces. As λ increases, the probability of trading at a tourist shop falls. Then, since local shops are less expensive, buyers carry less cash, as shown in the right panel. This lowers the surplus in both submarkets, suggesting welfare may fall with λ, but the net effect is positive at calibrated parameters. Increasing λ from 0 to 1 is worth between 3.7% and 15.6% of consumption, depending on the specification. However, we do not have to go all the way to $\lambda = 1$: in Model 1, welfare peaks at around $\lambda = 0.51$, which is sufficient to drive all tourist shops out of the market; in Model 2, this occurs at around $\lambda = 0.79$.

Instead of starting from $\lambda = 0$ we can start from the calibrated λ. Then increasing λ by enough to eliminate all tourist shops is worth between 1.2% and 2.7% of consumption. These numbers are driven by the effects on the mix between high- and low-markup firms in the DM. As λ increases the average DM markup falls due to changes in market composition, and the aggregate markup falls as the DM shrinks. Also, as regards price

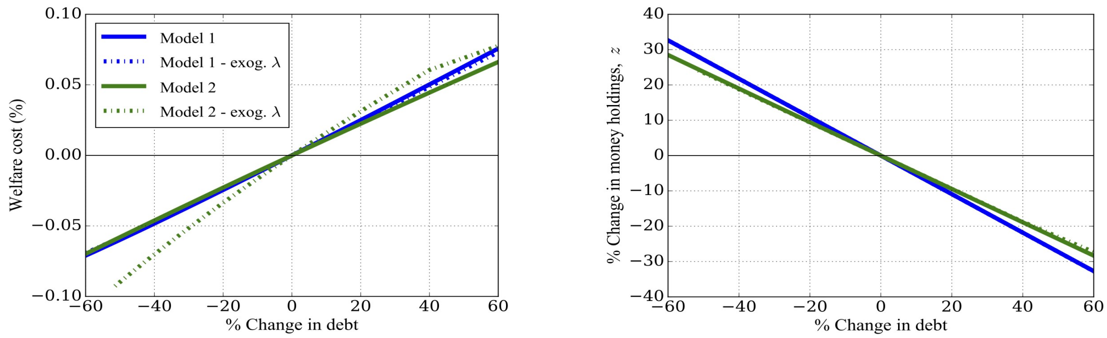


Figure 8: Effects of changes in debt induced by changes in the cost of credit.

dispersion, for low (high) values of λ it increases (decreases) with λ. Hence, while either positive or negative net effects of information on dispersion are possible in theory, at the calibrated parameters dispersion falls with λ.

3.4 Credit

Credit provides an alternative to cash, which matters for the impact of inflation. Symmetrically, cash provides an alternative to credit, which also matters. Fig. 8 plots the effects of changing credit conditions captured by C in the cost function $\gamma(d) = Cd^c$. Heuristically, higher-price shops use more credit, as well as more cash, and when C falls they are encouraged to enter. In response, buyers increase λ. They also decrease z (right panel) due to easier credit. Plus there is a novel second-best effect based on the idea that consumers tend to use too much credit and too little cash. The logic is that while both credit and cash are costly for individuals, the latter is not costly in aggregate because inflation revenue can be rebated to households, while the former entails deadweight loss.

Given all these effects, on net lower C actually hurts welfare (left panel). To see how this depends on money, consider changing credit conditions in a nonmonetary economy.[18] Without money, tighter credit is bad for welfare: raising C to get a 20% reduction in debt has a welfare cost of 0.1% and reduces output by 0.9%. Hence, not only the mag-

[18]For this we keep the parameters in Table 1 fixed, but to facilitate comparability add a costless credit limit $\bar{d}$ set equal to z from monetary equilibrium, .

nitude but also the sign of the effects change when one unrealistically ignores money. In general, these results should be words of caution to those trying to study credit in nonmonetary models.

3.5 Robustness

Here we sketch a few findings from the Supplementary Appendix. First, for many of the issues discussed above, it does not matter much if λ is endogenous or exogenous, which was not obvious ex ante. Second, it does not matter much if we recalibrate after reasonable changes in targets.[19] We also considered a version where consumers choose money holdings after their information state is revealed, which induces heterogeneous money holdings, and versions in which information types are permanent or there are multiple rounds of DM trade, which induce persistently heterogeneous money holdings. We also tried a formulation where better-informed agents get easier access to credit, suggested by a referee based on the idea that local customers are better known at local shops. None of this had a big impact on the results.

In general, most of the quantitative findings are fairly robust to alternative specifications – e.g., in Model 1 the welfare cost of 10% inflation varies between 0.6% and 1.1%; the gain to moving from the calibrated λ to $\lambda = 1$ varies between 1.1% and 1.8%; and the impact of changes in credit conditions varies hardly at all. Even having multiple rounds of DM trade was not too important for these numbers. Notice these statements are not about comparing across Models 1 and 2; rather they are about within each model the conclusions being not overly sensitive to reasonable changes in the formulation or calibration. Some differences across Models 1 and 2 are discussed below.

4 Conclusion

This paper developed a general equilibrium monetary framework with decentralized goods markets and used it to study the impact of inflation, credit conditions and in-

[19]As mentioned above, this includes the markup targets, which may be surprising, but is consistent with the results in Aruoba et al. (2011). To be clear, in that paper and in this one, it matters a lot whether the markup is 0 or, say, 5%, but not so much whether it is 5%, 10% or 15%.

formation frictions. The approach is tractable and delivers sharp analytic results on existence, uniqueness, efficiency and comparative statics. Some of the results are novel, like the possibility of better information raising prices in monetary economies, and the potential desirability of inflation above the Friedman rule, due to its impact on market composition (which is different from other papers showing inflation can be good for other reasons). Going beyond pure theory, the various specifications proved to be amenable to calibration based on standard observations, and generated results that we think are quantitatively relevant.

One result we find interesting is that inflation is considerably less costly than in papers with only random search and bargaining. Did those papers overestimate the negative impact of inflation? Not exactly. They were about the *cost* of inflation, and correctly pointed out that reduced-form papers underestimated this cost. None of those papers identified the *benefit* of inflation emphasized here, its positive impact on the types of sellers in the market. Another interesting result is that optimal inflation is -2.8% (just above the Friedman rule) in Model 1 and $+2.7\%$ (close to actual central bank targets) in Model 2. The big discrepancy is driven by differences in the information structure and the calibrated value of bargaining power, as explained in Section 3.2. This is consistent with a main theme of the project: information frictions and micro market structure are crucial for understanding many issues. That is something one obviously misses using only reduced-form theories with, say, money-in-utility or cash-in-advance assumptions, and is a reason we prefer an approach that strives for better microfoundations.

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Appendix

Proof of Lemma 1: Given $z = 0$, from (8)-(9) q_T and p_T are unique. If $p_T - q_T > k$ then submarket T is active. In this case there is a unique $n_T > 0$ solving $\alpha(n_T)(p_T - q_T) = k$, and submarket T is active at $\Gamma_T = (p_T, q_T, n_T)$. One can similarly show n_L and q_L are unique (see the proof of Lemma 4). By free entry p_L is also unique, and submarket L is active at Γ_L if the surplus is nonnegative. Since the terms of trade are efficient in submarket L and not submarket T, if the latter is active so is the former. Hence, a unique non-monetary equilibrium exists.

We now compare Γ_L and Γ_T when $N_L, N_T > 0$, which requires $\lambda > 0$. Immediately (2) and $N_L, \lambda > 0$ imply $n_L > n_T$. Since n_L solves (4), $n_L \leq \hat{n}$ where $\alpha(\hat{n}) = 1$. For $n_L > \hat{n}$ the objective function in (4) can be increased by lowering n_L. Then $\hat{n} \geq n_L > n_T$ implies $\alpha(n_L) > \alpha(n_T)$. Hence $\Pi_L = \Pi_T$ implies $p_L - q_L < p_T - q_T$. The results for P and R are obvious once we check q and p, so it remains to show $q_L > q_T$ and $p_L < p_T$. By (5)-(8), if q_j is big then p_j is small, so $p_L - q_L < p_T - q_T$ implies $p_L < p_T$ and $q_L > q_T$. ■

Proof of Lemma 2: In monetary equilibrium, the FOC holds at equality, $1 + i = V'(z)$. Hence, $i > 0$ implies either $\gamma'(d_L) > 0$ or $\gamma'(d_T) > 0$ by (26). This implies either $d_L > 0$ or $d_T > 0$, and so $z < \max\{p_L, p_T\}$. Then we can rewrite (11) using $u(q_T) - p_T - \gamma(d_T) = \theta[u(q_T) - q_T - \gamma(d_T)]$ from the bargaining solution as

$$\begin{aligned} V'(z) &= 1 + \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T}\theta\left[u'(q_T)q_T' - q_T' - \gamma'(d_T)(p_T' - 1)\right] \\ &= 1 + \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T}\theta\gamma'(d_T)(q_T' - p_T' + 1) \\ &= 1 + \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T}\frac{\theta\gamma'(d_T)}{1 + (1-\theta)\gamma'(d_T)}. \end{aligned} \tag{26}$$

The second line uses $u'(q_T) = 1 + \gamma'(d_T)$ by (8). The third uses $q_T' - p_T' + 1 = 1/[1 + (1-\theta)\gamma'(d_T)]$ which comes from differentiating (9) wrt z. Since ω_j and n_j are independent of z and γ is convex, $V''(z) \leq 0$ as long as $d_j = p_j - z$ decreases in z for $j = L, T$, and d_L falls in z, because p_L is not a function of z. Next, d_T falls in z by (8) and (9). Finally, $V''(z) < 0$ if either d_L or d_T falls strictly in z, and d_j falls strictly in z as long as it is strictly positive, and we already established $d_L > 0$ or $d_T > 0$. ■

The next Lemma, which is useful for proving additional results, is based on noticing that (2), (4) and (7) imply N_j and Γ_j can be solved as functions of z.

Lemma 4 *For $j = \{L, T\}$ and given $q_j < q^*$, Γ_j is differentiable wrt z with $\partial p_j/\partial z > 0$, $\partial q_j/\partial z > 0$, $\partial n_j/\partial z < 0$ and $\partial d_j/\partial z < 0$, while $\partial P_j/\partial z$ can go either way.*

Proof: Consider first tourist shops. If $p_T^* \le z$, then credit is not needed and all derivatives wrt z are 0. If $p_T^* > z$, we use the Implicit Function Theorem on (8)-(9) to define $p_T(z)$ and $q_T(z)$ as functions of z. Differentiating wrt z, we get

$$p_T'(z) = 1 - \left\{1 - \frac{\gamma''(d_T)}{u''(q_T)}\left[1 + (1-\theta)\gamma'(d_T)\frac{\gamma''(d_T) - u''(q_T)}{\gamma''(d_T)}\right]\right\}^{-1} \in [0,1) \quad (27)$$

$$q_T'(z) = [p_T'(z) - 1]\frac{\gamma''(d_T)}{u''(q_T)} > 0. \quad (28)$$

Since $p_T'(z) \in [0,1)$, we have $d_T'(z) = p_T'(z) - 1 < 0$. Next, the RHS of (9) rises in z, since $q_T'(z) > 0$ and $d_T'(z) < 0$, so $R_T'(z) > 0$. Therefore $n_T'(z) < 0$ by free entry.

Now consider local shops' problem (4). By a change of variable $y \equiv p - z - q$, using the constraint to eliminate p and z from (4), we have

$$\max_{n,q,y}\left\{\frac{\alpha(n)}{n}[u(q) - q - \gamma(y+q)] - \frac{k}{n}\right\} \text{ st } \alpha(n)(y+z) \ge k. \quad (29)$$

We can first choose q independent of n. Define $G(y) \equiv \max_q\{u(q) - q - \gamma(y+q)\}$. The solution for q satisfies $u'(q) - 1 = \gamma'(y+q)$. Thus, q decreases continuously and is differentiable in y. Then $G'(y) = -\gamma'(y+q) < 0$ and $G''(y) = \gamma''(y+q)u''(q)/[\gamma''(y+q) - u''(q)] < 0$. Eliminating y using the constraint from (29), we get

$$\max_n F(n,z) \equiv \max_n\left\{\frac{\alpha(n)}{n}G\left[\frac{k}{\alpha(n)} - z\right] - \frac{k}{n}\right\}.$$

Since $F(0,z) = -\infty$, $n = 0$ is not a solution. Since $\alpha(n) = 1$ and $\alpha'(n) = 0 \; \forall n > \hat{n}$, the solution is $n(z) \le \hat{n}$, as otherwise we can lower n to increase the objective function. Thus $n(z) \in (0, \hat{n}]$. If $n(z) < \hat{n}$, then $\partial F(n,z)/\partial n|_{n=n(z)} = 0$; otherwise $n(z) = \hat{n}$.

We now verify the SOC's and show $n(z)$ is unique. Consider

$$\begin{aligned}\frac{\partial F(n,z)}{\partial n} &= \left[\frac{\alpha'(n)}{n} - \frac{\alpha(n)}{n^2}\right]G\left[\frac{k}{\alpha(n)} - z\right] - \frac{k\alpha'(n)}{\alpha(n)n}G'\left[\frac{k}{\alpha(n)} - z\right] + \frac{k}{n^2} \\ &= \frac{1}{n^2}\left\{[n\alpha'(n) - \alpha(n)]G\left[\frac{k}{\alpha(n)} - z\right] - \frac{k\alpha'(n)n}{\alpha(n)}G'\left[\frac{k}{\alpha(n)} - z\right] + k\right\}.\end{aligned}$$

This derivative vanishes at an interior solution. For the SOC's, differentiate the expression in braces wrt n to get

$$\alpha''(n)nG\left[\frac{k}{\alpha(n)} - z\right] - \frac{k\alpha''(n)n}{\alpha(n)}G'\left[\frac{k}{\alpha(n)} - z\right] + \frac{\alpha'(n)^2nk^2}{\alpha(n)^3}G''\left[\frac{k}{\alpha(n)} - z\right].$$

At $n = n(z)$, $G[k/\alpha(n) - z] > 0$ and this is strictly because $\alpha'', G', G'' < 0$.

Hence the SOC's hold and the optimizer $n(z)$ is unique. Next we show it falls with z. Since

$$\frac{\partial^2 F(n,z)}{\partial z \partial n} = \frac{\alpha'(n)k}{\alpha(n)n} G'' \left[\frac{k}{\alpha(n)} - z \right] + \frac{\alpha(n)}{n^2} \left[1 - \frac{\alpha'(n)n}{\alpha(n)} \right] G' \left[\frac{k}{\alpha(n)} - z \right] \leq 0,$$

any interior $n(z)$ is differentiable with $n'(z) \leq 0$. Given $q_L < q^*$, we have $G'[k/\alpha(n) - z] < 0$ and $\partial^2 F(n,z)/\partial z \partial n < 0$ and $n'(z) < 0$.

Next we show q increases with z. Write (29) as a Lagrangian

$$\max_{n,y} \left\{ \frac{\alpha(n)}{n} G(y) - \frac{k}{n} + \zeta[\alpha(n)(y+z) - k] \right\},$$

with ζ the multiplier for free entry. Taking the FOC's wrt n and y, and eliminating ζ, we get

$$0 = \frac{1}{n} \left[\alpha'(n) - \frac{\alpha(n)}{n} \right] G(y) + \frac{k}{n^2} - \frac{G'(y)}{n} \alpha'(n)(y+z). \tag{30}$$

Since $n'(z)$ exists, $y'(z)$ and $q'(z)$ exist. To show $q'(z) > 0$, it is sufficient to show $y'(z) < 0$ because the solution for q falls and is differentiable in y. Consider (30). Since the elasticity of α is less than 1, the term in square brackets is negative. Since $G', G'' < 0$, the RHS of (30) rises strictly with z or y, so $y'(z) < 0$.

Finally, it is not hard to show by example that $\partial P_j / \partial z$ can go either way. ■

Lemma 5 *If monetary equilibrium is unique then z is almost everywhere differentiable wrt i with $\partial z / \partial i < 0$.*

Proof: Since $T(z)$ is continuous and $T(z)$ decreases in z, the solution for $T(z) = i$ falls continuously in i. For $N_L, N_T > 0$, by (26), (31) can be written

$$T(z) = \frac{\omega_L \alpha(n_L)}{n_L} \gamma'(p_L - z) + \omega_T \frac{\alpha(n_T)}{n_T} \frac{\theta \gamma'(d_T)}{1 + (1-\theta)\gamma'(d_T)}.$$

Clearly $T'(z)$ exists because γ'' exists and $(n_j, p_j, d_j, \omega_j)$ is differentiable in z by Lemma 4. For $N_L > N_T = 0$, $T'(z)$ exists by (32). So $T'(z)$ exists except when (31) equals (32). ■

Proof of Proposition 1: We seek $z > 0$ such that $T(z) = i$, where $T(z)$ is given by the RHS of (12). For $N_T(z) > 0$, this can be written

$$T(z) = \frac{\omega_L \alpha(n_L)}{n_L} \gamma'(p_L - z) + \frac{\omega_T \alpha(n_T)}{n_T} \left[u'(q_T) q'_T - p'_T - \gamma'(p_T - z)(p'_T - 1) \right]; \tag{31}$$

for $N_T(z) = 0$ it can be written

$$T(z) = \frac{\alpha(n_L)}{n_L}\gamma'(p_L - z). \tag{32}$$

We first show $T(z)$ is continuous. By Lemma 4, $q_j(z)$, $n_j(z)$, $N_j(z)$ and $p_j(z)$ are continuous, and thus (31) and (32) are continuous in z. When $\lambda = 1 - n_T(z)/n_L(z)$, (31) and (32) are identical, so $T(z)$ is continuous. At $i = 0$, by (3) the solution for z is big enough to sustain equilibrium with $N_T + N_L > 0$. Since $T(z)$ decreases in z, a unique monetary equilibrium exists when i is small by continuity.

We now compare Γ_L and Γ_T. As in the proof of Lemma 1, $n_L > n_T$. Since n_L solves (4), $n_L \leq \hat{n}$ where $\alpha(\hat{n}) = 1$. For $n_L > \hat{n}$ the objective function in (4) can be increased by lowering n_L. Since $\hat{n} \geq n_L > n_T$, $\alpha(n_L) > \alpha(n_T)$. Hence $\Pi_L = \Pi_T$ implies $p_L - q_L < p_T - q_T$. The results for P and R are obvious once we check q and p, so it remains to show $q_L \geq q_T$ and $p_L < p_T$. There are different cases. First suppose $p_L, p_T \geq z$. By (5)-(8), if q_j is big then p_j is small, so $p_L - q_L < p_T - q_T$ implies $p_L < p_T$ and $q_L > q_T$. Second suppose $p_L < z \leq p_T$. By (5)-(8), $q_L = q^* \geq q_T$. Third suppose $p_L \geq z > p_T$. By (5)-(8), $q_T = q^* \geq q_L$, but then $\Pi_L = \Pi_T$ implies $p_T \geq p_L$, contradicting the supposition $p_L \geq z > p_T$; so this case cannot occur. Finally suppose $p_L, p_T < z$. Then $q_L = q_T = q^*$, so $\Pi_L = \Pi_T$ implies $p_L < p_T$. Hence $p_L < p_T$ and $q_L \geq q_T$ in all cases. If $q_T < q^*$, then either the first or the second case holds, and $q_L > q_T$. ■

Proposition 2: Consider first the effect of i. When $\lambda = 0$, as in Aruoba et al. (2007), Ω is maximized at $i = 0$. Also, as in Rocheteau and Wright (2005), Ω is maximized when $i = 0$ when $\lambda = 1$. By continuity, for λ near 0 or 1, Ω falls with i. It remains to show Ω rises with i for $(\lambda, i) \in \mathcal{A}_1^*$.

Rewrite (13) as

$$\Omega = \frac{\omega_L \alpha(n_L)}{n_L}\left[u(q_L) - p_L - \gamma\left(p_L - z\right)\right] + \frac{\omega_T \alpha(n_T)}{n_T}\left[u(q_T) - p_T - \gamma(p_T - z)\right],$$

where the entry cost k does not show up since the terms in brackets are the buyer's (not total) surplus, and k cancels with the seller's surplus. Using (6), (9) and free entry, then using (1), (2) and $u'^{*}) = 1$, we get

$$\Omega = \frac{\omega_L}{n_L}\left(\frac{\eta_L k}{1-\eta_L}\right) + \frac{\omega_T}{n_T}\left(\frac{\theta k}{1-\theta}\right) = \frac{\lambda}{n_L - n_T}\left(\frac{\eta_L k}{1-\eta_L}\right) + \frac{(1-\lambda)n_L - n_T}{n_T(n_L - n_T)}\left(\frac{\theta k}{1-\theta}\right).$$

The RHS depends on i only through n_T and n_L. Since $\partial n_T/\partial i > \partial n_L/\partial i = 0$ in $\mathcal{A}_1$, $\partial\Omega/\partial i$ has the same sign as $\partial\Omega/\partial n_T$. Differentiating the RHS wrt n_T yields

$$\frac{\partial\Omega}{\partial n_T} = \frac{k}{(n_L - n_T)^2}\left\{\frac{\lambda\eta_L}{1-\eta_L} + \frac{\theta}{1-\theta}\left[(1-\lambda)\frac{n_L}{n_T}\left(2 - \frac{n_L}{n_T}\right) - 1\right]\right\}. \tag{33}$$

The sign of the RHS depends on the term in curly brackets, call it Φ. An equilibrium in $\mathcal{A}_1$ is in $\mathcal{A}_1^*$ iff $\Phi \geq 0$. At the intersection point of $\mathcal{A}_1$ and $\mathcal{A}_3$, $N_T = 0 \Leftrightarrow (1-\lambda)n_L = n_T$. In this situation, inflation enhances welfare:

$$\begin{aligned}
\Phi &\equiv \frac{\lambda\eta_L}{1-\eta_L} + \frac{\theta}{1-\theta}\left[(1-\lambda)\frac{n_L}{n_T}\left(2 - \frac{n_L}{n_T}\right) - 1\right] \\
&= \frac{\lambda\eta_L}{1-\eta_L} - \frac{\theta\lambda}{1-\theta}\frac{n_L}{n_T} \\
&= \frac{\lambda n_L}{k}\left\{\frac{\alpha(n_L)}{n_L}\left[u(q_L) - p_L - \gamma(d_L)\right] - \frac{\alpha(n_T)}{n_T}\left[u(q_T) - p_T - \gamma(d_T)\right]\right\} > 0.
\end{aligned}$$

The second equation uses $(1-\lambda)n_L = n_T$. The third uses (6), (9) and free entry. The inequality holds because a buyer receives a higher expected payoff in submarket L than submarket T. By continuity, there is an interval for λ where $\Phi > 0$ at $i = 0$. Therefore, by continuity there is a non-empty region $\mathcal{A}_1^*$ in the lower-right part of $\mathcal{A}_1$ such that Ω increases with i.

Next consider the effect of making credit more costly. First, we claim γ_2 is more costly than γ_1 using the definition in the text then $\gamma_2[\gamma_2'^{-1}(a)] < \gamma_1[\gamma_1'^{-1}(a)]$ $\forall a \geq 0$. To verify this, notice if $\gamma_2^{-1}(b)$ is flatter than $\gamma_1^{-1}(b)$ $\forall b > 0$ then

$$\frac{\partial\gamma_2^{-1}(b)}{\partial b} < \frac{\partial\gamma_1^{-1}(b)}{\partial b} \Leftrightarrow \frac{1}{\gamma_2'(\gamma_2^{-1}(b))} < \frac{1}{\gamma_1'(\gamma_1^{-1}(b))} \Leftrightarrow \gamma_1'(\gamma_1^{-1}(b)) < \gamma_2'(\gamma_2^{-1}(b)).$$

Since $\gamma'(d)$ and $\gamma^{-1}(b)$ are increasing, $\gamma'[\gamma^{-1}(b)]$ rises with b. Thus, the last inequality implies $\forall b_1, b_2$, $\gamma_2'[\gamma_2^{-1}(b_2)] = \gamma_1'[\gamma_1^{-1}(b_1)] \Rightarrow b_1 > b_2$. In other words, $a = \gamma_2'[\gamma_2^{-1}(b_2)] = \gamma_1'[\gamma_1^{-1}(b_1)] \Rightarrow \gamma_2[\gamma_2'^{-1}(a)] = b_2 < b_1 = \gamma_1[\gamma_1'^{-1}(a)]$. Therefore, given $a \geq 0$, $\gamma(\gamma'^{-1}(a))$ falls strictly as γ^{-1} grows flatter. This establishes the claim.

Now consider in turn (i) $\lambda = 0$ and (ii) $\lambda = 1$. In case (i), since $\gamma(d)$ is strictly increasing, strictly convex and differentiable $\forall d \geq 0$, $\gamma'(d)$ exists and rises in d. For any given $a = \gamma'(d)$, $\gamma(d)$ is an implicit function $\gamma[\gamma'^{-1}(a)]$. When $\lambda = 0$, only submarket T exists and by (31) and (26)

$$i = \frac{\alpha(n_T)}{n_T}\frac{\theta\gamma'(d)}{1 + (1-\theta)\gamma'(d)} = \frac{\alpha(n_T)}{n_T}\frac{\theta a}{1 + (1-\theta)a}. \tag{34}$$

By (8), q_T falls continuously in $\gamma'(d)$. Thus, we can also write q_T as an implicit function $q(a) \equiv u'^{-1}(1+a)$ where $a = \gamma'(d)$, and the inverse function exists since $u''(q) < 0$. This also implies $q(a)$ rises strictly in a. It is easy to verify $q(0) = q^*$. By (9) and free entry,

$$\frac{k}{(1-\theta)\alpha(n_T)} = u[q(a)] - q(a) - \gamma[\gamma'^{-1}(a)]. \tag{35}$$

Using (34) we define an implicit function $n_T = n_1(a)$ for any a because $\alpha(n_T)/n_T$ falls strictly in n_T. Similarly, we can define $n_T = n_2(a)$ by (35). Any a solving $n_1(a) = n_2(a)$ determines the terms of trade in equilibrium. Given equilibrium is unique by assumption here, the solution for $n_1(a) = n_2(a)$ is unique. To characterize it, note that at $a = 0$, $n_1(0) = 0$ by (34) and $n_2(0) = \alpha^{-1}\{k/(1-\theta)[u(q^*) - q^*]\} > 0$ by (35). Thus $n_2(a)$ cuts $n_1(a)$ from above once as a rises.

As the cost of credit increases, the implicit function $\gamma[\gamma'^{-1}(a)]$ falls $\forall a$ by the claim discussed above. In this case, $n_2(a)$ falls $\forall a$ by (35). This implies a and n fall, so there are more sellers and $\alpha(n_T)/n_T$ rises. Also, $q_T(a)$ rises as a falls because $u'[q_T(a)] = 1 + a$. Moreover, the surplus for sellers $p_T - q_T$ rises by free entry, and so p_T rises. The surplus for buyers $u(q_T) - q_T - \gamma(d_T)$ rises in $p_T - q_T$ from the bargaining solution. So buyers get a larger surplus and a higher matching probability, raising Ω. Finally, as the cost of using credit rises, the total expenditure on credit $\gamma(d_T) = \gamma[\gamma^{'-1}(a)]$ falls because a falls and $\gamma[\gamma'^{-1}(a)]$ falls $\forall a$. This means d_T falls because $\gamma(d)$ rises $\forall d > 0$ and $\gamma(d_T)$ falls. Since p_T rises and d_T falls, $z = p_T - d_T$ rises.

Now consider (ii) with $\lambda = 1$. Then (n, q, p) solves

$$\max_{n,q,p,z} \left\{ \frac{\alpha(n)}{n} [u(q) - q - \gamma(p - z)] - \frac{k}{n} - iz \right\} \text{ st } k = \alpha(n)(p - q).$$

We make several changes of variables. First, let $a = \gamma'(p - z)$ so $p = \gamma'^{-1}(a) + z$. Then, since the solution satisfies (5), q solves $q(a) \equiv u'^{-1}(1 + a)$. Then, by free entry $k = \alpha(n)(p - q)$, $z = p - \gamma'^{-1}(a) = q(a) - \gamma'^{-1}(a) + k/\alpha(n)$. Then, from the FOC $i = \gamma'(p_L - z)\alpha(n)/n$ we can express n as an implicit function $n(a)$ where $i = a\alpha[n(a)]/n(a)$. Substitute $a, q(a), n(a)$ and $z = q(a) - \gamma'^{-1}(a) + k/\alpha(n)$ into the problem to get

$$\max_a \frac{i}{a} \left\{ u[q(a)] - q(a) - \gamma[\gamma'^{-1}(a)] \right\} - \frac{k}{n(a)} - i \left\{ q(a) - \gamma'^{-1}(a) + \frac{k}{\alpha[n(a)]} \right\}.$$

Now we claim the solution for a falls strictly as γ becomes more costly. Let γ_2 be more costly than γ_1, so that γ_2^{-1} is weakly flatter than γ_1^{-1}. Let $F_j(a)$ be the objective function when the cost function is γ_j. Differentiating, we get

$$\frac{\partial F_2(a)}{\partial a} - \frac{\partial F_1(a)}{\partial a} = \frac{i}{a^2}\left\{\gamma_2[\gamma_2'^{-1}(a)] - \gamma_1[\gamma_1'^{-1}(a)]\right\} < 0,$$

using the claim concerning $\gamma(\gamma'^{-1}(a))$ discussed above. So a falls strictly in j by standard monotone comparative statics. Hence n falls as γ becomes more costly by (34). The rest of the proof is identical to part of (i).

Now we verify that for $\lambda \in \{0,1\}$, Ω rises as credit becomes more costly. Define $\bar{\gamma}(d) \equiv \gamma(bd)$ for $b > 1$, so $\bar{\gamma}^{-1}$ is flatter than γ^{-1}. As b rises, $\bar{\gamma}$ grows more costly and Ω rises by (i) and (ii) above.

The last step is to show Ω falls as credit becomes more costly in $\mathcal{A}_1^*$. First, we show that as γ becomes more costly Γ_L stays the same, p_T, q_T and ω_T rise, and n_T falls. By (1) and (2), $\omega_T = 1 - \lambda n_L/(n_L - n_T)$. Substitute this into (26) and let $a = \gamma(d_T)$ to get

$$i = \left(1 - \frac{\lambda n_L}{n_L - n_T}\right)\frac{\alpha(n_T)}{n_T}\frac{\theta a}{1 + (1-\theta)a}. \tag{36}$$

Any (n_T, a) solving (35) and (36) characterizes Γ_T. Now we claim that if $(\lambda, i) \in \mathcal{A}_1$ we stay in $\mathcal{A}_1$ as γ gets more costly. To show this, we assume we stay in $\mathcal{A}_1$ as γ grows more costly, then verify it. If we stay in $\mathcal{A}_1$, buyers have enough money to purchase q^* in submarket L, and Γ_L is constant. Now define $n_1(a)$ and $n_2(a)$ by (36) and (35), so Γ_T is characterized by $n_1(a) = n_2(a)$. One can show that n_T falls and p_T, q_T and z rise as γ gets more costly. Since z rises, buyers have enough z to get q^* in submarket L, and the equilibrium stays in $\mathcal{A}_1$. Then $\omega_T = 1 - \lambda n_L/(n_L - n_T)$ rises since n_T falls and n_L is constant.

Finally, if the equilibrium is in $\mathcal{A}_1^* \subset \mathcal{A}_1$, then n_T falls as γ gets more costly, as argued in the previous paragraph. Also, Ω rises in n_T in $\mathcal{A}_1^*$ by (33). Therefore Ω falls as γ grows more costly in $\mathcal{A}_1^*$. ■

Proof of Proposition 3. For $j \in \{L, T\}$ let $S_j(z)$ be the expected trade surplus in each submarket, namely

$$S_j(z) = \frac{\alpha(n_j)}{n_j}[u(q_j) - p_j - \gamma(p_j - z)].$$

Using the formula of ω_T in (1), equation (14) can be rewritten as

$$s'\left(1 - \frac{1}{(1-\hat{\omega}_T)(n_L/n_T - 1) + 1}\right) = [S_L(z) - S_T(z)]\hat{\omega}_T. \tag{37}$$

Clearly the left-side falls strictly in $\hat{\omega}_T$ and the right-hand side rises strictly in $\hat{\omega}_T$. When i is sufficiently small z is close to $z_U^* \equiv \theta u(q^*) + (1-\theta)q^*$. In this case $p_T > z > p_L$ and thus a change in z only affects submarket T. Therefore the difference $S_L(z) - S_T(z)$ falls strictly in z and the ratio n_L/n_T rises strictly in z. It follows that the left-hand side of (37) rises and the right-hand side falls in z. Hence (37) defines z as a strictly increasing implicit function of $\hat{\omega}_T$, call it $z_1(\hat{\omega}_T)$.

Using the bargaining solution (8) and (9), the FOC (12) can be written as

$$i = \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(p_L - z) + \omega_T \frac{\alpha(n_T)}{n_T}\left[\frac{\theta(u'(q_T) - 1)}{1 + (1-\theta)(u'(q_T) - 1)}\right]. \tag{38}$$

When z is sufficiently large $z > p_L$ and thus the first term is 0. By the bargaining solution (8) and (9) q_T rises and n_T falls in z. Therefore the right-hand side falls in z. By the definition of $\hat{\omega}_T$ and (1), we can write

$$\omega_T = (1-\lambda)\hat{\omega}_T = \frac{\hat{\omega}_T}{(1-\hat{\omega}_T)(n_L/n_T - 1) + 1}.$$

Therefore fixing n_T and n_L, ω_T rises in $\hat{\omega}_T$. It follows that the right-hand side of (38) rises in $\hat{\omega}_T$. As a result (38) defines z as an increasing function of $\hat{\omega}_T$, call it $z_2(\hat{\omega}_T)$. In equilibrium $\hat{\omega}_T$ solves $z_1(\hat{\omega}_T) = z_2(\hat{\omega}_T)$.

Since $z_1(\hat{\omega}_T)$ and $z_2(\hat{\omega}_T)$ both increase in $\hat{\omega}_T$, in general there could be multiple equilibria. But when $i = 0$ all buyers carry z_U^* and thus $z_2(\hat{\omega}_T) = z_U^*$ is constant in $\hat{\omega}_T$. By continuity, $z_2(\hat{\omega}_T)$ is very flat when i is small. Therefore $z_1(\hat{\omega}_T)$ and $z_2(\hat{\omega}_T)$ have at most one intersection point when i is small and $z_2(\hat{\omega}_T)$ must cut $z_1(\hat{\omega}_T)$ from above at the intersection. Hence equilibrium is unique when i is small.

Finally we argue that an equilibrium exists when i and $s'(0)$ are small. By (37) it is easy to check that $z_1(\hat{\omega}_T') = 0$ at some $\hat{\omega}_T' > 0$. If $z_2(0) > 0$, then $z_2(\hat{\omega}_T') > z_1(\hat{\omega}_T')$ because $z_2(\hat{\omega}_T)$ is an increasing function. When i is sufficiently small $z_2(0) \approx z_U^* > 0$.

It remains to show that $z_2(1) > z_1(1)$. Recall that $S_L(z) - S_T(z)$ falls in z when z is large (i.e. $p_T > z > p_L$). Therefore $z_2(1) > z_1(1)$ if and only if $s'(0) < S_L[z_2(1)] - S_T[z_2(1)]$ by (37). Therefore an equilibrium exists provided that $s'(0) < S_L[z_2(1)] - S_T[z_2(1)]$ and i is sufficiently small. ■

The next result is the analog of Lemmas 4 and 5 from the case of fixed λ.

Lemma 6 *If monetary equilibrium with endogenous λ is unique, as i rises, q_T, q_L, z and $\hat{\omega}_T$ fall, and n_T and n_L rise.*

Proof: Given the discussion in the proof of Proposition 3, an increase in i shifts $z_2(\cdot)$ down and leave $z_1(\cdot)$ unchanged. Therefore z and $\hat{\omega}_T$ rise in i. If $s'(\lambda)$ weakly rises at all λ, then $z_1(\cdot)$ shifts down and leave $z_2(\cdot)$ unchanged. It follows that z and $\hat{\omega}_T$ rise. ■

Proof of Proposition 4. As discuss in the proof of Proposition 2 $\mathcal{A}_1^*$ is the region where $i \approx 0$ and $\lambda \approx 1$. When λ is endogenous the equilibrium (i, λ) falls into $\mathcal{A}_1^*$ when $i \approx 0$ and $s'(\lambda)$ is sufficiently small $\forall \lambda$. Suppose $(i, \lambda) \in \mathcal{A}_1^*$. Now fixing λ, an increase in i raises Ω by Proposition 2. Moreover $\hat{\omega}_T$ increases in i by Lemma 6 which also raises Ω. Therefore Ω unambiguously increases in i when $i = 0$. ■

Lemma 7 *Buyers' expected payoff $\Sigma(R)\alpha(n(R))/n(R)$ is strictly quasi-concave in R and maximized at R_L.*

Proof. Since buyers prefer a smaller R in equilibrium, their expected payoff must fall in R or equivalently

$$\frac{d}{dR}\left\{\Sigma(R)\frac{\alpha\left[n\left(R\right)\right]}{n(R)}\right\} \leq 0.$$

Since $n(R)$ falls in R, this inequality holds if and only if

$$\frac{d}{dn}\max_p \left\{u[p - k/\alpha(n)] - p - \gamma(p - z)\right\}\frac{\alpha(n)}{n} \geq 0.$$

By the envelope theorem and free-entry, the inequality holds if and only if $\forall R \in [\underline{R}, \bar{R}]$

$$\frac{R}{\Sigma(R)} \geq \frac{1-\eta}{u'(q(R))\eta} \tag{39}$$

where $\eta \equiv \alpha'(n)n/\alpha(n)$ is the elasticity of the matching function. If this inequality binds at some R', then the share of surplus that buyers get is the same as that implied by a pure directed search market by (6). This implies $\{q(R'), p(R'), n(R')\}$ solve the same equations that pin down the terms of trade $\{q_L, p_L, n_L\}$ of the local submarket (i.e. free entry (4), efficiency (5), and surplus sharing (6)). But the solution to these equations is unique by the proof of Lemma 4. Therefore $R' = R_L \equiv p_L - q_L$. This implies the inequality (39) binds exactly once at R_L. For $R = R_T$, (39) is satisfied because

$$\frac{R_T}{\Sigma(R_T)} = \frac{1-\theta}{\theta} \geq \frac{1-\eta}{u'(q(R_T))\eta}.$$

The equation uses the definition of the Kalai bargaining solution. The inequality is true because $u'(q(R_T)) \geq 1$ and we have assumed $\theta \leq \eta$. It follows that (39) is satisfied $\forall R \in [R_L, R_T]$ and violated $\forall R < R_L$. ■

Lemma 3. It remains to show $\lambda^* \equiv 1 - n_T/n_L$. Suppose all sellers post R_L, the free entry condition implies $N = 1/n_L$. If a seller deviates to post any $R > R_L$, then the most profitable deviation is R_T. Since only uninformed buyers would visit the deviating seller, the buyer-to-seller ratio is $(1-\lambda)n_L$. If $n_T \geq (1-\lambda)n_L$ or equivalently $\lambda \geq 1 - n_T/n_L$, then no seller would deviate to post R_L. As discussed in the main text, it is never optimal for sellers to post $R < R_L$, therefore all sellers posting R_L is an equilibrium iff $\lambda \geq 1 - n_T/n_L$.

If $\lambda \geq 1 - n_T/n_L$, then the competitive search outcome is the only equilibrium. Suppose $F(R_L) < 1$, then the sellers that post $R > R_L$ can deviate to post R_L and the buyer-to-seller ratio would exceed n_L by (18) and $\lambda \geq 1 - n_T/n_L$. ■

Proposition 5. *Existence and Uniqueness:* When $i = 0$, all sellers produces q^* and $R_T = (1-\theta)(u(q^*) - q^*)$ and $R_L = (1-\eta_L)(u(q^*) - q^*)$. By Proposition 3 there is a unique solution for the distribution F. The money holding is determined by the seller that posts R_T, therefore $z^* = p(R_T) = (1-\theta)u(q^*) + \theta q^*$. In this case the equilibrium is clearly unique. Since one can derive F for any given z, we can consider the right-hand side of (23) as a function of z. Since it is weakly positive and continuous in z, it must fall strictly in z around the FR (i.e. when $z \approx (1-\theta)u(q^*) + \theta q^*$). It follows that there is a unique solution for z in (23), and thus equilibrium is unique near the FR. ■

Proposition 6. *Change in i:* As mentioned above, the right-hand side of (23) falls in z when $i \approx 0$. Therefore z falls in i. Next, we argue that the distribution of prices falls in the first-order stochastic dominance (FOSD) sense as z falls through two channels. First, given R, the price p falls as z falls by equation (16) and the envelope theorem. Second, the distribution of R falls in the FOSD sense as z falls. To see this, note that when $i = 0$ buyers carry $z = p_T > p_L$. In this case a small decrease in z affects the terms of trade with high price sellers but do not affect the terms of trade at the local shops. Therefore the surplus R_T falls as z falls by the Kalai bargaining solution while R_L remains constant as z falls. It follows that the distribution F falls in the FOSD sense by equation (19) and (21). The distribution of prices falls in the FOSD sense because p

falls as R falls by (16).

As i rises, buyers carry less money and it affects welfare Ω through three channels:

$$\frac{d\Omega}{di}=\frac{\partial z}{\partial i}\left(\frac{\partial\Omega}{\partial z}+\frac{\partial R_T}{\partial z}\frac{\partial\Omega}{\partial R_T}+\frac{\partial R_L}{\partial z}\frac{\partial\Omega}{\partial R_L}\right).$$

Since $\partial\Omega/\partial z=i$ by (23), it vanishes at the FR. Similarly, $\partial R_T/\partial z$ and $\partial R_L/\partial z$ vanishes at the FR by the envelope theorem. Altogether, $d\Omega/di=0$ at $i=0$.

Now consider $i\approx 0$. As discussed in the first paragraph, $\partial R_L/\partial z=0$. Therefore

$$\frac{d\Omega}{di}=\frac{\partial z}{\partial i}\left(\frac{\partial\Omega}{\partial z}+\frac{\partial R_T}{\partial z}\frac{\partial\Omega}{\partial R_T}\right).$$

Differentiating Ω and R_T with respect to z yields

$$\frac{d\Omega}{di}=\frac{\partial z}{\partial i}\left(\int\gamma'(p(R)-z)d\tilde{F}(R)+\frac{(1-\theta)\gamma'(p(R_T)-z)}{[(1-\theta)\gamma'(p(R_T)-z)+1]}\frac{\partial\Omega}{\partial R_T}\right).$$

Define an implicit function $\tilde{R}(z)$ by $p(\tilde{R}(z))=z$. Therefore buyers do no use costly credit when they meet a seller who posts $R<\tilde{R}(z)$ or equivalently $\gamma'(p(R)-z)=0$ $\forall R\leq\tilde{R}(z)$. We can rewrite the displayed equality as

$$\frac{d\Omega}{di}=\frac{\partial z}{\partial i}\gamma'(p(R_T)-z)\left(\int_{\tilde{R}(z)}^{\bar{R}}\frac{\gamma'(p(R)-z)}{\gamma'(p(R_T)-z)}d\tilde{F}(R)+\frac{(1-\theta)}{[(1-\theta)\gamma'(p(R_T)-z)+1]}\frac{\partial\Omega}{\partial R_T}\right).$$

When $i=0$, buyers do not use costly credit and thus $p(R_T)=z$, or equivalently $\tilde{R}(z)=R_T=\bar{R}$. It follows that the integral in the large bracket is 0 at $i=0$. The second term is strictly negative at $i=0$ because (i) the fraction reduces to $1-\theta>0$ at $i=0$ and (ii) the derivative $\partial\Omega/\partial R_T<0$ when $i=0$ because $\tilde{F}(R)$ strictly falls in the FOSD sense in R_T and the integrand in (24) falls strictly in R. Therefore the expression in the large bracket is strictly negative at $i=0$, and thus is strictly negative for $i\approx 0$ by continuity. Since $\gamma'(p_T-z)>0$ for $i>0$, $d\Omega/di>0$ when i is close to 0.

Change in λ: The distribution F decreases in λ in the strict FOSD sense by (19) and (21) and thus so does the distribution $\tilde{F}$ in (23). It follow that the right-hand side of (23) falls in λ because $n(R)$ rises and $p(R)$ falls in R. Since the right-hand side falls in z near the FR, z falls in λ to balance (23). Next, since z falls and λ rises, $\tilde{F}$ falls in the strict FOSD sense, and so does the distribution of prices $p(R)$, because $p(R)$ falls in R by (16). Moreover, fixing R, $p(R)$ falls as z falls by (16).

Finally, we argue that welfare Ω rises in λ. As λ rises, buyers carry less money and we can write the direct and indirect effects of λ as:

$$\frac{d\Omega}{d\lambda} = \frac{\partial\Omega}{\partial\lambda} + \frac{\partial z}{\partial\lambda}\frac{\partial\Omega}{\partial z}.$$

The first term is strictly positive because $\tilde{F}$ falls strictly in the FOSD sense in λ as argued above and the integrand in (24) falls strictly in R. The second term vanishes at $i = 0$ because $\partial\Omega/\partial z = 0$ at the FR as discussed above. Altogether, Ω rises in λ at $i = 0$. ■

Proposition 7. At $i = 0$, $z = z_U^* \equiv \theta u(q^*) + (1-\theta)q^*$. When $\lambda = 0$ or $\lambda = 1$ the distribution F is degenerate by Lemma 3 and therefore the marginal benefit of choosing a higher λ is zero. Therefore the right-hand side of (25) is zero when $\lambda \in \{0, 1\}$. Hence the right-hand side of (25) rises in λ when $\lambda \approx 0$ and falls when $\lambda \approx 1$. When $i = 0$ and the marginal cost $s'(\lambda)$ is sufficiently small at all λ, there is a unique $\lambda \in (0, 1)$ solving (25). ■

Proposition 8. When λ is exogenous, z falls in λ near $i = 0$ by Proposition 6. This defines z as a decreasing function of λ, call this $z_1(\lambda)$.

With endogenous λ, when equilibrium is unique the right side of (25) falls and the left side falls in λ. To derive the impact of a change in z on λ, it suffices to show the right-hand side of (25) rises in z. Define a buyer's ex-post surplus as

$$B(R, z) \equiv \frac{\alpha\left[n(R)\right]}{n(R)}\left\{u\left[q(R)\right] - p(R) - \gamma\left[p(R) - z\right]\right\}$$

and $B_1(R, z) = dB(R, z)/dR$. Then the right-hand side of (25) can be rewritten as

$$\begin{aligned}\int B(R, z)d\{[1 - F(R)]F(R)\} &= -\int [1 - F(R)]F(R)B_1(R, z)dR \\ &= -\int_0^1 (1-a)aB_1[F^{-1}(a), z]\frac{dF^{-1}(a)}{da}da. \qquad (40)\end{aligned}$$

The first equation is by integrating by parts. The second equation changes variable $a = F(R)$. By (21) one can show that

$$F^{-1}(a) = \frac{k}{\alpha(\alpha^{-1}(k/R_T)[\frac{2\lambda(1-a)}{1-\lambda} + 1])}$$

and it rises in R_T. One can check that $dF^{-1}(a)/da$ also rises in R_T. Since R_T rises in z by the definition of R_T, both $F^{-1}(a)$ and $dF^{-1}(a)/da$ rise in z. Next by directly differentiating $B(R, z)$, we have

$$B_1(R, z) = \frac{\alpha[n(R)]}{n(R)} \left(\frac{\{1 - \eta[n(R)]\}}{\eta[n(R)]} \frac{\Sigma(R)}{R} - u'[q(R)] \right) \leq 0.$$

The inequality is true as discussed in the proof of Lemma 7. This derivative falls in R by Proposition 6 and because $\eta(n)$ falls in n. It follows that (40) is positive and rises in z. Therefore (25) defines a positive relationship between z and λ, call this mapping $z_2(\lambda)$. An equilibrium is a solution to $z_1(\lambda) = z_2(\lambda)$. Since $z_1(\lambda)$ falls and $z_2(\lambda)$ rises in λ, there is only one intersection point. As i rises, $z_1(\lambda)$ drops at all λ by Proposition 6. Therefore z and λ fall in i near $i = 0$. ■

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Supplementary Appendix: Not for Publication

A Heterogenous Buyers

Here we sketch a model with heterogenous buyers, focusing on $i = \theta = 0$ to keep things simple. Suppose there are buyer types $j = 1, 2$, where ρ_j is the measure of type-j and $\epsilon_j u(q)$ is their DM utility function, with $\epsilon_1 < \epsilon_2$. Define q_j^* by $\epsilon_j u'(q_j^*) = 1$. Since $i = 0$, all buyers carry enough money to purchase q_j^*.

First, suppose λ is exogenous (it is endogenous below). When a type-j buyer is in a tourist shop, the price comes from bargaining with $\theta = 0$, $p_T^j = \epsilon_j u(q_j^*)$. Market tightness is determined by entry,

$$\alpha(n_T)\frac{(1-\lambda_1)\rho_1(p_T^1 - q_1^*) + (1-\lambda_2)\rho_2(p_T^2 - q_2^*)}{(1-\lambda_1)\rho_1 + (1-\lambda_2)\rho_2} = k \tag{A.1}$$

where λ_j is the fraction of type-j buyers that are informed, and the denominator is the total measure uninformed buyers. If a local shop caters to type-j buyers then $\{q_L^j, n_L^j, p_L^j\}$ solves

$$\max_{q_L^j, n_L^j, p_L^j} \left\{ \frac{\alpha(n_L^j)}{n_L^j}[\epsilon_j u(q_L^j) - p_L^j] \right\} \text{ st } \alpha(n_L^j)(p_L^j - q_L^j) = k.$$

Given $\{n_T, n_L^1, n_L^2\}$ and the measures of uninformed and informed buyers, we can solve for the measure of all sellers N and local sellers N_L^j from

$$N = \frac{(1-\lambda_1)\rho_1 + (1-\lambda_2)\rho_2}{n_T}, \quad N_L^j = \frac{\lambda_j \rho_j}{n_L^j}. \tag{A.2}$$

The measure of tourist shops is then $N_T = N - N_L^1 - N_L^2$. The probability of an uninformed buyer entering the tourist submarket is N_T/N and the probability of entering a local shop catering to type-j buyers is N_L^j/N.

Let U_j^* be the expected payoff of type-j conditional on entering the correct local submarket,

$$U_j^* \equiv \frac{\alpha(n_L^j)}{n_L^j}\left[\epsilon_j u(q_L^j) - p_L^j\right].$$

The expected payoff for a type-1 uninformed buyer in the DM is

$$\frac{N_T}{N}0 + \left(1 - \frac{N_T}{N}\right)\left(\frac{N_L^1}{N_L}U_1^* + \frac{N_L^2}{N_L}U_1^2\right); \tag{A.3}$$

type-2 is similar. Notice here that the information externality can be negative:

Proposition 9 *An increase in λ_2 raises N_L^1 and N_L^2, lowers N_T and N, and raise the expected payoff of both types of uninformed buyers. An increase in λ_1 raises N_T, N, N_L^1/N and N_L^2/N, provided $\alpha(n)$ is sufficiently inelastic, and lowers the expected payoff of both types uninformed buyers.*

Proof: By (A.1) a small increase in λ_1 reduces n_T. If $\alpha(n)$ is sufficiently inelastic, the drop in n_T would be arbitrarily large and N in (A.2) rises by an arbitrary amount. Since N_L^1 increases at the same rate as λ_1 by (A.2) and N_L^2 remains constant, N_L^1/N and N_L^2/N fall and N_T/N rises if $\alpha(n)$ is sufficiently inelastic. Since N_T/N can rise by an arbitrarily amount, a type 1 uninformed buyer' payoff drops by (A.3) (the ratio N_L^1/N_L and N_L^2/N_L change in λ_1 but their effect would be dominated by that of N_T/N). One can use the same logic to show a type-2 uninformed buyer' payoff falls in λ_1 provided $\alpha(n)$ is sufficiently inelastic. ■

Now suppose λ is endogenous, and consider equilibrium where informed buyers can always find the relevant local shops (i.e., ignore the trivial equilibrium where buyers do not acquire information because they think submarket L is empty, and submarket L is empty because sellers think there is no informed buyers). Then stationary equilibrium is unique. Depending on the size of s, there are four possible cases:

Proposition 10 *There are four types of equilibrium*

1. *If $s \geq U_2^*$, then $\lambda_1 = \lambda_2 = 0$.*

2. *If $U_2^* > s > U_2^* \left(\frac{U_1^* - U_1^2}{U_2^* - U_1^2}\right)$, then $\lambda_2 > \lambda_1 = 0$.*

3. *If*
$$U_2^* \left(\frac{U_1^* - U_1^2}{U_2^* - U_1^2}\right) \geq s \geq \frac{(U_1^* - U_1^2)(U_2^* - U_2^1)}{U_1^2 + U_2^1},$$
then $\lambda_1, \lambda_2 > 0$.

4. *If*
$$s < \frac{(U_1^* - U_1^2)(U_2^* - U_2^1)}{U_1^2 + U_2^1},$$

then $\lambda_1 = \lambda_2 = 1$.

Proof: We consider each case in turn.

Case 1: $\lambda_1 = \lambda_2 = 0$. When s is large, no buyer acquires information and only the tourist submarket is open. Since $\theta = 0$, buyers' DM payoff is 0. Then the value of information to a type-j buyer is U_j^*. If $s \geq U_j^*$ for $j = 1, 2$, no buyer acquires information. It is easy to show $U_2^* \geq U_1^*$. Thus, equilibrium with $\lambda_1 = \lambda_2 = 0$ exists if and only if $s \geq U_2^*$.

Case 2: $\lambda_2 > \lambda_1 = 0$. For $s \leq U_2^*$, some type-2 buyers acquire information. Since $\lambda_2 \in (0, 1)$, they are indifferent between acquiring information or not. Therefore,

$$U_2^* - s = \frac{N_T}{N} 0 + \left(1 - \frac{N_T}{N}\right) U_2^* \implies \frac{N_T}{N} = \frac{s}{U_2^*}.$$

Given N_T/N, one can solve for λ_2 by (A.2).

To sustain $\lambda_1 = 0$, we need to make sure type-1 buyers do not acquire information. Letting U_j^ℓ be the payoff of a type j buyer at a local shop catered to a type ℓ agent, we need

$$U_1^* - s < \frac{N_T}{N} 0 + \left(1 - \frac{N_T}{N}\right) U_1^2 \implies s > U_1^* - \left(1 - \frac{N_T}{N}\right) U_1^2.$$

Using $N_T/N = s/U_2^*$, one can show this equilibrium exists when

$$U_2^* > s > U_2^* \left(\frac{U_1^* - U_1^2}{U_2^* - U_1^2}\right).$$

Case 3: $\lambda_2, \lambda_1 > 0$. When s is small, both types acquire information. The expected DM payoff for a type 1 uninformed buyer is

$$\frac{N_T}{N} 0 + \left(1 - \frac{N_T}{N}\right) \left(\frac{N_L^1}{N_L} U_1^* + \frac{N_L^2}{N_L} U_1^2\right).$$

It follows that the indifference conditions are

$$\begin{aligned} U_1^* - s &= \left(1 - \frac{N_T}{N}\right) \left(\frac{N_L^1}{N_L} U_1^* + \frac{N_L^2}{N_L} U_1^2\right) \\ U_2^* - s &= \left(1 - \frac{N_T}{N}\right) \left(\frac{N_L^2}{N_L} U_2^* + \frac{N_L^1}{N_L} U_2^1\right). \end{aligned}$$

Using these we can solve for

$$\frac{N_T}{N} = \frac{(U_1^2 + U_2^1)s - (U_1^* - U_1^2)(U_2^* - U_2^1)}{U_2^*(U_2^1 + U_1^2) - (U_2^* - U_2^1)(U_2^* - U_1^2)} \tag{A.4}$$

$$\frac{N_L^1}{N_L} = \frac{(U_1^* - U_1^2) - (U_2^* - U_1^*)\frac{N_T}{N_L}}{U_1^2 + U_2^1}. \tag{A.5}$$

As s decreases, N_T/N falls and N_L^1/N_L rises. An equilibrium exists when $N_L^1/N_L, N_T/N > 0$. One can show that this holds when

$$U_2^* \left(\frac{U_1^* - U_1^2}{U_2^* - U_1^2} \right) \geq s \geq \frac{(U_1^* - U_1^2)(U_2^* - U_2^1)}{U_1^2 + U_2^1}.$$

Case 4: $\lambda_2 = \lambda_1 = 1$. When s is sufficiently small, all buyers are informed and the tourist submarket vanishes. By (A.4), $N_T/N < 0$ when

$$s < \frac{(U_1^* - U_1^2)(U_2^* - U_2^1)}{U_1^2 + U_2^1}. \qquad \blacksquare$$

B Alternative Specification

Here we show that with a different way to model endogenous information $i = 0$ is always suboptimal. Suppose buyers are informed iff they pay $s > 0$. Then the two types of buyers, informed and uninformed, choose real balances z_I and z_U. Local shops post terms of trade to attract informed buyers. Assume (as in Rocheteau and Wright 2005) local shops post terms of trade to maximize informed buyers' expected payoff subject to free entry:

$$\max_{q_L, n_L, p_L, z_L} \left\{ \frac{\alpha(n_L)}{n_L} (u(q_L) - p_L - \gamma(p_L - z_L)) - i z_I \right\}$$
$$s.t. \quad \alpha(n_L)(p_L - q_L) = k.$$

Note that local shops have no restriction $p_L \leq z_I$, because they have first-mover advantage that can affect informed buyers' z.

Given z_U, the terms of trade $\{q_T, n_T, p_T\}$ at tourist shops solve Kalai bargaining and free entry:

$$
\begin{aligned}
u'(q_T) &= 1 + \gamma'(p_T - z_U) \\
z_U - q_T &= (1-\theta)(u(q_T) - q_T - \gamma(p_T - z_U) \\
\alpha(n_T)(z_U - q_T) &= k.
\end{aligned}
$$

At $i = 0$ buyers carry enough cash to get q^*, $z_I^* = (1-\eta)u(q^*) + \eta q^*$ and $z_U^* = (1-\theta)u(q^*) + \theta q^*$. Hence, $\theta < \eta$ implies $z_U \geq z_I$ for $i = 0$, and, by continuity also for $i \approx 0$. Similarly, buyers strictly prefer not to bargain at local shops for $i \approx 0$. It follows that z_U maximizes the expected DM payoff,

$$V(z) = W(z+\tau) + \omega_L \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z)] + \omega_T \frac{\alpha(n_T)}{n_T}[u(q_T) - p_T - \gamma(p_T - z)].$$

The chance of entering each submarket is

$$\omega_L = \frac{N_L}{N_L + N_T} = \frac{\lambda n_T}{(1-\lambda)(n_L - n_T)} \quad \text{and} \quad \omega_T = 1 - \omega_L. \tag{B.1}$$

The choice of z_U is given by the FOC

$$i = \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(p_L - z) + \omega_T \frac{\alpha(n_T)}{n_T}[u'(q_T)\frac{dq_T}{dp_T} - 1].$$

From the bargaining solution, this becomes

$$i = \omega_L \frac{\alpha(n_L)}{n_L}\gamma'(p_L - z) + \omega_T \frac{\alpha(n_T)}{n_T}\left[\frac{\theta(u'(q_T) - 1)}{1 + (1-\theta)(u'(q_T) - 1)}\right]. \tag{B.2}$$

If a buyer pays s then the buyer sees all (p, q) with probability a; otherwise, he sees one (p, q). He chooses z after learning his information status. The payoffs for informed and uninformed buyers are

$$V_I = \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z_I)] - iz_I \tag{B.3}$$

$$
\begin{aligned}
V_U = {} & \omega_L \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z_U)] \\
& + \omega_T \frac{\alpha(n_T)}{n_T}[u(q_T) - p_T - \gamma(p_T - z_U)] - iz_U.
\end{aligned} \tag{B.4}
$$

Finally, for $\lambda \in (0, 1)$ we require $s = V_I - V_U$.

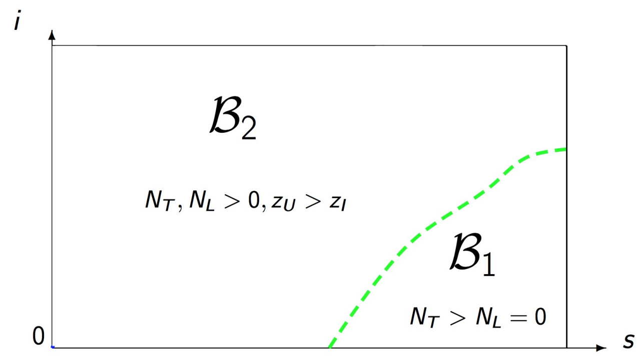

Figure 9: Equilibrium regions with endogenous information.

Definition 5 *A stationary monetary equilibrium with endogenous λ is a nonnegative list $\langle \lambda, \Gamma_L, \Gamma_T, z_I, z_u \rangle$ solving the above conditions with $z_U, z_I > 0$.*

let us focus on equilibria with $\lambda > 0$. In Fig. 9, in region $\mathcal{B}_1$ the cost s is so high that no buyer becomes informed and only submarket T is open, $\lambda = N_L = 0$. In $\mathcal{B}_2$ both submarkets are open and $z_U > z_I$. Note that submarket T is always open, since for all $s > 0$ we have $\lambda < 1$ for the usual reason: $\lambda = 1$ implies all shops are the same, so it is not a best response to pay s. The next result establishes existence and gives a sufficient condition for uniqueness.

Proposition 11 *Monetary equilibrium with endogenous $\lambda \in (0,1)$ exists and it is unique if i and s are not too big and $\alpha(n)/n$ is sufficiently inelastic.*

Proof. Given i we can solve for the terms of trade in the local submarket Γ_L and z_I. Note that Γ_L does not depend on s, Γ_T or z_U.

Consider (B.2) and assume q_T is a function of n_T as defined by the free entry condition and bargaining solution of submarket T. Then (B.2) defines ω_T as an implicit function of n_T. Since q_T falls in n_T, the left-hand side of (B.2) rises in n_T when $\alpha(n_T)/n_T$ is sufficiently inelastic. Hence (B.2) defines ω_T as a falling function of n_T, call it $\omega_{T1}(n_T)$.

Next consider (B.4). Again assume q_T and p_T are determined by the bargaining solution and assume z_U is chosen to maximize V_U. Given V_U, (B.4) defines ω_T as a falling function of n_T by the envelope theorem, call it $\omega_{T2}(n_T)$. When $\alpha(n)/n$ is sufficiently inelastic $\omega_{T2}(n_T)$ falls very slowly in n_T. An equilibrium with $\lambda \in (0,1)$

is a pair of (ω_T, n_T) that solves $\omega_{T1}(n_T) = \omega_{T2}(n_T)$. Since ω_{T1} falls in n_T and ω_{T2} is constant in n_T when $\alpha(n)/n$ is sufficiently inelastic, there is a unique solution for (ω_T, n_T).

When $s = 0$ there clearly exists an equilibrium with $\lambda > 0$. When s is sufficiently large this equilibrium disappears because no buyer would acquire information and $\omega_T = 1$. As shown in Proposition 12 ω_T rises in s and therefore there is a cutoff for s such that $\omega_T \leq 1$ iff s is below the cutoff. ■

Next we derive comparative statics for equilibrium in $\mathcal{B}_2$, which is the interesting region, because information and money are substitutes in the sense that informed buyers carry less cash.

Proposition 12 *If monetary equilibrium with endogenous λ is unique, in $\mathcal{B}_2$, as i rises, q_T, q_L, z_U, z_I and ω_T fall, and n_T and n_L rise. Moreover, as s rises, q_T, z_U and ω_T rise, n_T falls, and q_L, n_L and z_I remain unchanged.*

Proof. As i rises, naturally q_L, z_I fall and n_L rises, see Choi (2017) for a proof. Next subtract (B.3) from (B.4) to get

$$-s = -V_I + \omega_L \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z_U)] + \omega_T \frac{\alpha(n_T)}{n_T}[u(q_T) - p_T - \gamma(p_T - z_U)] - iz_U.$$

Note that $dV_I/di = -z_I$ by the envelope theorem. Fixing n_T and ω_T, the derivative of the last three terms is $-z_U$ also by the envelope theorem. Fixing ω_T, as i increases z_U falls and n_T rises, which leads to a even bigger drop in the right-hand side. When i is sufficiently small $z_U > z_I$ and thus the right side falls in i. Therefore $\omega_{T2}(n_T)$ falls in i for any given n_T.

By (B.2) $\omega_{T1}(n_T)$ rises for all n_T as i rises. Therefore ω_T falls and n_T rises in equilibrium. By the free entry condition q_T, p_T and z_T all fall in i.

As s rises, Γ_L, z_I and V_L are unchanged. Then V_U must drop. It follows that $\omega_{T2}(n_T)$ rises for all n_T. Therefore ω_T rises and n_T falls in s in equilibrium. By the free entry condition and bargaining solution of the tourist submarket q_T, p_T and z_U rise in s. ■

Now consider welfare. Using the envelope condition one can show

$$\frac{\partial\Omega}{\partial i}_{|i=0} = (1-\lambda)(z_U^* - z_I^*) > 0.$$

Hence, $i = 0$ is *not* optimal. This is because higher i makes buyers carry less cash and hence more willing to pay more to avoid tourist shops. While raising i from 0 reduces real balances, the loss is second-order by the envelope theorem, while higher λ implies fewer tourist shops and that is a first-order gain. This is relevant because people consider it a puzzle that the Friedman rule is optimal in many models and yet central banks rarely if ever target $i = 0$. While not the first model to provide a reason why $i > 0$ may be desirable, we think our effect is novel and compelling.

Proposition 13 *With endogenous $\lambda \in (0,1)$, $i > 0$ is optimal.*

Proof. The total welfare is $\Omega = -s\lambda + \lambda V_I + (1-\lambda)V_U + \tau i$, where $\tau = \lambda z_I + (1-\lambda) z_U$. Substitute τ and $s = V_I - V_U$ to get

$$\Omega = V_I - s + i[\lambda z_I + (1-\lambda) z_U],$$

and derive

$$\frac{\partial\Omega}{\partial i} = (1-\lambda)(z_U - z_I) + i\frac{\partial[\lambda z_I + (1-\lambda)z_U]}{\partial i}.$$

This uses $\partial V_I/\partial i = -z_I$ by the Envelope Theorem. This expression is strictly positive at $i = 0$ because $z_U = p_T^* > p_L^* = z_I$ at $i = 0$. ■

C Quantitative Robustness

Here we consider alternative models or calibration strategies. The results are reported in Figure 10.

Markup and Standard Deviation Targets Consider halving the target markup and standard deviation of prices, to 20% for the former and of 7.5% for the latter (other targets stay the same). This yields similar values for the utility and cost parameters, $b = 0.67$, $B = 0.645$, $c = 5.4$ and $C = 12.5$. The lower standard deviation implies

$\theta = 0.87$ rather than 0.72, and the lower markup implies $\lambda = 0.19$ rather than 0.38. The markups in the submarkets L and T come out as 13.7% and 27.6%. The welfare cost of inflation falls slightly: going from 10% to π^* is worth 0.9% of consumption rather than 1.1%. The welfare gain of going to $\lambda = 1$ from the calibrated value is 1.2% rather than 1.0%, while the gain of going to $\lambda = 1$ from 0 is 1.9% rather than 3.5%. The impact of changing credit conditions is essentially unchanged.

Heterogeneous Credit Costs Suppose informed agents not only know the terms of trade at all shops, but also know the owners well enough to get a lower cost of credit. Thus, the credit cost function is $\gamma_j(d) = C_j d^c$, where $j = I, U$ and $C_I < C_U$. For the sake of illustration, if $C_I = C_U/3$ then calibration yields $C_U = 33$ and $C_I = 11$, and while this is somewhat arbitrary, to give a sense of magnitude, it implies this: the informed use credit for 70% of their expenditures, while for the uninformed the number is 42%. The remaining parameters are basically unchanged. The welfare cost of inflation, information, and credit conditions change little. The cost of 10% inflation is around 0.8%. The gain of going to $\lambda = 1$ from 0 is 1.8% and the gain of going to $l = 1$ from the calibrated value is 4.2%. The impact of changing credit conditions remains small.

Heterogeneous Money Holdings Suppose in Model 1 agents choose money holdings after learning if they are informed. This implies that since information leads to better ex-post terms of trade, informed agents will in general hold fewer real balances, $z_I \leq z_U$. The calibration strategy remains the same leading to parameters $b = 0.76$, $B = 0.52$, $c = 5.8$, $C = 25.1$, $\theta = 0.65$, $k = 0.02$ and $E = 0.049$. The welfare cost of inflation, information, and credit conditions change little. The cost of 10% inflation is around 0.6%. The gain of going to $\lambda = 1$ from 0 is 4.0% and the gain of going to $\lambda = 1$ from the calibrated value is 1.9%. The impact of changing credit conditions remains small.

Multiple DM's Consider a model with multiple – here, two – rounds of DM trade, where the informed are informed in both rounds and the rest uninformed in both rounds.

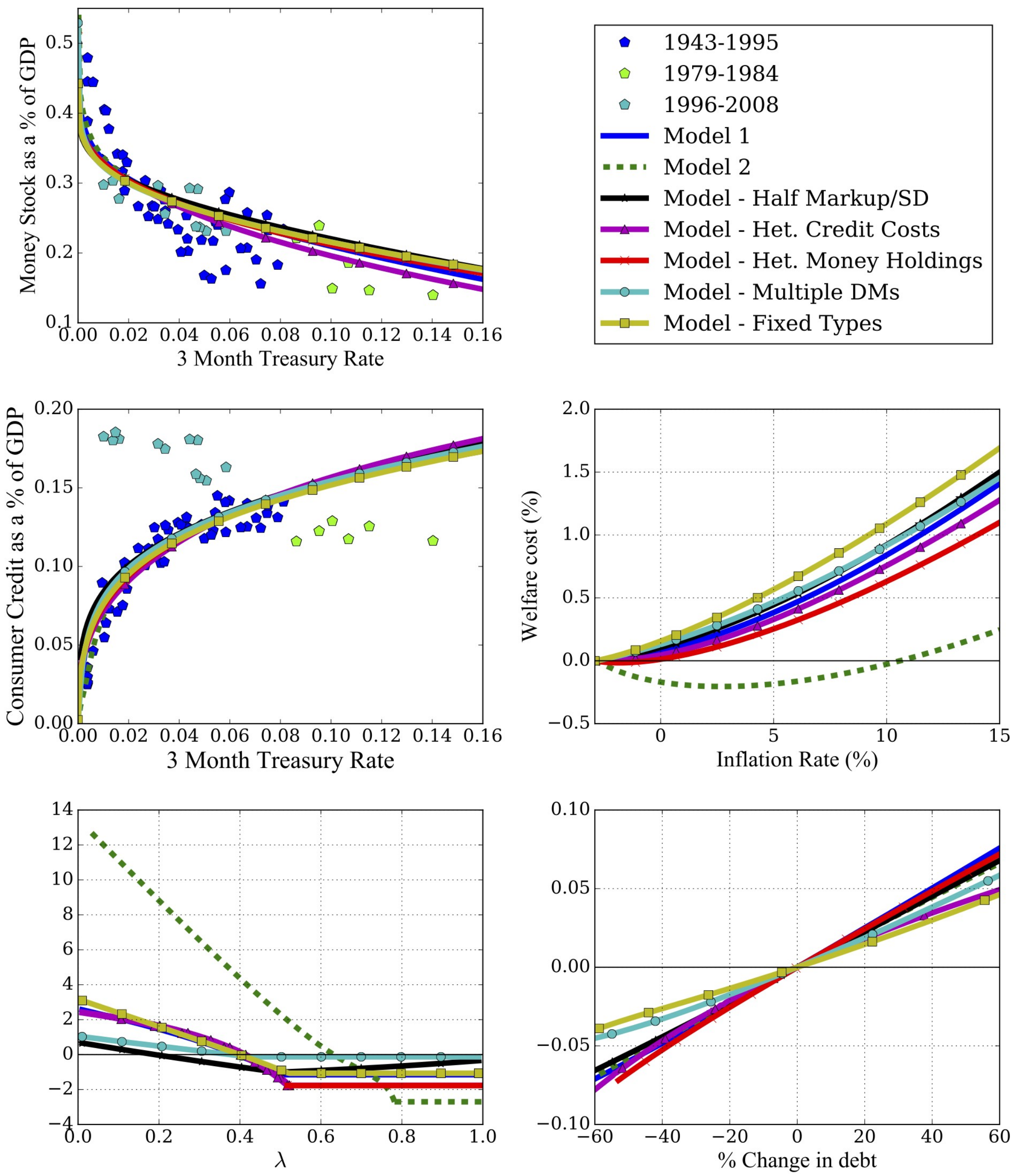

Figure 10: Robustness: (from top-left to bottom-right) money demand, credit demand, and the welfare cost of, respectively, i, λ, and C across models.

Here we assume λ is exogenous (although endogenous λ gives similar results). For simplicity, each buyer makes only 1 purchase per period, so if he trades in DM1 he skips DM2. Hence, there are exactly two types of buyers (informed and uninformed) in each DM. This gets in persistence in money holdings and information (across DM's) that has consequences for the distribution of prices and entry of tourist shops. In DM1, buyers have an outside option to trade in DM2, which lowers the surplus of high-price sellers. Hence, tightness in submarket is T higher in DM1 than DM2, and for calibrated parameters tourist shops are open in DM2 but not DM1. The calibrated utility and cost parameters are similar to the baseline: $b = 0.69$, $B = 0.55$, $c = 5.4$ and $C = 14.9$. Also, $\theta = 0.65$ and $\lambda = 0.37$.

The measure of tourist shops falls from 42% in the baseline specification to 7%. However, their markup increases from 65% to 100%. So while buyers are less likely to run into tourist shops, they get it worse when they do. The cost of 10% inflation does not change much, falling only to 0.9%. The gain of going to $\lambda = 1$ from 0 is 1.2% and of going to $\lambda = 1$ from the calibrated value is 0.2%.

Fixed Permanent Information Types Suppose information types are permanent. Of course, buyers choose money given their type. However, now changes in inflation or variables do not alter λ. Calibration yields $b = 0.76$, $B = 0.52$, $c = 5.8$, $C = 24.81$, $\theta = 0.81$ and $\lambda = 0.40$. The cost of inflation and incomplete information are essentially unchanged.