Optimal Credit Fluctuations*

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S1. Core and competitive equilibrium

Recall that an allocation, $\mathcal{L} = \{(y(i), \ell(i)), (y(j), \ell(j)) : i \in \mathbb{B}, j \in \mathbb{S}\}$, where $(y(i), \ell(i))$ denotes buyer i's DM and CM consumptions and $(y(j), \ell(j))$ denotes seller j's DM and CM consumptions, is in the core if there is no blocking (finite) coalition, $\mathcal{I} \subset \mathbb{B} \cup \mathbb{S}$, such that each agent in $\mathcal{I}$ enjoys at least the same utility as his allocation in $\mathcal{L}$, but at least one of them is strictly better off. Now we show that the only core allocation is the competitive outcome, with debt limit, d, is given by the symmetric allocation, (y, ℓ), such that $\ell = \eta(y) \equiv v'(y)y$ and $y = \min\{y^*, \eta^{-1}(d)\}$.

First notice that, by standard arguments, the competitive outcome is in the core. For necessity, we restrict ourselves to symmetric allocations. For a justification of such assumption, see Mas-Colell et al. (1995). Note that to be in the core, $u(y) \geq \ell \geq v(y)$. First we show that $\ell = v'(y)y$. Suppose, by contradiction, $\ell \neq v'(y)y$. Assume that $\ell < v'(y)y$. The other direction has a similar proof. Let ε be so small that

$$[v'(y) - \varepsilon]y > \ell. \tag{1}$$

Consider a coalition with m buyers and n sellers such that with $\delta = m/n < 1$, we have

$$\frac{v(y) - v(\delta y)}{(1 - \delta)y} > v'(y) - \varepsilon. \tag{2}$$

Consider the following allocation: each buyer consumes y and issues an IOU with face value ℓ, and each seller produces δy and receives an IOU with face value $\delta\ell$. Note that such allocation is feasible:

$$my = n\delta y \text{ and } m\ell = n\delta\ell.$$

Now, each buyer enjoys the same utility as before, but each seller has a higher utility: combining (1) and (2),

$$v(y) - v(\delta y) > [v'(y) - \varepsilon](1 - \delta)y > (1 - \delta)\ell,$$

and hence

$$\delta\ell - v(\delta y) > \ell - v(y).$$

This proves $\ell = v'(y)y = \eta(y)$. Finally, if $y < \min\{y^*, \eta^{-1}(d)\}$, then a buyer and a seller can form a coalition to increase surplus.

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S2. Recursive formulation of the mechanism design problem

Under large-group meeting, the program that selects the best PBE is

$$\max_{\{y_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \alpha \left[u(y_t) - \upsilon(y_t)\right] \tag{3}$$

$$\text{s.t.} \qquad \lambda\eta(y_t) \leq \alpha \sum_{s=1}^{+\infty} \beta^s \left[u(y_{t+s}) - \eta(y_{t+s})\right] \tag{4}$$

$$y_t \leq y^* \text{ for all } t = 0, 1, 2, ... \tag{5}$$

recursively. First we show that recursive formulation with promised utility as a state variable is equivalent to the original sequence problem.

Lemma 1 *A sequence $\{y_t\}_{t=0}^{\infty}$ satisfies (4) and (5) if and only if there is a sequence $\{\omega_t\}_{t=0}^{\infty}$ such that, for all $t = 0, 1, 2, ...$,*

$$\omega_t \leq \alpha \left[u(y_t) - \eta(y_t)\right] + \beta\omega_{t+1}, \tag{6}$$

$$\eta(y_t) \leq \beta\omega_{t+1}/\lambda, \tag{7}$$

$$y_t \in [0, y^*], \tag{8}$$

$$\omega_t \in [0, \bar{\omega}]. \tag{9}$$

Proof. Suppose that $\{y_t\}_{t=0}^{\infty}$ satisfies (4) and (5). Then, define, for each $t = 0, 1, 2, ...$,

$$\omega_t = \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})]. \tag{10}$$

The right side of (4) is equal to $\beta\omega_{t+1}$ for each t. Hence, $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (7). By definition of $\hat{y}$,

$$u(y_t) - \eta(y_t) \leq u(\hat{y}) - \eta(\hat{y}) \text{ for all } t \in \mathbb{N}_0.$$

It follows from (5) that $\{\omega_t\}_{t=0}^{\infty}$ satisfies (9). Finally, by (10),

$$\omega_t = \alpha[u(y_t) - \eta(y_t)] + \beta \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s+1}) - \eta(y_{t+s+1})] = \alpha[u(y_t) - \eta(y_t)] + \beta\omega_{t+1,0} \,.$$

for all $t \in \mathbb{N}$. Hence, $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (6).

Conversely, suppose that $\{\omega_t, y_t\}_{t=0}^{\infty}$ satisfies (6)-(9). Then, $\{y_t\}_{t=0}^{\infty}$ satisfies (5) by (8). To show (4), define, for each $t \in \mathbb{N}_0$,

$$\omega_t' = \sum_{s=0}^{\infty} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})]. \tag{11}$$

By (7), it suffices to show that $\omega_t \leq \omega_t'$ for all $t \geq 0$. Let t be given. We show by induction on T that

$$\omega_t \leq \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1}\omega_{T+1}. \tag{12}$$

When $T = 0$, this follows from (6). Suppose that it holds for T. Then,

$$\begin{aligned}
\omega_t &\leq \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1}\omega_{T+1} \\
&= \sum_{s=0}^{T} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+1} \left\{\alpha\left[u(y_{T+1}) - \eta(y_{T+1})\right] + \beta\omega_{T+2}\right\} \\
&= \sum_{s=0}^{T+1} \beta^s \alpha[u(y_{t+s}) - \eta(y_{t+s})] + \beta^{T+2}\omega_{T+2}.
\end{aligned}$$

This proves (12). Now, because, by (9), $\omega_{T+1} \leq \bar{\omega}$ for all T, it follows from the limit by taking T to infinity in (12) that $\omega_t \leq \omega_t'$. ■

Because of Lemma 1, we may replace the constraints (4) and (5) by (6)-(9). Note that the initial condition for the promised utility, ω_0, is also a choice variable.

Define the planner's value function, $V(\omega)$, as follows:

$$V(\omega) = \max_{\{y_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \alpha \left[u(y_t) - \upsilon(y_t) \right]$$

subject to (6)-(9) with $\omega_0 = \omega$. From the Principle of Optimality V satisfies the Bellman equations.

References

[1] Mas-Colell, Andreu, Michael Whinston, and Jerry Green (1995). Microeconomic Theory. Oxford University Press.