# Unemployment and the Distribution of Liquidity*

Zachary Bethune
University of Virginia

Guillaume Rocheteau
University of California, Irvine

January 2022

**Abstract**

We develop a New-Monetarist model of unemployment in which distributional considerations matter. Households who lack commitment are subject to both employment and expenditure risks. They self-insure by accumulating assets with different liquidity and returns. Inflation affects unemployment through two channels: an aggregate-demand channel according to which inflation reduces households' liquid wealth, and an interest rate channel through which inflation lowers firms' discount rate. We calibrate the model and show that it can match both cross-sectional and time series moments. Quantitatively, the two channels are large in magnitude but work in nearly equal, but opposite, directions, leading to a vertical long-run Phillips curve.

**JEL Classification Numbers**: D83, E24, E40, E50
**Keywords**: unemployment, money, inflation, distributional effects.

*Contacts: Zachary Bethune: bethune@virginia.edu, Guillaume Rocheteau: grochete@uci.edu. We are thankful for useful suggestions and comments from conference and seminar particpants at Penn State University, University of California - Irvine, the University of Virginia, the Soceity of Economic Dynamics meeting in Minneapolis, the virtual Money, Payments, Banking and Finance Workshop, the Search and Matching, the virtual Search and Matching Congress, the Southern Economics Association conference in Houston, and the virtual COCONUTS workshop.

# 1 Introduction

How does money creation affect equilibrium unemployment? The answer to this core question in macroeconomics has proven elusive empirically.[1] The question is also challenging theoretically as it requires a model with frictions in both goods and labor markets so as to make money essential and to generate unemployment in equilibrium. In the confines of pure currency economies, Berensten et al. (2011) – BMW thereafter – constructs such a model and shows that a higher rate of money creation lowers the rate of return on currency, thereby reducing consumers' holdings of liquid assets, firms' revenue, and job openings, i.e., the long-run Phillips curve is positively sloped.[2] For tractability, however, BMW omits the distributional effects of monetary policy – there is evidence that such effects are quantitatively important (e.g., Doepke and Schneider, 2006, and Auclert, 2019) – and assumes that households are neutral to unemployment risk. Moreover, BMW assumes that the ownership of firms is distributed evenly across consumers and cannot be traded, thereby shutting down a main channel from the incomplete-market models, namely, that the rate at which firms discount future profits is endogenous and depends on both public and private liquidity. The objective of this paper is to construct and calibrate a framework that unharnesses the ex-post heterogeneity resulting from both idiosyncratic expenditure and employment risks and that allows households to self-insure with both public and private liquidity in order to tease out and quantify the mechanisms through which money creation affects unemployment and welfare.

Our model is a two-good version of a Bewley (1980, 1983) economy with multiple assets where risk-averse households are unable to commit and hence cannot borrow. Following the banking literature, we label the two consumption goods as *early* (because consumption takes place before labor income and asset returns are paid) and *late*, where preferences over the goods are subject to idiosyncratic shocks. The distinction between early and late consumption has two purposes. First, the endogenous relative price between the two goods provides a channel through which the distribution of households' liquid wealth affects firms' revenue and job creation decisions. Second, it allows us to differentiate assets (money, government bonds, stocks, housing) according to their degree of liquidity. Specifically, while all assets can be liquidated in the late stage of each period, assets differ according to the ease with which they can be liquidated in the early stage, which we take as our notion of asset liquidity. The coexistence of multiple (partially) liquid assets will allow us to study different schemes to alter public liquidity (e.g., helicopter drops of money, inflation-financed unemployment benefits, changes in the supply of government bonds). In terms of market structures, goods markets are competitive and open in sequence. The labor market is frictional, with workers and jobs being matched bilaterally according to a time-consuming process, which creates an idiosyncratic unemployment risk.

[1] Benati (2015), using both classical and Bayesian structural VARs, shows that the data cannot reject positively or negatively sloped long-run Phillips curves.

[2] A related model was first proposed by Shi (1998) where large households are composed of a continuum of members who pool their money and share their resources. Our model will be closer to the version in BMW.

We start by studying a simplified version of our model that is analytically tractable and directly comparable to BMW. Relative to BMW, liquid wealth in our model is composed of both publicly-supplied liquidity (money and government bonds) and privately-supplied liquidity (stock mutual funds and housing). We identify two main channels through which changes in the money growth rate affect the economy. As in BMW, there is an aggregate demand channel according to which the revenue of the firm increases with the liquid wealth of consumers. Because we have a broader notion of liquid wealth, this channel is magnified in our model: an increase in public liquidity has a multiplier effect on aggregate liquidity through the valuation of stocks. There is a separate interest-rate channel according to which the discount rate of firms is endogenous and depends on the supply of liquidity. We show that the two channels work in opposite directions: an increase in the money growth rate or, equivalently, a decrease in the supply of private liquidity, raises unemployment according to the aggregate demand channel but reduces it according to the interest rate channel. In versions of our model with ex-post heterogeneity, we analytically show that distributional effects amplify the interest-rate channel.

The second part of the paper explores the quantitative implications of our full model by calibrating it to match standard labor market moments, the interest-rate elasticity of the demand for liquid assets, interest rate spreads, and household portfolio shares from the Survey of Consumer Finances. We take the view that early consumption represents large, unplanned household expenditures, such as vehicle or home repairs and medical expenses. Despite that the two main shocks in the model (employment and expenditure risks) are two-state Markov chains, the calibrated model can fit the distribution of liquidity well. The model matches qualitative features of the distribution of financial and housing wealth, e.g. wealthier households hold a larger share of their wealth in financial assets, but slightly underperforms quantitatively. However, the model performs well in matching untargeted moments such as the average consumption decline upon job loss and the cross-section of marginal propensities to consume.

Using the calibrated model, we illustrate the effects of changes in the rate of return of liquid assets through money growth. When money creation is transferred lump-sum to households, inflation reduces aggregate demand, increases unemployment and leads to a decline in output. The effects, however, are quantitatively negligible so that the long-run Phillips curve is almost vertical. A key finding is that even though the response of unemployment to anticipated inflation is small, it is the result of two strong channels of opposite sign. According to the aggregate demand channel, an increase in inflation from 0 to 10% raises the steady-state unemployment rate by more than one percentage point. According to the interest rate channel, the same increase in inflation reduces the unemployment rate by a little more than one percentage point. For our calibration, these two channels happen to cancel each other.

The inflation rate that maximizes steady-state welfare is between 5% and 10% and society's welfare is almost flat between these two values. The welfare gains arise from improved risk-sharing as a result of two dominant channels.

First, inflation via lump-sum money transfers is a progressive transfer that provides risk-sharing benefits. Second, even though inflation raises the cost for households to self-insure against expenditure risk, it improves households' ability to self-insure against employment risk by increasing returns on financial wealth and housing. The calibration implies that optimal inflation produces welfare benefits for all households except the most wealthy. Low-wealth households (although not the lowest wealth) benefit the most, and, conditional on wealth, the unemployed benefit less than the employed.

While the long-run Phillips curve is almost vertical for our benchmark calibration, we show that technological advances that make financial assets more liquid or changes in the implementation of monetary policy can affect the sign and size of the Phillips curve. If financial assets can be liquidated more often to finance expenditure shocks, then the interest channel of monetary policy becomes stronger and the long-run Phillips curve becomes negatively sloped, i.e., an increase in anticipated inflation reduces unemployment. On the contrary, if the 'helicopter drops' are targeted so that they are received by the unemployed only, then the long-run Phillips curve is upward sloping because the insurance provided by targeted transfers reduces the precautionary demand for financial assets, thereby reducing the strength of the interest-rate channel.

## 1.1 Literature

Our model has a similar structure as in BMW that extends the quasi-linear environment of Lagos and Wright (2005) to include a frictional labor market.[3] Relative to BMW, goods markets are competitive, as in Rocheteau and Wright (2005); we generalize preferences to make households risk averse so that unemployment risk is relevant; claims on firms' profits are tradable and their rate of return is endogenous; the set of assets is broader and includes partially liquid government bonds and housing. These elements have been studied individually in models of unemployment with degenerate wealth distributions. Liquid claims have been incorporated by Rocheteau and Rodriguez-Lopez (2014) and Branch and Silva (2020). Our description of housing is similar to He et al. (2015) and Branch et al. (2016).

New-Monetarist models with non-degenerate distributions of money holdings include Molico (2006), Green and Zhou (1998), Chiu and Molico (2010, 2011), Menzio et al. (2013), and Rocheteau et al. (2018), among others.[4] Our approach is closer to Rocheteau et al. (2021), which includes both expenditure and unemployment risks. However, our model is more general in terms of preferences and asset structure.

Our model where goods markets are competitive and households are price takers can be interpreted as a two-sector Bewley (1980, 1983) economy with multiple assets. Related Bewley economies include Hansen and Imrohoroglu (1992) who study optimal unemployment insurance and Algan et al. (2011) who study temporary and permanent

[3] In Rocheteau et al. (2007) only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. The BMW model has been extended to incorporate credit and various forms of liquidity, e.g., Bethune et al. (2015) and Branch et al. (2016).

[4] See also Jin and Zhu (2019, 2020) and Kam and Lee (2021).

changes in money growth. Frictional labor markets have been added to incomplete-market models by Krusell et al. (2010) and Eeckhout and Sepahsalari (2021), among others. A key difference in our approach is the distinction between early and late consumption that allows us to differentiate assets according to their liquidity following Williamson (2012), Venkateswaran and Wright (2013), and many other contributions to the New Monetarist literature surveyed in Lagos et al. (2017).

The recent Heterogeneous Agent New Keynesian (HANK) literature pioneered by Kaplan et al. (2018) also includes assets with different degrees of liquidity. In their model, the lack of liquidity of an asset is formalized through transaction costs to deposit or withdraw from an illiquid account. Kekre (2021) and Graves (2021) study business cycles in HANK models with frictional unemployment and two assets (bonds and capital). Kekre (2021) focuses the role of time-varying unemployment benefits as macroeconomic stabilization while Graves (2021) studies how the composition of assets is important for the amplification of shocks. In both environments, financial discount rates do not affect firm entry because either firms are owned by risk-neutral entrepreneurs instead of risk-averse households or because monetary policy fixes the real return of capital. Our focus on the long-run implications of inflation also differs from theirs.

## 2 Environment

Time is discrete and is indexed by $t \in \mathbb{N}$. The economy is populated by a unit measure of infinitely-lived households. Each period of time is divided into three stages. The first stage is a frictional labor market. The second and third stages have markets for goods and assets opening sequentially. There are three perishable goods and services: an early-consumption good produced in the second stage, a late-consumption good produced in the last stage, and housing services. We take the late-consumption good as the numeraire.

The lifetime expected utility function of a household is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t v(y_t) + U(c_t, h_t, e_t)\right], \tag{1}$$

where $\beta = (1+\rho)^{-1} \in (0,1)$, $y_t \in \mathbb{R}_+$ is early (second-stage) consumption, $\varepsilon_t \in \{0,1\}$ is a preference shock for early consumption, $c_t \in \mathbb{R}_+$ is late (third-stage) consumption, $h_t \in \mathbb{R}_+$ is housing services, and $e_t \in \{0,1\}$ is the worker's employment status. The utility functions, $v(y_t)$ and $U(c_t, h_t, e_t)$, are bounded, twice continuously differentiable, strictly increasing, and concave in $(c,h)$ and $y$, respectively. Since $v(y)$ is bounded below, we adopt the normalization $v(0) = 0$. The utility functions satisfy the following Inada conditions: $U_c(0, h_t, e_t) = U_h(c_t, 0, e_t) = +\infty$, $v'(0) = +\infty$, and $v'(\infty) = 0$. Preferences shocks, $\{\varepsilon_t\}_{t=0}^{+\infty}$, are i.i.d. across agents and time with $\Pr[\varepsilon_t = 1] = \alpha$ and $\Pr[\varepsilon_t = 0] = 1-\alpha$. So a household wishes to consume early with probability $\alpha$. The price of early-consumption in terms of the numeraire is $p^y$ and the rental price of housing is $p^h$.

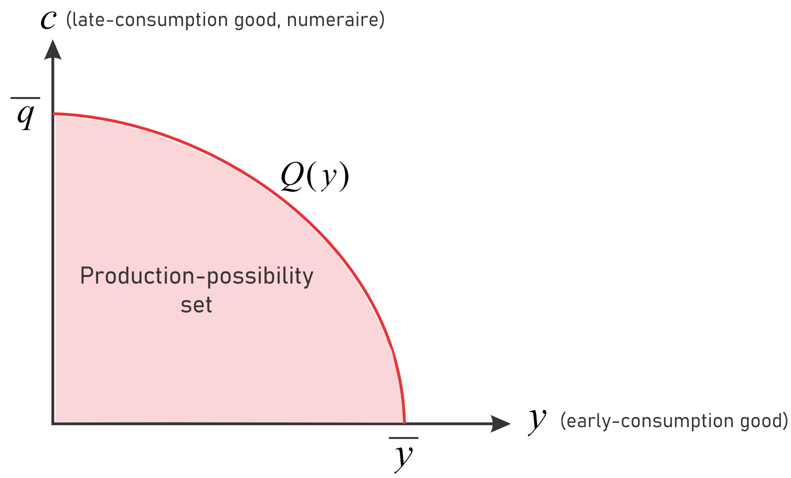


Figure 1: Production possibility frontier between early and late consumption

A firm is a technology to produce the second-stage good (early consumption) and the numeraire with one unit of labor as the only input. This technology is represented by the production-possibility frontier, $Q(y)$, that specifies the amount of numeraire a firm can produce if it has already produced $y$ units of the second-stage good. The production-possibility frontier satisfies $Q(0) = \bar{q} > 0$, $Q(\bar{y}) = 0$ for some $\bar{y} > 0$, $Q'(y) < 0$, $Q''(y) < 0$, $Q'(0) = 0$, and $Q'(\bar{y}) = -\infty$. See Figure 1. We define the opportunity cost of producing $y$ as $\kappa(y) \equiv Q(0) - Q(y)$. Hence, $\kappa(0) = 0$, $\kappa'(y) > 0$, $\kappa''(y) > 0$, $\kappa'(0) = 0$, and $\kappa'(\bar{y}) = +\infty$.

In order to create a job in period $t$, firms must open a vacant position, which costs $k^f > 0$ in terms of the numeraire in $t-1$. The measure of matches between vacant jobs and unemployed workers in period $t$ is given by $\mathcal{M}(s_t, o_t)$, where $s_t$ is the measure of job seekers and $o_t$ is the measure of job openings. The matching function, $\mathcal{M}$, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $\mathcal{M}(0, o_t) = \mathcal{M}(s_t, 0) = 0$ and $\mathcal{M}(s_t, o_t) \leq \min(s_t, o_t)$. The job finding probability for a worker is $\lambda_t = \mathcal{M}(s_t, o_t)/s_t = \mathcal{M}(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. The vacancy filling probability for a job is $\mathcal{M}(s_t, o_t)/o_t = \mathcal{M}(1/\theta_t, 1) = \lambda_t/\theta_t$. An existing match is destroyed at the beginning of a period with probability $\delta^f$. A worker who loses his job in period $t$ is unemployed in period $t$ and becomes a job seeker in period $t+1$. Therefore, $s_{t+1} = u_t$. The employment rate (measured after the matching phase at the beginning of the second stage) is denoted $n_t$ and the unemployment rate is $u_t$. Therefore, $u_t + n_t = 1$.

A household's income, $w_e$, is a function of its employment status and is decomposed into two components: a transfer (or tax) from the government, $\tau_e$, and a non-transfer income, $\bar{w}_e$. Hence, $\bar{w}_1$ is the wage in terms of the numeraire good paid in the last stage. We either take $\bar{w}_1$ as exogenous or we adopt some ad hoc wage determination rule. The non-transfer income (endowment) of the unemployed, $\bar{w}_0$, can be interpreted as income from limited self-employment opportunities.

Households are anonymous (i.e., their employment and trading histories are private) and cannot commit to honor future obligations. Hence, idiosyncratic expenditure and employment shocks are uninsurable through credit, which creates a need for precautionary savings. There are four types of assets, indexed in $\mathbb{A} \equiv \{m, g, f, h\}$. Fiat money is perfectly divisible, storable, and non-counterfeitable. Its supply, $M_t$, grows as the constant rate $\pi$. The price of money in terms of the numeraire is $\phi^m$. There is a fixed supply of one-period real government bonds, $A^g$. Each bond issued in the third stage is a claim to one unit of numeraire in the third stage in the following period. The third type of asset corresponds to shares in fully diversified investment funds that mutualize claims to firms' profits. The supply of these claims is endogenous and equal to the market capitalization of all firms. Finally, the fourth asset is housing. Its endogenous supply is denoted $H$, and its price in terms of the numeraire is $\phi^h$. The cost in terms of numeraire to produce a unit of housing in the third stage is $k^h > 0$ and there is free entry into the creation of homes. The depreciation rate of housing is $\delta^h$. We denote the gross rate of return of asset $j \in \mathbb{A}$ as denoted $R^j = 1 + r^j$.

Assets are subject to resalability constraints in the second stage. We denote $\Omega_{\mathbb{A}}$ the set of all non-empty subsets of $\mathbb{A}$. Conditional on $\varepsilon = 1$, the set of assets that are acceptable to finance early consumption is $\omega \in \Omega_{\mathbb{A}}$ with probability $\alpha_\omega/\alpha$ where $\sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega = \alpha$.[5] For instance, if $\omega = \{m\}$, then $\alpha_\omega$ is the probability of a preference shock for early consumption where only money is accepted as a means of payment. If $\omega = \{m, b\}$, then money and bonds are accepted. And so on. We denote $\chi^j_\omega = \mathbb{I}_{\{j \in \omega\}} \in \{0, 1\}$ as the acceptability of asset $j \in \mathbb{A}$ in the event where $\omega \in \Omega_{\mathbb{A}}$ is realized. In the last stage, all assets are equally acceptable.

# 3 Equilibrium

We characterize steady-state equilibria where the distribution of asset portfolios, the rates of return of assets, and the relative price of consumption goods and services are constant over time.

## 3.1 Households

We first describe the household's consumption and asset portfolio problem taking the price of early consumption, $p^y$, the rental price of housing, $p^h$, and the gross rates of return of assets, $\left\{R^j\right\}_{j \in \mathbb{A}}$, as given. The state of a household when entering the last stage is composed of its employment status, $e \in \{0, 1\}$, and its total wealth expressed in the numeraire, $a = \sum_{j \in \mathbb{A}} a^j$, where $a^j$ denotes real holdings of asset $j$. The household's value function is given by:

$$W_e(a) = \max_{c,h,\hat{\mathbf{a}}} \left\{U(c, h, e) + \beta \mathbb{E}_e V_{e'}(\hat{\mathbf{a}})\right\} \text{ s.t. } c + p^h h + \mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e, \tag{2}$$

where all control variables are subject to nonnegativity constraints, $V_{e'}$ is the value function of the household in the employment state $e' \in \{0, 1\}$ at the start of the second stage, and $\mathbb{E}_e$ is the expectation operator with respect to $e'$

[5] This idea is formalized, e.g., in Lester et al. (2012) and Li et al. (2012).

conditional on its current employment state, $e$. The transition from $e$ to $e'$ occurs in the first stage. According to (2), the household chooses its current consumption, $c$, housing services, $h$, and next-period's portfolio, $\hat{\mathbf{a}} = (\hat{a}^m, \hat{a}^g, \hat{a}^h, \hat{a}^f)^\intercal$, in order to maximize its current utility plus its discounted continuation value in the following period. The budget identity specifies that total consumption of goods and housing services and the next-period discounted asset portfolio are equal to current income and wealth. The vector of discount factors for the different types of assets is denoted $\mathbf{R}^{-1} = (1/R^m, 1/R^g, 1/R^h, 1/R^f)$ and $\mathbf{R}^{-1}.\hat{\mathbf{a}} = \sum_{j\in\mathbb{A}} \hat{a}^j/R^j$.

The value function at the beginning of the second stage solves:

$$V_e(\hat{\mathbf{a}}) = \sum_{\omega\in\Omega_{\mathbb{A}}} \alpha_\omega \max_y \left[v(y) + W_e\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y\right)\right] + (1-\alpha)W_e(\mathbf{1}.\hat{\mathbf{a}}) \text{ s.t. } p^y y \in [0, \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}]\,, \tag{3}$$

where $\boldsymbol{\chi}_\omega = (\chi_\omega^j)_{j\in\mathbb{A}}$ is the vector of asset acceptability in state $\omega$ and $\mathbf{1} = (1,1,1,1)$. With probability $\alpha_\omega$, the household wishes to consume early but can only liquidate assets in $\omega$ to finance its consumption level, $y$. Formally, the total expenditure, $p^y y$, cannot exceed the household's resalable wealth, $\boldsymbol{\chi}_\omega.\hat{\mathbf{a}} = \sum_{j\in\omega} \hat{a}^j$. With probability, $1-\alpha$, the household does not wish to consume early and enters the third stage with total wealth $\mathbf{1}.\hat{\mathbf{a}} = \sum_{j\in\mathbb{A}} \hat{a}^j$. We combine (2) and (3) to obtain a single Bellman equation:

$$W_e(a) = \max_{c,h,\mathbf{y},\hat{\mathbf{a}}} \left\{ U(c,h,e) + \beta\mathbb{E}_e\left\{ \sum_{\omega\in\Omega_{\mathbb{A}}} \alpha_\omega\left[v(y_{\omega e'}) + W_{e'}\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}\right)\right] + (1-\alpha)W_{e'}(\mathbf{1}.\hat{\mathbf{a}})\right\}\right\} \tag{4}$$
$$\text{s.t. } c + p^h h + \mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e \text{ and } p^y y_{\omega e'} \le \boldsymbol{\chi}_\omega.\hat{\mathbf{a}},\ \ e' \in \{0,1\}.$$

The household makes plans for its next-period early-consumption contingent on its future employment status and asset resalability, $\mathbf{y} = (y_{\omega e})$. In the last stage, all assets are perfectly fungible in total wealth, which is represented by $a \in \mathbb{R}_+$. In the second stage, assets differ in their acceptability and, hence, the asset portfolio is represented by a vector, $\hat{\mathbf{a}} \in \mathbb{R}_+^4$. Proposition 1 gaurantees there is a unique solution to (2)-(3). All proofs are provided in Appendix A in Section 8.

**Proposition 1** ***(Households' Value Functions.)*** *There is a unique pair of value functions, $(W_e, V_e)$, solutions to (2)-(3) in the space of continuous and bounded functions. Moreover, $W_e$ and $V_e$ are increasing, concave, and continuously differentiable with $W_e'(a) = U_c\left[c_e(a), h_e(a), e\right]$ and*

$$\frac{\partial V_e(\hat{\mathbf{a}})}{\partial \hat{a}^j} = \sum_{\omega\in\Omega_{\mathbb{A}}} \alpha_\omega \left\{ \chi_\omega^j \frac{v'\left[y_{\omega e}(\hat{\mathbf{a}})\right]}{p^y} + (1-\chi_\omega^j)W_e'\left[\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}})\right]\right\} + (1-\alpha)W_e'(\mathbf{1}.\hat{\mathbf{a}}), \tag{5}$$

*for all $j \in \mathbb{A}$, where $c_e(a)$ is the last-stage policy function that specifies late-consumption as a function of total wealth and employment status; and $y_{\omega e}(\hat{\mathbf{a}})$ is the second-stage policy function that specifies early-consumption as a function of the portfolio at the start of the second stage, $\hat{\mathbf{a}} = (\hat{a}^m, \hat{a}^g, \hat{a}^h, \hat{a}^f)^\intercal$, employment status, and resalability event.*

The optimal portfolio choice in the last stage obeys Euler equations obtained by substituting $c = a + w_e - \mathbf{R}^{-1}.\hat{\mathbf{a}} - p^h h$ into (2) and taking first-order conditions:

$$-U_c(c,h,e) + R^j \beta \mathbb{E}_e \frac{\partial V_{e'}(\hat{\mathbf{a}})}{\partial a^j} \leq 0, \quad \text{“} = \text{”} \text{ if } \hat{a}^j > 0, \text{ for all } j \in \mathbb{A}. \tag{6}$$

According to (6), the marginal utility of late consumption, $U_c(c,h,e)$, is equalized to the discounted marginal benefit of asset $j$ in the following period. In order to understand the role of the resalability coefficients for asset pricing, it is instructive to substitute $\partial V_{e'}(\hat{\mathbf{a}})/\partial a^j$ by its expression given by (5), i.e.,

$$-U_c(c,h,e) + R^j \beta \mathbb{E}_e \left[ \sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega W'_{e'} \left[ \mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}(\hat{\mathbf{a}}) \right] + (1-\alpha) W'_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right]$$
$$+ R^j \beta \mathbb{E}_e \left[ \sum_{\omega \in \Omega^j_{\mathbb{A}}} \alpha_\omega \left\{ \frac{v' \left[ y_{\omega e'}(\hat{\mathbf{a}}) \right]}{p^y} - W'_{e'} \left[ \mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}(\hat{\mathbf{a}}) \right] \right\} \right] \leq 0, \tag{7}$$

with an equality if $\hat{a}^j > 0$ and where $\Omega^j_{\mathbb{A}} = \{\omega \in \Omega_{\mathbb{A}} : j \in \omega\}$. The only term that can account for differences in rates of return is the last term on the left side that depends of $\Omega^j_{\mathbb{A}}$, the acceptability of asset $j$. It can be interpreted as the expected nonpecuniary return from investing in an additional unit of asset $j$ that can serve as means of payment for early consumption for all resalability events $\omega \in \Omega^j_{\mathbb{A}}$.

We now turn to the optimality conditions for goods and services. The first-order condition for the choice of housing services is:

$$p^h U_c(c,h,e) = U_h(c,h,e). \tag{8}$$

The left side is the opportunity cost in term of foregone consumption valued according to the marginal utility of late consumption. The right side is the marginal utility of housing services. The choice of early consumption is obtained from (3) and the associated first-order condition:

$$v'(y_{\omega e}) \geq p^y W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}) \quad \text{“} = \text{”} \text{ if } p^y y_{\omega e} < \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}. \tag{9}$$

From (9) the optimal early-consumption choice is based on the comparison of the household's marginal utility from spending a unit of pledgeable wealth in the second stage, $v'(y)/p^y$, and the marginal value of wealth in the last stage, $W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y)$. The two terms are equal provided that $p^y y \leq \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}$ does not bind. If it binds, then the household spends all its resalable wealth.[6] Hence, even wealthy households can face binding liquidity constraints if the amount they invested in resalable assets is low.

## 3.2 Distribution of asset holdings

We characterize the steady-state distributions of asset holdings across households recursively following the logic of the Bellman equations (2) and (3). We denote $G_e(a)$ the measure of households in state $e \in \{0,1\}$ holding at most $a$

[6] As in Rocheteau et al. (2018, 2021), for each employment status, $e \in \{0,1\}$, there exists a threshold for the resalable wealth, $\underline{\ell}_e > 0$, such that if $\boldsymbol{\chi}_\omega.\hat{\mathbf{a}} < \underline{\ell}_e$ then $y_{\omega e}(\hat{\mathbf{a}}) = \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}$.

units of wealth at the start of the last stage (before late consumption) in period $t$. It solves:

$$\begin{aligned} G_e(a) &= \int \left[ \sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega \mathbb{I}_{\{\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}}) \le a\}} + (1-\alpha) \mathbb{I}_{\{\mathbf{1}.\hat{\mathbf{a}} \le a\}} \right] dF_e(\hat{\mathbf{a}}) && (10) \\ G(a) &= G_0(a) + G_1(a), && (11) \end{aligned}$$

where $\int \mathbb{I}_{\mathcal{A}} dF_e(\hat{\mathbf{a}})$ is the measure of households in employment state $e$ with portfolio $\hat{\mathbf{a}} \in \mathcal{A} \in \mathcal{B}(\mathbb{R}^4_+)$ at the start of the second stage, and where $\mathcal{B}$ is the Borel algebra on $\mathbb{R}^4_+$. According to the integrand on the right side of (10), the measure of households who hold at least $a$ in the third stage is equal to the measure of agents who hold $a$ in the second stage and do not consume, with probability $1-\alpha$, plus the measure of households who had an opportunity to consume early and whose post-trade wealth, $\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}})$, is less than $a$.

The distribution of asset portfolios at the start of the second stage, $F_{e'}(\hat{\mathbf{a}})$, is obtained recursively from $G_e$ as follows:

$$\int \mathbb{I}_{\mathcal{A}} dF_{e'}(\hat{\mathbf{a}}) = \sum_{e \in \{0,1\}} \gamma_{e,e'} \int \mathbb{I}_{\{(x,e): \hat{\mathbf{a}}(x,e) \in \mathcal{A}\}} dG_e(x) \;\; \text{for all } \mathcal{A} \in \mathcal{B}(\mathbb{R}^4_+) \tag{12}$$

where $\gamma_{e,e'}$ is the transition probability from $e$ to $e'$, e.g., $\gamma_{0,1} = \lambda$ and $\gamma_{1,0} = \delta^f$, and $\hat{\mathbf{a}}(x,e)$ is the portfolio choice conditional on holding $x$ units of wealth in employment state $e$. The measure of households with a portfolio in the second stage belonging to set $\mathcal{A}$ and an employment status $e'$ is equal to the measure of households whose wealth, $x$, and employment status, $e$, in the last stage is such that the optimal portfolio choice is $\hat{\mathbf{a}}(x,e) \in \mathcal{A}$ and who transitioned to employment status $e'$ in the first stage. The marginal cumulative distributions for each asset are $F^j_e(x) = \int \mathbb{I}_{\hat{a}^j \in [0,x]} dF_e(\hat{\mathbf{a}})$. Moreover, $F^j(x) = F^j_0(x) + F^j_1(x)$.

## 3.3 Pricing jobs and homes

**Creation and value of jobs** The value of a filled job at the beginning of the second stage solves:

$$\phi^f = q - \bar{w}_1 + (1-\delta^f)\frac{\phi^f}{R^f}. \tag{13}$$

It is equal to the expected revenue from early and late sales expressed in terms of the numeraire, $q$, net of the wage, $\bar{w}_1$, plus the expected discounted profits of the job if it is not destroyed, with probability $1-\delta^f$, where the gross discount rate, $R^f$, corresponds to the gross real rate of return on the shares of the mutual funds. The revenue of a job expressed in terms of the numeraire is:

$$q(p^y) = \max_{y \in [0,\bar{y}]} \{p^y y + Q(y)\} = \bar{q} + \max_{y \in [0,\bar{y}]} \{p^y y - \kappa(y)\}, \tag{14}$$

where the second equality is obtained by using that $\kappa(y) = \bar{q} - Q(y)$. The first term on the right side is the firm's total output in terms of numeraire. The second term represents the firm's profits from selling to early consumers. Using that $\kappa'(\bar{y}) = +\infty$, the solution is interior and the optimal supply of goods in the retail market is

$$y_t^s = \kappa'^{-1}(p^y). \tag{15}$$

The price of early consumption is equal to the firm's marginal cost from producing early. Hence, $q$ can be re-expressed as:

$$q = \bar{q} + [\mu(y^s) - 1]\,\kappa(y^s), \tag{16}$$

where $\mu(y^s) \equiv \kappa'(y^s)y^s/\kappa(y^s) \geq 1$. When the cost function is strictly convex, $\mu$ is akin to a gross markup over average cost.

The investment fund that mutualizes claims on firms' profits finances the entry of new firms as long as the following condition holds:

$$-\,k^f + \frac{\lambda(\theta)}{\theta}\frac{\phi^f}{R^f} \leq 0, \text{ " = " if } \theta > 0. \tag{17}$$

The first term on the right side of (17) is the cost of opening a vacancy in terms of numeraire while the second term is the expected discounted value of a job, where the vacant job is filled with probability $\lambda/\theta$. Substituting $\phi^f$ from (13), market tightness solves:

$$-\frac{\theta k^f}{\lambda(\theta)} + \frac{q - \bar{w}_1}{r^f + \delta^f} \leq 0, \text{ " = " if } \theta > 0, \tag{18}$$

where $r^f = R^f - 1$. We take $\bar{w}_1$ as exogenous but we impose the participation constraints $W_1(a) \geq W_0(a)$ and $\bar{w}_1 < q$. Employment, or measure of firms, at the steady state is

$$n = \frac{m(1,\theta)}{\delta^f + m(1,\theta)}. \tag{19}$$

**Supply and price of homes** The price of homes before rental revenue is paid, $\phi^h$, solves:

$$\phi^h = p^h + (1 - \delta^h)\frac{\phi^h}{R^h}. \tag{20}$$

Akin to the value of a filled job, the lifetime value of a home is equal to the expected rental revenue plus the discounted future revenue if the home is not destroyed, with probability $1 - \delta^h$, discounted by the gross real rate of return on shares of the mutual fund over the stock of homes, $R^h$. Since a new home costs $k^h$ in terms of numeraire in the third stage, free entry in the construction sector gives:

$$k^h = \frac{\phi^h}{R^h}. \tag{21}$$

Combining (20) and (21), $p^h = (r^h + \delta^h)k^h$. The rental price of housing must be equal to the construction cost multiplied by a user cost composed of the real rate of return on housing and its depreciation rate.

### 3.4 Market clearing and steady-state equilibrium

The market clearing conditions are:

$$ny^s = \sum_{(\omega,e)\in\Omega_{\mathbb{A}}\times\{0,1\}} \alpha_\omega \int y_{\omega e}(\hat{\mathbf{a}})dF_e(\hat{\mathbf{a}}) \tag{22}$$

$$H = \sum_{e\in\{0,1\}} \int h_e(a)dG_e(a) \tag{23}$$

$$A^j = \int xdF^j(x) \text{ for all } j \in \mathbb{A}, \tag{24}$$

where

$$A^m = \phi_t^m M_t,\ A^h = \phi^h H, \text{ and } A^f = n\phi^f. \tag{25}$$

Equation (22) is the market clearing condition for early consumption. The left side is the aggregate supply from a measure $n$ of firms each producing $y^s$. The right side is the aggregate demand arising from the measure, $\alpha$ households with a preference for early consumption. Equation (23) is the market clearing condition for rental services. On the right side, $h_e(a)$ is the demand for rental services from a household with $a$ units of wealth and employment status $e$ at the beginning of the third stage. Equation (24) is the market-clearing condition for asset $j$. The left side of (24) is the aggregate demand for asset $j$ while the right side is its supply. Using that $A^m$ is constant in a steady-state equilibrium, $R^m = M_t/M_{t+1} = (1+\pi)^{-1}$.

Finally, taxes and transfers must be such that the budget constraint of the government is satisfied, which requires:

$$n\tau_1 + (1-n)\tau_0 = \pi\phi_t M_t + \left(\frac{1}{R^g} - 1\right) A^g. \tag{26}$$

So net transfers to households are financed with money creation and the issue of new bonds net of the redemption of old ones. We now have the different components to define an equilibrium.

**Definition 1** *A steady-state monetary equilibrium is composed of: (i) Value functions, $W_e$ and $V_e$, and policy functions satisfying (2) and (3); (ii) Distributions of asset holdings, $(G_e, F_e)$, satisfying (10)-(11) and (12); (iii) Market tightness, $\theta$, satisfying (18); (iv) Prices of early consumption, $p^y$, and housing services, $p^h$, satisfying (22) and (23); (v) Asset prices, $\{\phi_t^m\}$, $\phi^h$, $\phi^f$ and $\{R^j\}$, that satisfy market-clearing conditions, (24) and (25) and asset pricing conditions, (13) and (20); (vi) Housing supply that satisfies free entry in construction, (21); (vii) Transfers that satisfy the government budget constraint (26).*

# 4 A simplified model

In this section, we describe the main channels through which policy affects equilibrium outcomes (an interest rate channel, an aggregate demand channel, and a distributional channel) in a simplified, slightly-modified, version of our model that is easily comparable to the literature. Preferences are given by $U(c_t, e_t) = c_t + (1 - e_t)\ell$. We ignore housing services and housing wealth. We set $\kappa(y) = y$, which means that the production $\bar{q} = \bar{y}$ is perfectly storable across stages. We assume that early consumption is sold at a markup $\mu > 1$ over the opportunity cost of selling late, i.e., the price of early consumption is $p^y = \mu$. We replace the market-clearing condition, (22), with a feasibility condition

$$\alpha \sum_{e \in \{0,1\}} \int y_e(\hat{a}) dF_e(\hat{a}) \leq n\bar{y}. \tag{27}$$

This condition requires that aggregate early consumption is no greater than the total output produced by firms. We assume that the early consumption is divided evenly across firms and the wage $\overline{w}_1$ is a constant. Finally, all assets are equally pledgeable, $\chi = 1$, so that the two forms of public liquidity, government bonds and money, are perfect substitutes. We focus on equilibria with bonds only and denote $a^g = A^g/n$ the supply of one-period real bonds per employed household.

### 4.1 Equilibria with degenerate distributions

We start with equilibria where the constraint $c_t \geq 0$ does not bind and the distribution of wealth is degenerate. Relative to BMW, the real interest is endogenous and depends on public liquidity. From Proposition 1, $W_e'(a) = 1$. Combining (5) and (6) with $p^y = \mu$, the household's choice of asset holdings solves:

$$\frac{\alpha \left[v'(a/\mu) - \mu\right]^+}{\mu} = \frac{\rho - r}{1 + r}, \tag{28}$$

where $[x]^+ = \max\{x, 0\}$. The left side is the marginal benefit of liquid wealth to finance early consumption while the right side is the cost of holding assets. Equation (28) specifies the household's optimal asset holdings, $a^\star(r)$, where $a^\star$ is independent of $e$, increases with $r$ and $\alpha$, but decreases with $\mu$. The constraint $c \geq 0$ does not bind if the household with no assets and no job can accumulate the optimal wealth target in a single period, $Rw_0 \geq a^\star$.

From (18), assuming an interior solution, market tightness solves

$$\frac{(r + \delta^f)\theta k^f}{\lambda(\theta)} = \bar{q} + \alpha \left(\frac{\mu - 1}{\mu}\right) \frac{\min\{a, \mu y_\mu^\star\}}{n} - \bar{w}_1, \tag{29}$$

where $y_\mu^\star$ solves $v'(y_\mu^\star) = \mu$ and $n$ is a function of $\theta$ given by (19). The aggregate demand channel is captured by the second term on the right side of (29). The term $\alpha \min\{a, \mu y_\mu^\star\}$ is the amount of assets spent by the measure $\alpha$ households with an opportunity for early consumption. If firms have market power, $\mu > 1$, then their revenue increases (weakly) with the amount of liquid assets held by households, $\partial\theta/\partial a \geq 0$. We denote $\theta^\star$ the solution to (29) when households' liquidity needs are satiated, in which case $r = \rho$ and $\min\{a, \mu y_\mu^\star\} = \mu y_\mu^\star$, i.e.,

$$\frac{(\rho + \delta^f)\theta^\star k^f}{\lambda(\theta^\star)} = \bar{q} - \bar{w}_1 + \alpha (\mu - 1) y_\mu^\star \left[1 + \frac{\delta^f}{\lambda(\theta^\star)}\right]. \tag{30}$$

We assume $(\rho+\delta^f)k^f < \bar{q}-\bar{w}_1$ so that $\theta^\star > 0$ for all $\mu$.

The real interest rate adjusts to clear the asset market:

$$a^\star(r) = \frac{(1+r)\theta(r)k^f}{\delta^f + \lambda\left[\theta(r)\right]} + A^g, \tag{31}$$

where $\theta(r)$ is defined implicitly by (29). The left side is households' asset demand. The right side is the supply of assets in the form of shares of mutual funds (first term) and government bonds (second term). From (17) and (19), the market capitalization of the mutual funds is $n\phi^f = R\theta k^f / \left[\delta^f + \lambda(\theta)\right]$. The interest rate channel, captured by (31), specifies that the interest rate at which firms' profits are discounted, $r$, depends on the supply of private and public liquidity.

An equilibrium is a triple, $(a, \theta, r)$, that solves (28), (29), and (31) with $a^\star(r) = a$. The textbook MP model corresponds to the special case when $\alpha = 0$, i.e., there is no demand for early consumption. In that case, from (28), $r = \rho$. From (30), market tightness solves $(r+\delta^f)\theta^\star k^f/\lambda(\theta^\star) = \bar{q} - \bar{w}_1$. In the MP model, the aggregate-demand and interest-rate channels are inoperative.

#### Abundant liquidity

In the class of equilibria with degenerate distributions, one can distinguish two subclasses. The first subclass is when liquidity needs are satiated, $a \geq \mu y_\mu^\star$. From (28) assets have no liquidity value and $r = \rho$. Given $r$, market tightness is uniquely pinned down by (29). The occurrence of this regime necessitates that the supply of private and public assets, $n\phi^f + A^g$, is larger than households' demand for assets, $a^\star$. If this condition holds, a change in $A^g$ has no effect on $r$ and $\theta$.

#### Scarce liquidity

The second subclass of equilibria is when liquidity is scarce, $a < \mu y^\star$. In such equilibria, asset prices exhibit a liquidity premium, $r < \rho$. From the job creation condition, (29), the aggregate supply of private assets is equal to

$$n\phi^f = \frac{(1+r)\theta k^f}{\delta^f + \lambda(\theta)} = \frac{1+r}{r+\delta^f}\left[\frac{\lambda(\theta)}{\delta^f+\lambda(\theta)}\left(\bar{q}-\bar{w}_1\right) + \alpha\left(\frac{\mu-1}{\mu}\right)a\right]. \tag{32}$$

The middle term is obtained from the left term by using that $n = \lambda(\theta)/\left[\delta^f + \lambda(\theta)\right]$ and $\phi^f = R\theta k^f/\lambda(\theta)$. The right side corresponds to the discounted sum of profits from all firms. It has two components: the profits if all the output was sold to late consumers and the profits arising from early sales at a markup. Substituting $n\phi^f$ by its expression given by (32) into the total asset supply, $a = n\phi^f + A^g$, and solving for $a$, we obtain $a = \bar{A}(\theta, r, a^g)$, where

$$\bar{A}(\theta, r, a^g) = \frac{\lambda(\theta)}{\delta^f + \lambda(\theta)} \frac{\left[(1+r)\left(\bar{q}-\bar{w}_1\right) + \left(r+\delta^f\right)a^g\right]}{r+\delta^f - (1+r)\,\alpha\mu^{-1}\left(\mu-1\right)}. \tag{33}$$

There is a unique $\bar{A}(\theta, r, a^g) \in (0, +\infty)$ provided that $(1+r)\alpha\mu^{-1}(\mu - 1)/(r + \delta^f) > 0$. If $\mu > 1$, then

$$\frac{\partial \bar{A}}{\partial A^g} = \frac{r + \delta^f}{r + \delta^f - (1+r)\,\alpha\mu^{-1}(\mu - 1)} > 1,$$

i.e., there is a multiplicative effect of public liquidity on aggregate liquidity. If public liquidity increases, then the value of firms increases through the aggregate demand channel, which raises private liquidity and hence the overall liquidity. The asset market clearing condition, (31), becomes

$$a^{\star}(r) = \bar{A}(\theta, r, a^g). \tag{34}$$

Since $\bar{A}(\theta, r, a^g)$ is increasing in $\theta$ but decreasing in $r$, (34) gives a positive relationship between $r$ and $\theta$. As $\theta$ increases, the supply of assets increases, which requires a higher real interest rate to clear asset markets.

We now turn to the determination of $\theta$. The wealth dedicated to the accumulation of private assets is $\bar{A}(\theta, r, a^g) - na^g$. Hence, $\bar{A}(\theta, r, a^g)/n - a^g = \phi^f = R\theta k^f/\lambda(\theta)$, which can be reexpressed as

$$\frac{\bar{q} - \bar{w}_1 + \alpha\mu^{-1}(\mu - 1)\,a^g}{r + \delta^f - (1+r)\,\alpha\mu^{-1}(\mu - 1)} = \frac{\theta k^f}{\lambda(\theta)}. \tag{35}$$

This condition gives a negative relationship between $\theta$ and $r$. An equilibrium with scarce liquidity can now be reduced to a pair, $(\theta, r)$, solution to (34) and (35). In Figure 2, the upward-sloping curve representing (34) is labelled EE (for Euler Equation) while the downward-sloping curve representing (35) is labelled FE. There is a unique equilibrium at the intersection of the two curves.

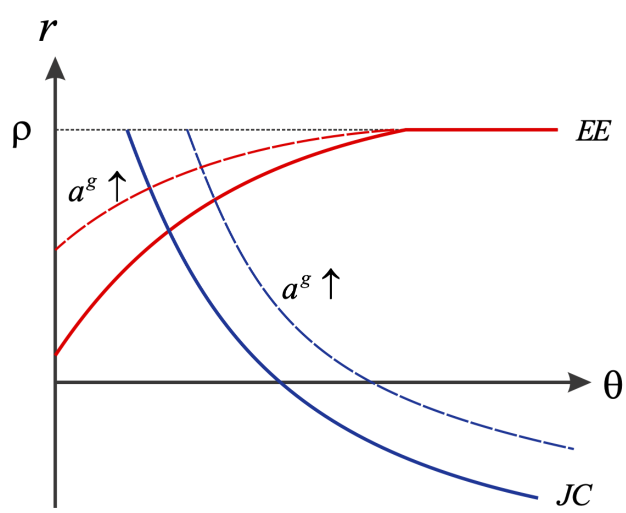


Figure 2: Equilibrium with degenerate distributions

**Channels of monetary policy**

The channels of monetary policy can be illustrated graphically. First, an increase in $a^g$ raises the supply of liquidity, which, for given $\theta$, raises $r$. Graphically, the EE curve moves upward. Second, an increase in $a^g$ raises households' expenditure on early consumption , which raises firms' profits when $\mu > 1$, and induces more job creation. Graphically, the JC curve moves to the right. The overall effect is positive on $r$ but ambiguous on $\theta$. We summarize our results so far in the following proposition.

**Proposition 2** *Assume $U_{e_t}(c_t) = c_t + (1 - e_t)\ell$.*

1. ***Abundant liquidity.*** *If*
$$\frac{(1+\rho)\theta^\star k^f}{\delta^f + \lambda(\theta^\star)} + A^g \geq \mu y^\star_\mu, \tag{36}$$
*then $y = y^\star_\mu$ and $r = \rho$. Moreover, $\partial\theta/\partial A^g = \partial r/\partial A^g = 0$.*

2. ***Scarce liquidity.*** *If (36) does not hold, then $y < y^\star_\mu$ and $r < \rho$. Moreover, $\partial r/\partial A^g \geq 0$ but $\partial\theta/\partial A^g \lesseqgtr 0$.*

We describe two special cases where each case has a single channel operative. Suppose first that the early consumption is priced competitively, $\mu = 1$.[7] Equations (34) and (35) become:

$$\begin{aligned} a^\star(r) &= \frac{(1+r)\,\theta k^f + \lambda(\theta) a^g}{\delta^f + \lambda(\theta)} \\ \frac{\theta k^f}{\lambda(\theta)} &= \frac{\bar{q} - \bar{w}_1}{r + \delta^f}. \end{aligned}$$

Only the interest rate channel is operative and $\partial\theta/\partial A^g < 0$. In that case public liquidity crowds out private liquidity, which generates an increase in the real interest rate.

The second special case is when the utility for early consumption is linear, $v(y) = \bar{v}y$ where $\bar{v}$ is constant. We consider equilibria where $c \geq 0$ does not bind. (We provide a full characterization of the model with linear utility in Appendix B.) From (28)

$$r = \frac{\rho\mu - \alpha\,(\bar{v} - \mu)}{\alpha\,(\bar{v} - \mu) + \mu}.$$

The interest rate channel is inoperative and only the aggregate demand channel prevails when $\mu > 1$, $\partial\theta/\partial A^g > 0$. An increase in public liquidity raises the wealth in households' hands, which raises firms' revenue from their sales to early consumers.

## 4.2 Equilibria with distributional effects

We now consider equilibria where $c \geq 0$ binds, i.e., households only consume early, and the distribution of asset holdings is nondegenerate. We focus on such equilibria where, in the event of an expenditure shock for early consumption, households deplete their asset holdings in full, i.e., $y_e(a) = a$ for all $a$ in the support of $F_e$.[8] From (9) this is the case when $v'(a/\mu) \geq \mu W_e'(0)$, i.e., the marginal utility of consumption is larger than the marginal of wealth when assets are depleted. Households' target for asset holdings is $a^\star$, defined as a solution to (28).

[7] This first case is analogous to Rocheteau and Rodriguez-Lopez (2014).

[8] Similar equilibria have been studied in detail in Rocheteau et al. (2021). Relative to this paper, we endogenize the real interest rate, $r$.

### Two-point distribution

The simplest equilibrium with a non-degenerate distribution has two mass points. Employed workers accumulate $a^\star$ in a single period, which requires $(1+r)w_1 > a^\star$. In contrast, it takes two periods for unemployed workers with depleted asset holdings to reach their target. This requires $R < a^\star/w_0 < R\,(1+R)$. In a steady state, a measure $\alpha u$ of households own $(1+r)w_0$ assets, those households that are unemployed and received an expenditure shock in the previous period, while the remaining $1-\alpha u$ households own $a^\star$. Hence, the distribution of asset holdings is

$$F(a) = \alpha u \mathbb{I}_{\{a \geq (1+r)w_0\}} + (1-\alpha u)\mathbb{I}_{\{a \geq a^\star\}}. \tag{37}$$

The distribution of asset holdings depends directly on both expenditure and unemployment risk through $\alpha$ and $u$. It also depends on the income of the unemployed, $w_0$. Hence, money creation implemented through transfers to households will affect the distribution through both $r$ and $\tau_0$. The relation between the supply of public liquidity and transfers is given by the budget constraint of the government, (26),

$$(1+r)\,(n\tau_1 + u\tau_0) = -rA^g. \tag{38}$$

The asset market clearing condition, (34), becomes:

$$(1-\alpha u)\,a^\star(\overset{+}{r}) + \alpha u(1+r)w_0 = \bar{A}(\overset{+}{\theta}, \overset{-}{r}, \overset{+}{a^g}), \tag{39}$$

where $\bar{A}(\theta, r, a^g)$ is given by (33) and the signs above the variables represent partial derivatives. An equilibrium is a pair, $(\theta, r)$, that solves (35) and (39).

As before, a change in $a^g$ triggers the aggregate demand and interest channels. In addition, via the government budget constraint (38), it is associated to a change in transfers. Consider an increase in $\tau_0$ in isolation (independently from $A^g$). From (39) it generates a fall in $r$ for given $\theta$. Indeed, if unemployed households receive a higher income, then they can accumulate assets faster and so aggregate asset holdings are larger. This lowers the real interest rate and induces firms to open more jobs. In general equilibrium, there is an amplification mechanism as the decrease in $u$ raises $r$ further.

### Three-point distribution

Consider another class of tractable equilibria where both employed and unemployed workers need two periods to reach their targeted real balances, $Rw_1 < a^\star$ and $w_0R(1+R) > a^\star$. Households have incentives to deplete their asset holdings in full provided that $Rw_e$ is sufficiently close to $a^\star$. There are now $\alpha n$ employed and $\alpha u$ unemployed households who save their full income in order to self-insure against the expenditure risk. The distribution of asset holdings is

$$F(a) = \alpha u \mathbb{I}_{\{a \geq (1+r)w_0\}} + \alpha n \mathbb{I}_{\{a \geq (1+r)w_1\}} + (1-\alpha)\mathbb{I}_{\{a \geq a^\star\}}. \tag{40}$$

The distribution of asset holdings depends on both $w_0$ and $w_1$, and hence on $\tau_0$ and $\tau_1$. Asset market clearing is given by

$$\alpha u R w_0 + \alpha n R w_1 + (1-\alpha)a^\star = \bar{A}(\overset{+}{\theta}, \overset{-}{r}, \overset{+}{a^g}). \tag{41}$$

The real interest rate is a decreasing function of both $w_0$ and $w_1$. If we combine (41) with the budget constraint of the government, (38), i.e., $\alpha R\,(n\tau_1 + u\tau_0) = -\alpha r A^g$, then the asset market clearing condition becomes

$$\alpha u R \bar{w}_0 + \alpha n R \bar{w}_1 + (1-\alpha)a^\star = \bar{A}(\theta, r, a^g) + \alpha r n a^g.$$

The transfers reinforce the effects of $a^g$ on $r$.

While our simplified model allows us to identify different channels of monetary policy, we had to impose several restrictive assumptions to achieve some amount of tractability: households are risk neutral relative to late consumption, all assets are equally liquid, and we could only study the class of equilibria where households deplete all their wealth in the event of an expenditure shock. Our general model will relax all these restrictions and will quantify the effects at work.

# 5 Quantitative Analysis

## 5.1 Baseline Calibration

We choose the time period to be one month and set $\beta = 0.95^{1/12}$. Preferences are given by $v(y) = A(y^{1-a} - 1)/(1-a)$ and $U(c,h) = ln(c) + Bln(h)$ (relative to Section 2, the function $U$ is not bounded). Targeting a 15% share of housing services in total non-durable consumption gives $B = 0.2$. The matching function takes the form $\mathcal{M}(s,o) = so/(s^\nu + o^\nu)^{1/\nu}$. Schaal (2017) estimates that $\nu = 1.6$. We set the separation rate, $\delta^f = 0.035$, to imply a quarterly job destruction rate of 10% and set vacancy costs, $k^f$, to imply an average monthly job finding rate of 30%, consistent with the evidence in Hall and Schulhofer-Wohl (2018). The calibration results in $k^f = 0.51$.[9] The production possibility frontier is given by $Q(y) = \bar{q} - \kappa(y)$. We let $\kappa(y) = y^\eta/\eta$ with $\eta = 1.3$ in order to target a 30% markup over average costs in the market for early consumption, an estimate of markups in the retail trade sector.[10] The parameter $\bar{q}$ is discussed later. We assume wages are proportional to a firm's productivity and let $\bar{w}_1 = \mu q$, where $q$ is given by (14). We choose $\mu = 0.85$ to target a profit share of 15%. Income of the unemployed is set based on a replacement

[9] This implies average vacancy costs in the model are 3.78% of the quarterly wage bill. Silva and Toledo (2009) estimate that it costs 3.6% of a recruiter's monthly wage to hire one worker. Average unemployment duration in the model is 3.3 months or 14.4 weeks, which is slightly below the average in the decade before the Great Recession between 1999 and 2008 of 16.5 weeks.

[10] See www.census.gov/econ/retail for more information. If one interprets early and late consumption goods as the same physical good sold to consumers with different valuations, our model generates a standard deviation of prices of 11.3%, which is in line with evidence of retail price dispersion in Kaplan et al. (2019).

rate of 40% following Shimer (2005), $\bar{w}_0 = .4\bar{w}_1$.[11] The decline in income upon job loss generates an average fall in consumption of 6.7%, slightly below the evidence in Browning and Crossley (2001) of 16%, Hurd and Rohwedder (2010) of 11%, and Ganong and Noel (2019) of 9%.

We make the following assumptions regarding asset liquidity. Liquid wealth is universally accepted while other assets are only partially acceptable. Moreover, bonds and stocks are equally liquid, which implies they yield the same rate of return $R^g = R^f \equiv R^i$. We assume that conditional on an expenditure shock, households face one of three acceptability events. They can access liquid wealth and housing with probability $\alpha_{m,h}$, they can access liquid wealth, bonds, and stocks with probability $\alpha_{m,g,f}$, or they can only access liquid wealth with probability $1 - \alpha_{m,h} - \alpha_{m,g,f}$.

There are 9 remaining parameters to calibrate $(A, a, \alpha, \alpha_{m,h}, \alpha_{m,g,f}, \delta^h, k^h, A^g, \overline{q})$. We set these to target moments related to household wealth and asset liquidity. We use portfolio holdings in the 1992-2013 Survey of Consumer Finances (SCF) and map household wealth into the four assets in the model: liquid wealth, housing equity, bonds, and financial equity. We interpret liquid assets as zero maturity wealth held by households, which essentially corresponds to transaction accounts (checking, saving, money market, and brokerage cash). We set the annual real rate of return on liquid balances to -1.5$\%$, equal to the average real rate of return of zero maturity assets (MZM) between 1999 and 2008 reported by the Federal Reserve Bank of St. Louis. We interpret housing equity as net worth directly held in residential structures and consumer durables, including vehicles. We set the depreciation rate to $\delta_h = 0.003$ to match an annual depreciation rate of $3.6\%$ according to the Bureau of Economic Analysis (BEA) fixed asset tables.[12] Financial wealth in the model corresponds to households' direct and indirect holdings of publicly-traded stocks and bonds as well as business equity.

We take the view that early consumption represents large, unplanned household expenditures, such as vehicle repairs or out-of-pocket medical costs, and set $\alpha$ and $A$ to match evidence on the frequency and average size of these expenses.[13] We use evidence from the Survey of American Family Finances (SAFF), a nationally-representative survey administered by The Pew Charitable Trusts.[14] Respondents were asked if in the past 12 months the family had experienced one of 6 types of unexpected expenses, including medical expenditures, divorce, and vehicle repairs or replacement. We set $\alpha$ to target the average number of shocks in a year of 1.2 which generates $\alpha = 0.1$.[15] Respondents were also asked the cost of the most expensive shock they received during the year. The median value was $2,000 so we

[11] While we match the income fall upon job loss using replacement rates, we assume $w_0$ is an endowment and not a transfer.

[12] The depreciation rate is the value-weighted average of depreciation of $2.3\%$ on residential structures and $19.7\%$ on consumer durables.

[13] Expenditure shocks are listed as a common reason that households choose to save. For instance in the 2013 SCF, 28% of households reported their most important reason for saving was for emergencies, unexpected events, or illness and medical/dental expenses. The two most common reasons given were saving for emergencies or unexpected needs and savings for retirement. See variables X3006, X3007, X7513, X7515, and X6848.

[14] See https://www.pewtrusts.org/~/media/assets/2015/03/fsm-poll-results-methodology_artfinal_v2.pdf for background and methodology of the survey and https://www.pewtrusts.org/en/research-and-analysis/issue-briefs/2015/10/the-role-of-emergency-savings-in-family-financial-security-how-do-families for a summary and data tables.

[15] The survey also gives the histogram of the number of shocks received in a year. Targeting $\alpha = 0.1$ implies that the probability of receiving one shock in a year is 38% while in the data it is 27%, receiving two shocks in a year is 23% while in the data it is 18%, and receiving three or more shocks in a year is 10% while in the data it is 15%.

set $A = 1.2$ to generate a median cost of early consumption of $2,000 or what amounts to a $60\%$ increase in monthly consumption. For comparison, the 2017 Consumer Expenditure Survey reports that households spent an average of $2,570 on home and vehicle repairs in a year and the OECD reports that average out-of-pocket medical expenses in 2017 were $1,121, so our measure of the cost of unplanned expenditures is in line with how much households spent in these categories in other surveys. We set the curvature parameter $a$ to target the aggregate elasticity of liquid-assets-to-GDP with respect to inflation. In the model, the aggregate holdings of liquid assets are $\int x dF^m(x)$ and aggregate output is $nq - k^f\theta(1-n)$. In the data, the semi-log elasticity of zero maturity assets (MZM) to GDP with respect to inflation over our sample period is $-0.048$. This procedure leads to $a = 0.26$.

We discipline the liquidity of housing and financial wealth, $\alpha_{m,h}$ and $\alpha_{m,g,f}$, using evidence in the SCF about the frequency of home equity cash outs and retirement withdrawals. The survey reports that, on average, 15% of households held balances in home equity lines of credit or initiated cash-out refinances during a year and 9% of households took some form of withdrawal from a retirement account or borrowed against their pension during the year. Hence, we set $\alpha_{m,h} = 1 - (1 - 0.15)^{1/12} = 0.013$ and $\alpha_{m,g,f} = 1 - (1 - 0.09)^{1/12} = 0.008$. This implies that, conditional on a preference shock, housing wealth can be liquidated 13% of the time and financial wealth can be liquidated 8% of the time.[16]

Finally, we set asset supply parameters ($A^g$, $\bar{q}$, $k^h$) to jointly target three moments: the share of financial wealth held in firm equity, the liquidity premium on financial wealth, and the price-to-rent ratio on housing. In the SCF, households hold 60% of their financial wealth in direct- and indirectly-held firm equity positions, including privately-owned businesses. Our measure of the liquidity premium on financial wealth is the real user cost of holding MZM assets. The user cost is calculated as the spread between the own rate of return on MZM and the Baa bond yield and averaged 4.0% over our time period.[17]. Since the model features no aggregate risk, we measure the liquidity premium in the model as $R^f$-$R^m$. Sommer et al. (2013) estimate a range of the annual price-to-rent ratio on U.S. residential housing between $8$ and $15.5$. Using (20), this range maps to a model-implied annual return on housing between $2.9\%$ and $9.4\%$. Since there is no clear measure of what fraction of the return on housing compensates for liquidity versus aggregate risk, we choose to target the lower bound of the return or, equivalently, the upper bound on the price-to-rent ratio of $15.5$. While the model is not able to match median housing wealth to income (0.05 in the model and 0.72 in the data), the model does well in terms of the other components of household wealth. Median liquid wealth to annual income is $0.055$ in the model compared to $0.043$ in the SCF data. Median financial wealth to annual income is $0.247$ in the model compared to $0.227$ in the data. We discuss in detail how well the model matches household wealth in Section 5.2.

[16] The SCF does not report the frequency of liquidating financial wealth held directly (e.g., not through retirement accounts). However, the median household holds their financial wealth almost exclusively through retirement accounts.

[17] See Anderson and Jones (2011) for details.

Table 1 provides a summary of the calibrated parameters and targeted moments in the model and the data.

| **Parameter** | **Value** | **Moment** | **Data** | **Model** |
|---|---|---|---|---|
| *Parameters Set Directly* | | | | |
| discount rate, $\beta^{12}$ | $0.950$ | | | |
| rate of return on liquid wealth, $R^{m^{12}}$ | $0.985$ | annual real return of MZM | $-1.5$ | $-1.5$ |
| matching curvature, $\nu$ | $1.600$ | Schaal (2017), fit of Beveridge curve | | |
| replacement rate, $\bar{w}_0/\bar{w}_1$ | $0.400$ | 40% replacement rate | $0.40$ | $0.40$ |
| worker share of revenue, $\mu$ | $0.850$ | profit share of 15% | $0.15$ | $0.15$ |
| job depreciation rate, $\delta^f$ | $0.035$ | quarterly separation rate, BEA | $10\%$ | $10\%$ |
| housing utility level, $B$ | $0.200$ | share of housing services in consumption | $20\%$ | $20\%$ |
| housing depreciation rate, $\delta^h$ | $0.003$ | annual depreciation rate, BEA | $3.6\%$ | $3.6\%$ |
| production cost curvature, $\eta$ | $1.300$ | average markup, Retail Trade Survey | $30\%$ | $30\%$ |
| expenditure shock, $\alpha$ | $0.100$ | prob. of no expense shocks in a year, SAFF | $48.6\%$ | $48.6\%$ |
| acceptability of housing, $\alpha_{m,h}$ | $0.013$ | home equity usage rate, SCF | $15\%$ | $15\%$ |
| acceptability of financial wealth, $\alpha_{m,f}$ | $0.008$ | withdrawal rate on retirement accounts, SCF | $9\%$ | $9\%$ |
| *Parameters Calibrated Jointly* | | | | |
| early consumption utility curvature, $a$ | $0.260$ | elas. of liquid wealth to inflation | $-0.048$ | $-0.025$ |
| early consumption utility level, $A$ | $1.225$ | median cost of expenditure shock | $\$2,000$ | $\$2,000$ |
| bond supply, $A^g$ | $0.204$ | annual real user cost of MZM | $4.0\%$ | $4.0\%$ |
| output per job, $\overline{q}$ | $0.310$ | equity share of financial wealth, SCF | $60\%$ | $60\%$ |
| fixed cost of housing, $k^h$ | $55.0$ | price-to-rent ratio, Sommer et al. (2013) | $15.5$ | $15.5$ |
| fixed cost of vacancy posting, $k^f$ | $0.512$ | monthly job finding rate | $30\%$ | $30\%$ |

Table 1: Calibration: parameters and target moments

## 5.2 Untargeted heterogeneity in wealth, MPCs, and unemployment risk

The model produces heterogeneity along several dimensions that are not directly targeted in the calibration but are consistent with empirical evidence. Figure 3 illustrates the heterogeneity in the composition of household wealth. From left to right, the panels show the share of total wealth held in liquid, financial, and housing assets, as a function of the household's percentile of total wealth and employment status. The red and blue curves represent the employed and unemployed, respectively, in the model, while the green dots represent the data.

The share of wealth held in liquid assets in the cross section matches the data well; the share falls with total wealth and ranges from near $100\%$ for the poorest households to around $15\%$ for the most wealthy. Further, the model predicts that unemployed households hold more of their wealth in liquid assets, both unconditional and conditional on total wealth. The average liquid share for unemployed households in the model is $8.6$ percentage points higher than for employed households (shares of $29.1\%$ and $20.5\%$ for the unemployed and employed, respectively). Estimates from the 1992-2013 SCF suggest that the liquid share for households with an unemployed head is $3.8$ percentage points higher than for those with an employed head; $16.2\%$ versus $12.4\%$ for the unemployed and employed, respectively. While the model does not do as well in quantitatively matching the cross section of the share of wealth held in financial and housing assets, it does produce the correct patterns. As total wealth increases, the share held in financial assets rises monotonically while the share held in housing assets is hump-shaped.

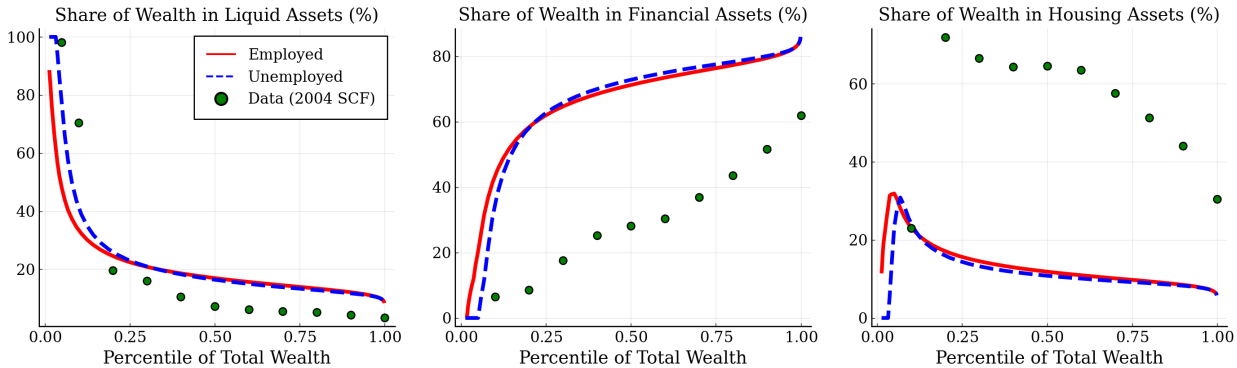


Figure 3: Cross section of household portfolio shares: model vs. data

The model is also consistent with cross-sectional differences in how households adjust the composition of their wealth between liquid and illiquid assets in response to interest rate changes. Figure 4 illustrates the elasticity of the share of wealth held in liquid assets with respect to annual returns on liquid and financial wealth for various percentiles of the total wealth distribution. The solid-blue and dashed-red lines represent the elasticities in the model while the dots illustrate the same elasticities from the 1992-2013 SCF data.[18] The model predicts that poor households around the 10th percentile of total wealth are the most responsive to interest rate changes, consistent with the data. As total wealth increases beyond the 10th percentile, households are less and less responsive to interest rate changes. Households at the very bottom or top of the wealth distribution hardly respond.

Figure 5 illustrates the marginal propensity to consume out of liquid wealth (right panel), the average consumption fall upon job loss (middle panel), and the wealth distribution (left panel). We measure the three-month marginal propensity to consume (MPC) from a one-time unanticipated transfer of liquid wealth of \$500.[19] This experiment is comparable to evidence in Parker et al. (2013) from spending responses to one-time stimulus payments. The average MPC in the model is 18.7%, in the middle of the range of responses of 12% to 30% found in Parker et al. (2013) on non-durable spending, but below the range of 50 to 90% found when including the purchase of durables. Along the cross section, the model matches the pattern of spending responses by wealth and income from the data. MPCs fall in total wealth, ranging from 70% for the poorest, unemployed households to 16% for the wealthiest, employed households. These responses are within the range given from survey evidence in the 2010 Italian Survey of Household

[18] Specifically, we take the sequence of real returns in the SCF data and simulate the model, computing the steady-state equilibrium price of consumption and housing, $p^y$ and $p^h$, for a given set of returns, $\{R^m, R^h, R^f\}$. We then estimate the elasticities in both the model and annual data using a linear specification $y_t = \beta_0 + \beta_m R_t^m + \beta_h R_t^h + \beta_f R_t^R + \epsilon$, where $y_t$ is the liquid share of wealth for a given total wealth percentile.

[19] Computationally, we calculate the change in total consumption, between a household that received the unanticipated transfer versus the same household type that did not, as a fraction of the size of the transfer. We start the experiment at the beginning of the period in the steady-state equilibrium. For each household type, $(e, \mathbf{a})$ we simulate their sequence of expense and employment shocks over the three periods using the equilibrium stochastic processes.

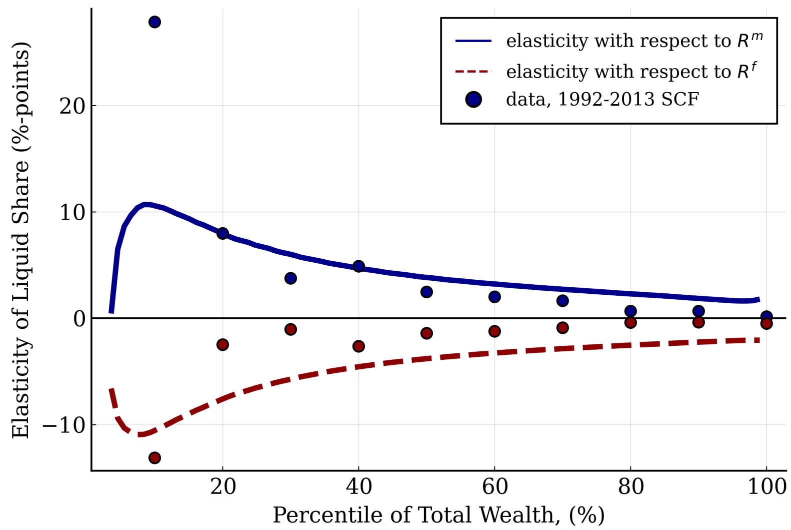


Figure 4: Elasticities of the share of wealth held in liquid assets with respect to the real returns of liquid and financial wealth: model vs. data

Income and Wealth (SHIW), reported in Jappelli and Pistaferri (2014). The lowest percentile of wealth reports an average MPC of around 70% while the highest reports an average MPC around 35%.[20] In terms of income, the model yields an average MPC of 17.2% for employed households and 27.9% for unemployed households, a difference of 10.7 percentage points. Kekre (2021) reports a difference of 25 percentage points using the 2010 SHIW.

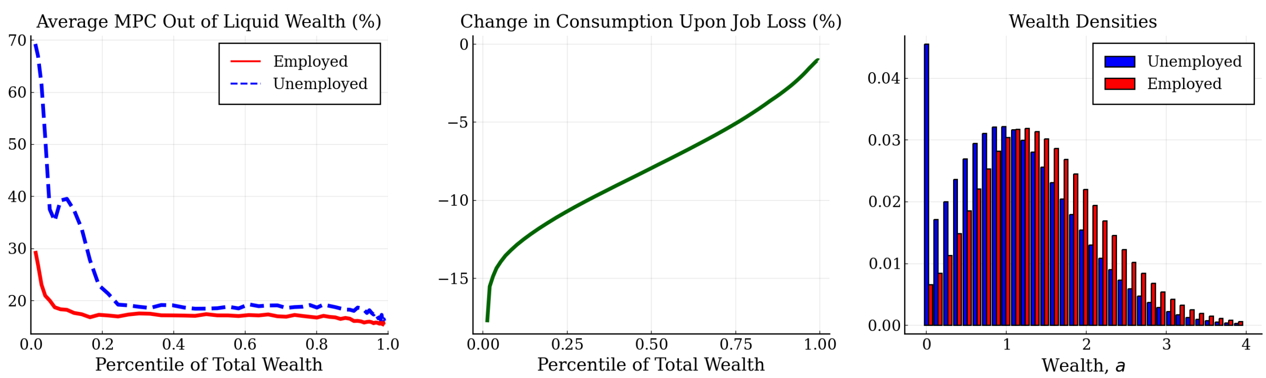


Figure 5: Model cross section of MPCs, consumption responses to job loss, and wealth densities

The model also produces heterogeneity in the consumption responses upon job loss, a measure of how well households are insured against employment risk. The middle panel of Figure 5 shows the percentage change in total consumption (early and late, and housing services) in the first month after a job-loss for a household with a given percentile of total wealth. The median consumption decline is 7.7%, but the response to job loss varies strongly with wealth. For the wealthiest agents, consumption falls by around 1% while for the poorest agents, consumption falls by 17%.[21]

[20] See Figure 2 in Jappelli and Pistaferri (2014). They report percentiles of "cash-on-hand" equal to household total disposable income plus financial wealth, net of consumer debt.

[21] Unfortunately, there is limited empirical evidence on the effect of wealth on the consumption response to job loss because surveys typically do not feature rich enough data on consumption, labor market outcomes, and wealth.

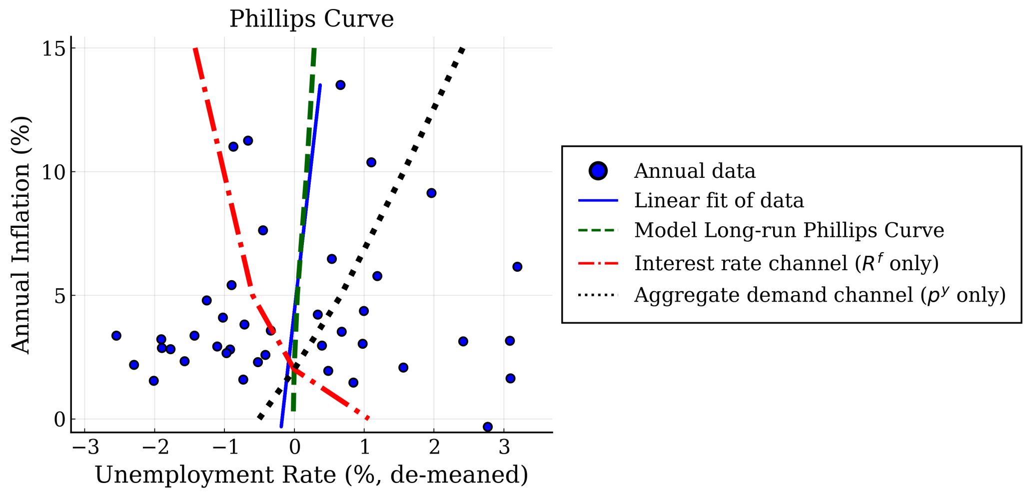


Figure 6: Inflation and unemployment: model vs. data

# 6 'Helicopter' money, unemployment, asset returns, and welfare

In this section, we explore the effect of a change in the rate of money creation, $\pi$, on aggregate unemployment, asset returns, prices, and welfare. Following Friedman's (1969) 'helicopter money experiment', changes in the money supply are distributed to all households in a lump-sum fashion.[22] Figure 6 plots the model-implied relationship between inflation and unemployment (dashed-green line) as well as the raw data with a linear fit (blue dots and solid-blue line, respectively).[23] Consistent with the data, the model predicts a positively-sloped, but almost vertical long-run Phillips curve.

## 6.1 The channels behind the long-run Phillips curve

The muted impact of long-run inflation on aggregate unemployment masks the magnitude of the interest-rate and aggregate-demand channels that work in opposite directions. In the context of our model, we separate these two channels as follows. From the free-entry condition, (17), and the Beveridge curve, (19), unemployment can be written as a function of two prices: (i) the price of financial wealth, $1/R^f$, and (ii) the price of early consumption, $p^y$. The interest rate channel is captured by changes in $R^f$ while the aggregate-demand channel is captured by changes in $p^y$. We represent each channel in isolation in Figure 6 as the red-dash-dotted line and black-dotted line, respectively.

First, consider the interest-rate channel. In accordance with a Tobin effect, inflation induces a substitution towards illiquid assets, which causes their rate of return to fall, firm entry to increase, and unemployment to fall. Quantitatively,

[22] In Friedman's (1969) words: "Let us suppose now that one day a helicopter flies over this community and drops an additional $1,000 in bills from the sky, which is, of course, hastily collected by members of the community."

[23] We choose to compare the model against the raw, un-filtered data instead of taking a stand on the correct way to estimate the long-run trend component of each series. However, this makes little difference in practice; regardless of the filter chosen the relationship between the long-run trend components remains weakly positive. The unemployment rate in the data is given by the Bureau of Labor Statistics U-3 measure of unemployment, FRED series UNRATE.

the interest-rate channel leads to a strong negative relationship between long-run inflation and unemployment.

The effect of inflation on interest rates in the model is consistent with the data, as shown in the left and middle panels of Figure 7 for financial and housing wealth, respectively. The model is represented as the green-dashed line while the relationship in the data is shown as the blue-solid line. As in the data, inflation and the real rate of return on financial wealth are negatively related. Quantitatively, the model matches the slope of the relationship in the data for low inflation rates, but under predicts it for high inflation rates. We find the pass-through from inflation to rates of return is weaker for housing wealth, also consistent with the data.

The aggregate-demand channel works on the opposite direction from the interest rate channel and leads to a strong negative relationship between long-run inflation and unemployment. In the model, inflation reduces aggregate effective liquidity (market capitalization of each asset weighted by its acceptability), tightens households' liquidity constraints, lowers the demand for early consumption, and decreases $p^y$, illustrated in the right panel of Figure 7. The fall in $p^y$, lowers firms' expected revenue and profits, reduces entry, and increases unemployment. Quantitatively, we find the aggregate demand and interest rate channels essentially off-set each other.

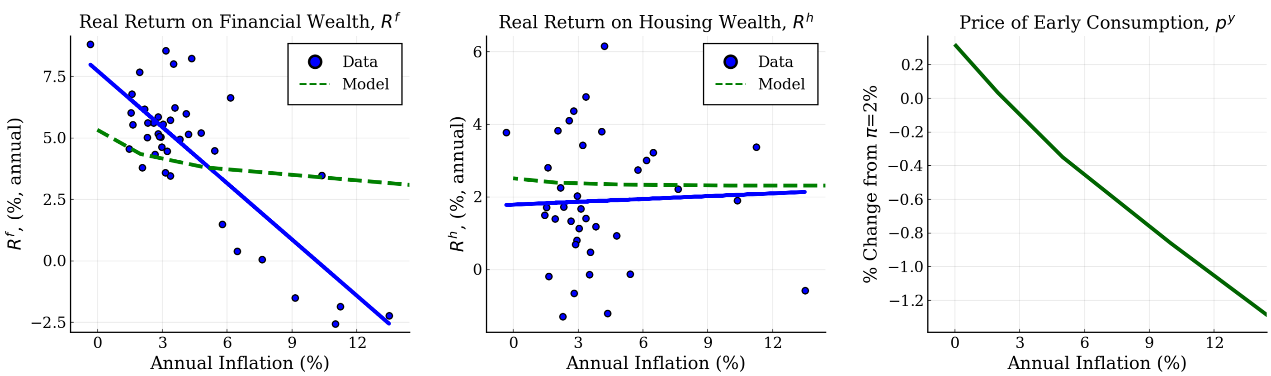


Figure 7: Inflation, asset returns, and prices

## 6.2 More on the slope of the long-run Phillips curve

For our calibration, the long-run Phillips curve is almost vertical, i.e., the unemployment rate is largely unresponsive to a change in anticipated inflation. As we emphasized, this unresponsiveness dissimulates strong channels of money creation working in opposite directions on the unemployment rate. We now show that changes in fundamentals or policy could make one channel dominant with quantitative implications for the long-run trade-off between inflation and unemployment.

**Liquidity of financial assets and the long-run Phillips curve** In the benchmark calibration, conditional on a preference shock, financial wealth can be liquidated 8% of the time. Suppose now that innovations in the finance and banking

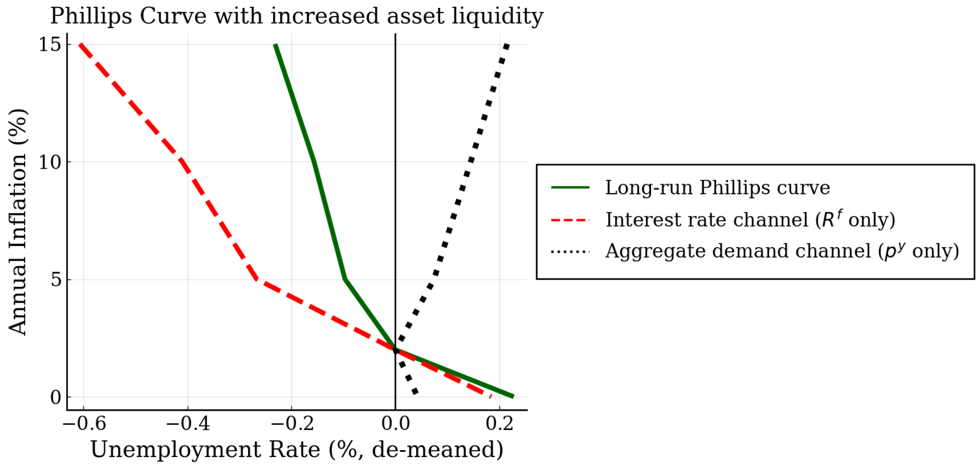


Figure 8: Long-run Phillips curve when asset liquidity is higher, $\alpha^{m,f} = \alpha^{m,h} = \alpha^m = 1/3$.

industry makes it easier to liquidate and transfer financial wealth in order to allow households to finance unexpected expenditures. We capture this idea by assuming that financial and housing wealth are more liquid than in the baseline, while keeping the same rate of expenditure shocks $\alpha = 0.1$. We set $\alpha^{m,f}/\alpha = \alpha^{m,h}/\alpha = \alpha^m/\alpha = 1/3$. Figure 8 illustrates how the long-run Phillips curve, and the strength of the aggregate demand and interest rate channels, change under these assumptions.

Increasing the liquidity of financial and housing wealth leads to a negatively-sloped long-run Phillips curve, illustrated as a solid-green line in Figure 8. Quantitatively, an increase in the inflation rate from 0 to 10% reduces unemployment by about 0.4 percentage points. When the liquidity of financial and housing wealth increases, these assets become more substitutable with money. As a result, the Tobin effect associated with inflation strengthens the interest-rate channel, as illustrated in the dashed-red line. At the same time, as more assets become liquid, the aggregate demand channel weakens since inflation has a smaller effect on households' total effective liquidity.

**Targeted 'helicoper drops'** In our benchmark experiment, the proceeds of money creation are rebated lump-sum to all households. We now consider a change in policy according to which the 'helicopter' drops target the unemployed, i.e., money creation is distributed lump sum to unemployed households only. Equivalently, unemployment benefits are financed with the inflation tax. Formally, the transfers conditional on employment status are equal to $\tau_0 = \pi\phi_t M_t/(1-n) + (1/R^g - 1)A^g$ and $\tau_{1=}(1/R^g - 1)A^g$. It means that taxes required to service government debt affect all households, but money creation is only distributed to the unemployed. This transfer scheme captures the possibility of income-progressivity in monetary transfers.

When money creation is used to finance unemployment benefits, the slope of the long-run Phillips curve increases relative to the baseline, as illustrated in the solid-green line in Figure 9. An increase of the inflation rate from 0

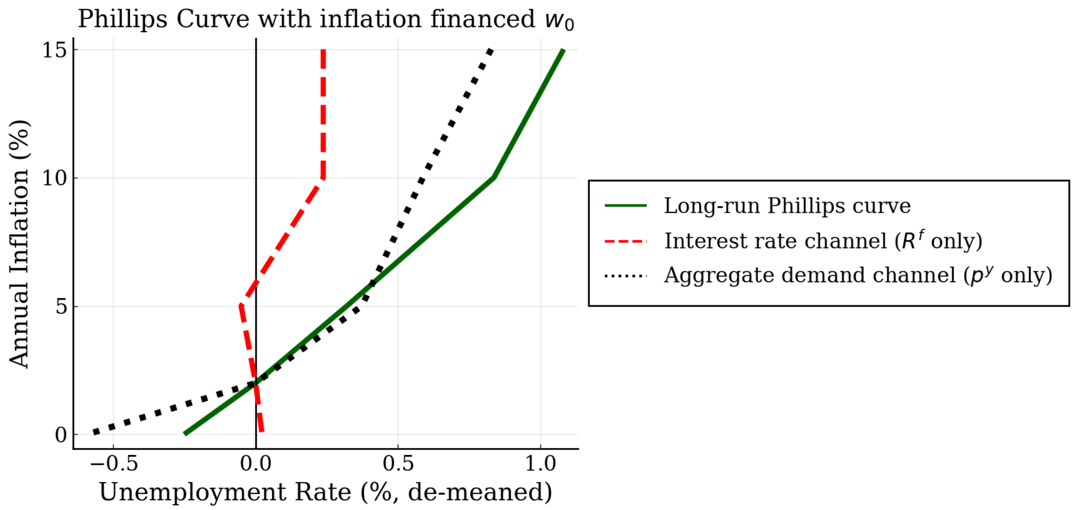


Figure 9: Long-run Phillips curve when money creation is distributed lump-sum to unemployed households

to 10 percent raises equilibrium unemployment by about one percentage point. The insurance provided by targeted transfers reduces households' precautionary demand for higher-return, less-liquid wealth, which weakens the interest-rate channel and strengthens the aggregate demand channel. This result shows that the slope of the long-run Phillips curve cannot be evaluated independently from the implemation scheme of monetary policy. A change in that scheme changes the long-run trade-off between anticipated inflation and unemployment.

### 6.3 Inflation and aggregate welfare

We now turn to the normative implications of money creation. The left panel of Figure 10 plots the welfare cost of inflation, measured as the percentage change in consumption that agents would be willing to incur, on average, to avoid moving from a steady state with inflation rate $\pi$ to a new steady state with inflation rate $\hat{\pi}$, as in Lucas (2000).[24] Formally, given any steady-state equilibrium collection of value functions, decision rules, and prices associated with inflation rate $\pi$, we define aggregate welfare as $\mathcal{W}(\pi, \Delta) = \sum_{e \in \{0,1\}} \int W_e(a, \Delta) dF_e(a)$, where

$$W_e(a, \Delta) = U(\Delta c, \Delta h, e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega \left[ v(\Delta y_{\omega e'}) + W_{e'} \left( \mathbf{1}.\hat{\mathbf{a}} - p^y \Delta y_{\omega e'} \right) \right] + (1 - \alpha) W_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right\}.$$

Notice, we consider a consumption equivalent across the 3 types of goods: early consumption, late consumption, and housing services. The welfare cost is then given by $1 - \Delta$, where $\Delta$ is the solution to $\mathcal{W}(\pi, \Delta) = \mathcal{W}(\hat{\pi}, 1)$. The left-panel of Figure 10 shows the welfare cost relative to a base inflation rate of $\pi = 2\%$.

The welfare cost is non-monotone as the inflation rate varies between 0% and 15%. Reducing inflation from 2% to zero harms aggregate welfare, i.e., agents would be willing to pay 0.25% of their total consumption to avoid constant prices. Increasing inflation above 2%, however, improves aggregate welfare. The optimal inflation rate is between 5%

[24] In these calculations, we ignore the costs and benefits along the transition path to a different steady-state equilibrium.

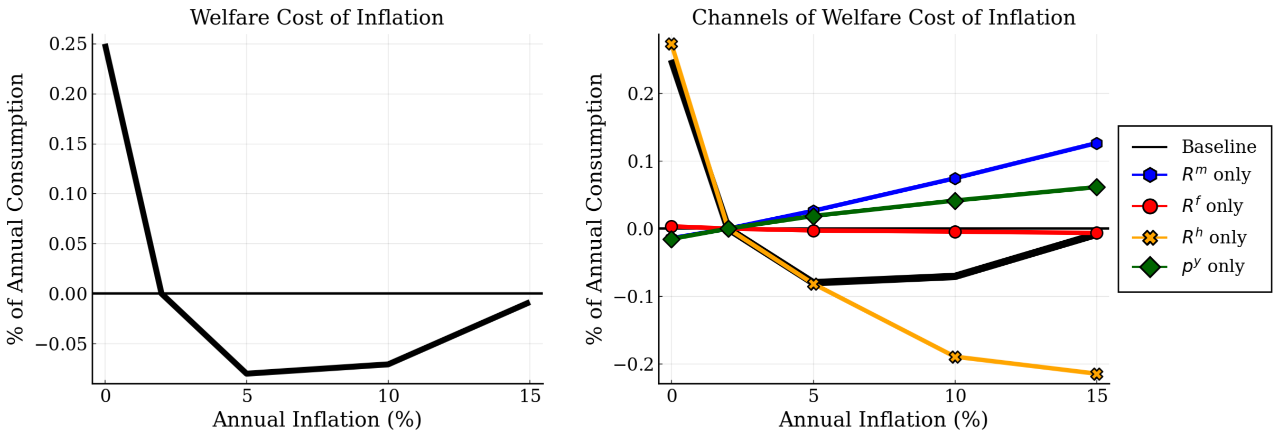


Figure 10: The aggregate welfare cost of inflation (left) and its decomposition (right).

and 10%, although, quantitatively, the aggregate welfare effects are small throughout the entire range shown.

The right panel of Figure 10 decomposes the welfare cost into components associated with the response of a particular asset return or price, $(R^m, R^f, R^h, p^y)$. Specifically, for each line we only allow one of these four equilibrium objects to vary at time, while keeping the remaining three constant. However, we also allow transfers, labor market tightness, and wages to vary when applicable.

Consider the welfare cost associated with the return on liquid wealth, $R^m$, illustrated by the blue line with hexagons. Inflation reduces the return on liquid wealth which unequivocally lowers welfare. However, inflation through money creation increases lump-sum transfers that have redistributive effects, which can lead to net welfare gains. This channel leads to a modest welfare cost that is in line with those from pure monetary models with non-degenerate distributions in the literature, e.g., Chiu and Molico (2010) find that increasing inflation from 0% to 10% reduces welfare by $0.62\%$, while we find it reduces welfare by and even smaller $0.10\%$.

The green line plots the welfare cost associated with the change in the price of early consumption, $p^y$. Inflation reduces $p^y$, which improves welfare as early consumption becomes cheaper. However, the fall in $p^y$ also decreases firms' expected revenue, firm entry, and labor market tightness. This reduces wages and increases unemployment duration, which lowers welfare. On net, the latter dominates, and the aggregate demand channel harms welfare.

The last two lines capture the interest rate channels of inflation through $R^f$ and $R^h$, illustrated in Figure 10 as red-circled and orange-crossed lines, respectively. Inflation causes a substitution towards higher-return, less-liquid assets that lowers $R^f$ and $R^h$. This effect unequivocally lowers welfare. However, there are also off-setting channels that improve welfare. As $R^f$ falls, firm entry and labor market tightness increase. This increases the job finding probability and lowers unemployment duration, improving welfare. Further, interest payments on government debt and, as a result, taxes fall. Again this improves welfare. On net, the interest rate channel through $R^f$ leads to essentially no welfare cost over the range we consider. In terms of the return on housing, a fall in $R^h$ causes the housing supply to increase and reduces the price of housing services, $p^h$, which improves welfare.

## 6.4 The effects of money creation in the cross-section

### 6.4.1 Inflation and asset portfolios

Figure 11 illustrates the effect of inflation on the composition of asset portfolios, by wealth and income, by plotting the change in the steady-state share of wealth in the form of liquid, financial, and housing assets (shown from the left panel to the right) induced by an increase in inflation from 2% to 15%. We illustrate the change by the percentile of total wealth in the baseline, separately for the employed (red-solid lines) and the unemployed (blue-dashed lines).

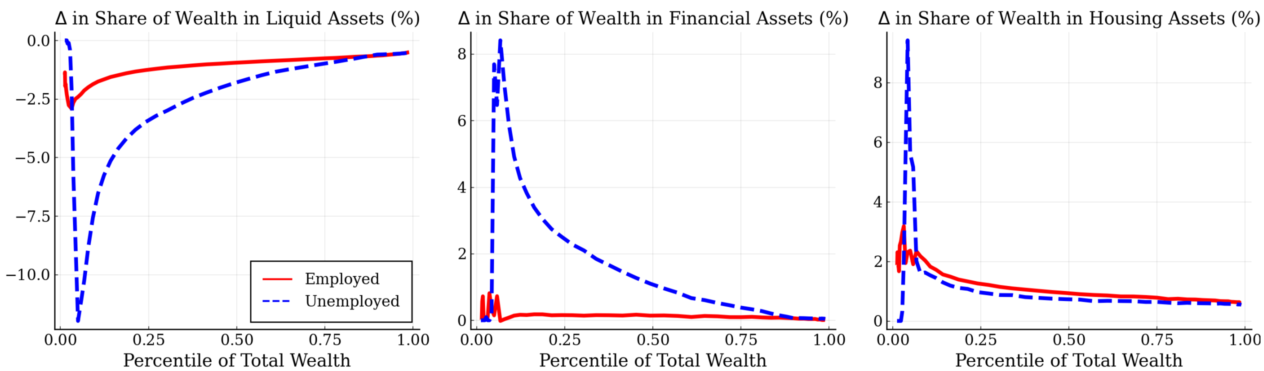


Figure 11: The effect of increasing inflation from 2% to 15% on portfolio shares, by wealth and income

Inflation has no effect on the portfolio choices of unemployed households at the very bottom of the wealth distribution. These households hold their entire wealth in liquid assets when inflation is low and continue to do so when inflation rises. Unemployed households with low to median wealth, however, respond strongly to inflation by reallocating their wealth towards less liquid assets. Households near the 10th percentile decrease their share of wealth held in liquid assets by 12% and increase their shares of financial and housing wealth by roughly equal amounts. Employed households are less responsive to inflation compared to the unemployed, but still reallocate toward less liquid assets. They predominately readjust their portfolio by substituting liquid wealth for housing wealth (the second-most liquid asset), regardless of their total wealth.

### 6.4.2 Inflation and consumption

The change in the composition of wealth induced by inflation is not necessarily indicative of how consumption responds. Figure 12 illustrates the change in early consumption (left panel), late consumption (middle panel), and housing services (right panel) by wealth. Most households respond to the rise of inflation by decreasing their early consumption. However, the low-wealth unemployed increase their early consumption despite holding less liquid wealth. This illustrates a 'hot potato' effect of inflation. Precautionary savings also falls in the last stage as late consumption and housing services rise for all households.

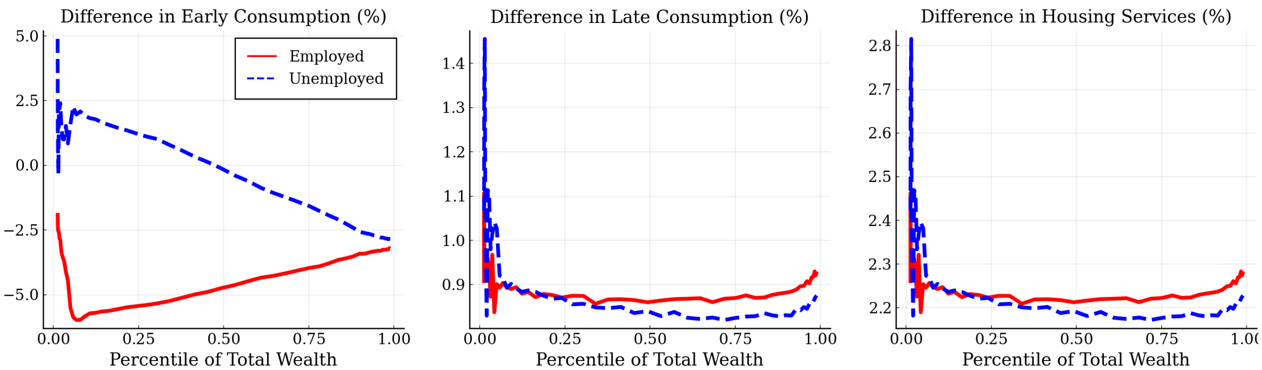


Figure 12: The effect of increasing inflation from 2% to 15% on the components of total consumption, by wealth and income

### 6.4.3 Inflation and welfare

Figure 13 reports the welfare cost of increasing inflation from 2% to 15%, by wealth and income. The dotted-green line represents the welfare cost for a given percentile of total wealth in the baseline steady state with $\pi = 2\%$, aggregating across employed and unemployed households. Inflation with lump-sum transfers has a strong redistributive effect. For a given level of income, the welfare cost of inflation is increasing in total wealth. However, inflation is a net positive for most households, except for the most wealthy. In terms of employment status, for any given wealth level, inflation is more costly for the unemployed compared to the employed. The households that gain the most are the lowest wealth employed while the households that bear the greatest cost are highest wealth unemployed.

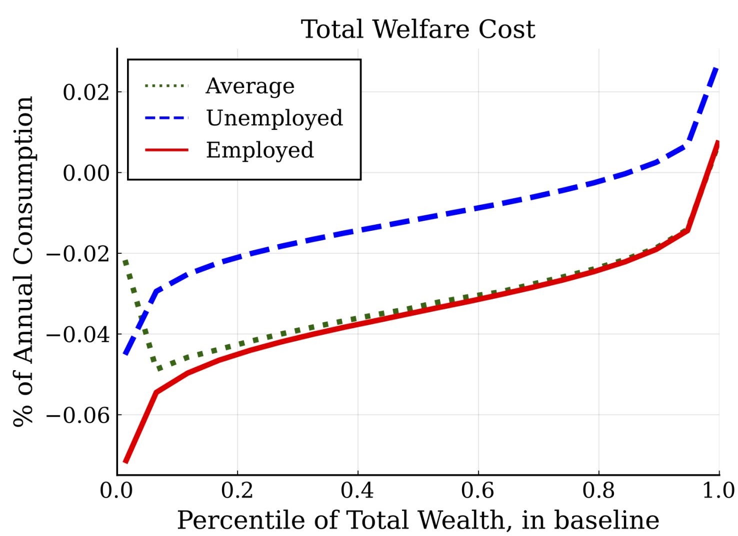


Figure 13: The welfare cost of increasing inflation from 2% to 15%, by wealth and income

In Figure 14, we show a decomposition of the welfare cost of inflation across the various interest rate and aggregate demand channels. In each panel, we plot the welfare cost of 15% inflation relative to 2% inflation, induced by changes in a particular interest rate or price, $(R^m, R^f, R^h, p^y)$, keeping the other three objects constant. We allow all other endogenous objects to change, for instance labor market tightness and transfers.

The top-left panel shows the welfare cost of inflation induced by the fall in $R^m$. Inflation is in part a regressive tax on poorer households that predominately hold their wealth in liquid assets. However, money creation through lump-sum transfers is progressive. On net, the latter effect dominates and the welfare cost of inflation through the fall in $R^m$ rises in total wealth. For a given level of wealth, however, the fall in $R^m$ is more costly for the unemployed than the employed. The top-right panel illustrates the distributional effects of inflation through the interest rate channel on financial wealth, $R^f$. The fall in $R^f$ disproportionately taxes wealthier households that hold a larger share of their wealth in stocks and bonds. It also promotes firm entry and increases job finding probabilities that disproportionately affect the unemployed and lower-wealth households. On net, the interest rate channel represents a welfare loss for unemployed households, regardless of their total wealth, and a welfare gain for almost all employed households except for the most wealthy.

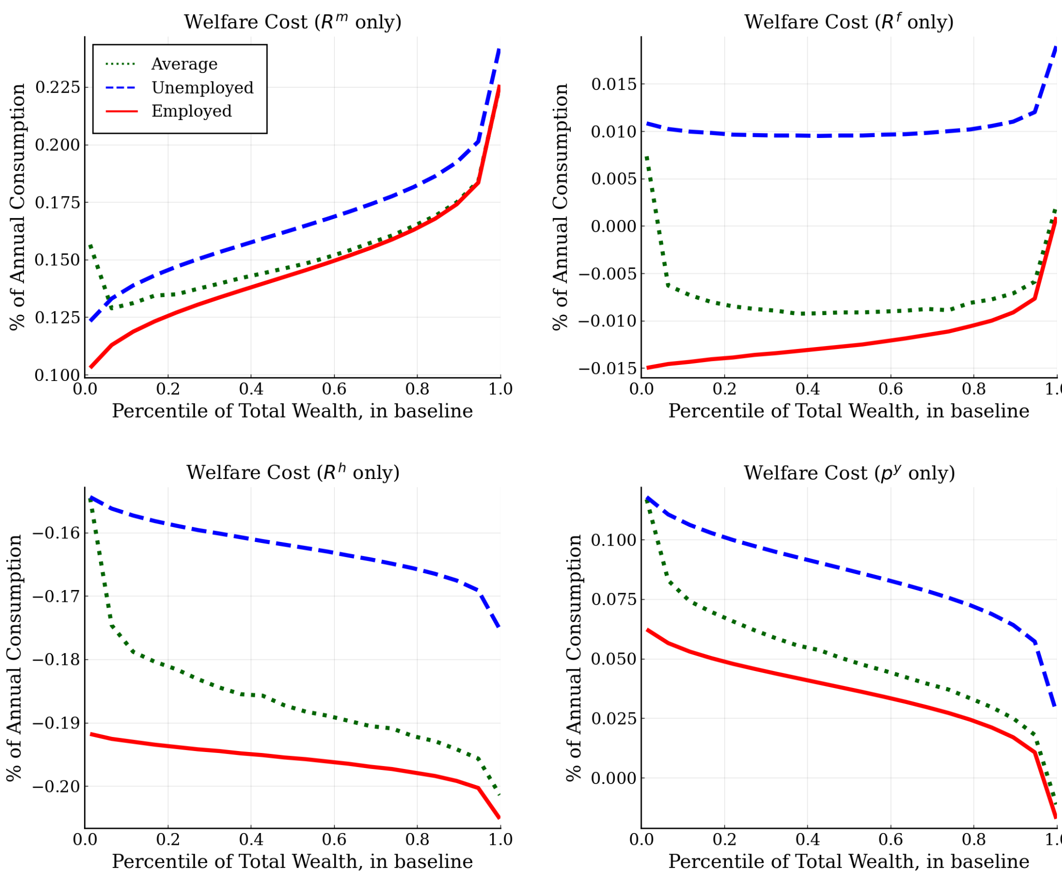


Figure 14: The channel of the welfare cost of increasing inflation from 2% to 15%, by wealth and income

The bottom-right panel of Figure 14 plots the welfare cost induced by the aggregate demand channel through the fall in $p^y$. Households benefit from lower $p^y$, however the decline in firms expected revenue reduces labor market tightness and wages. On net, the aggregate demand channel produces a welfare cost that falls most heavily on poorer households and the unemployed. Only the wealthiest employed households benefit from the fall in $p^y$. Finally, the bottom-left panel plots the welfare cost induced by the fall in the return on housing wealth, $R^h$. The fall in $R^h$ leads to a increase in the supply of houses and a fall in the price of housing services. On net, this benefits all households, however more so for the wealthier and employed.

# 7 Conclusion

We constructed a New-Monetarist model with competitive goods and asset markets opening sequentially and a frictional labor market described as in Mortensen and Pissarides (1994). Households, who are risk averse, face two types of uninsurable idiosyncratic risk, an endogenous employment risk and an expenditure risk. They self-insure against these risks by accumulating a portfolio of multiple assets (money, bonds, stocks, and real estate) with various degrees of liquidity. We applied our model to the study of money creation through lump-sum transfers to households, a.k.a., "helicopter money", and its effects on unemployment, households' asset portfolios and rates of return. We showed that money creation affects the economy through a variety of channels that we identified and quantified. First, there is an aggregate demand channel according to which the consumption of households who receive expenditure shocks increases with aggregate liquidity – a weighted-average of the market capitalization of all assets. Second, there is an interest-rate channel according to which anticipated inflation generates a substitution toward financial assets and real estate, thereby lowering their rates of return. Third, there is a distributional effect working through the proceeds of money creation that are redistributed lump-sum to all households.

Our calibrated model shows that anticipated inflation has a modest positive effect on equilibrium unemployment – the long-run Phillips curve is positively sloped but almost vertical. However, the underlying channels through which inflation operates are quantitatively large but operate in opposite directions, almost canceling each other. This finding suggests that changes in fundamentals that would affect the relative strengths of these channels, e.g., changes in the liquidity of financial assets, the progressivity of monetary transfers, or frequency of large expenditure shocks, could have important implications, qualitatively and quantitatively, for the long-run Phillips curve. For instance, we show that a somewhat modest increase in the acceptability of financial assets makes the Phillips curve downward sloping, i.e., an increase in anticipated inflation lowers the steady-state unemployment rate.

The optimal rate of money creation – in the sense of maximizing steady-state welfare – is around 5 percent. By disaggregating the effects of inflation across households at different points of the wealth distribution, we showed that transfers financed with money creation raises the welfare of all households except the most wealthy.

We also investigate the effects of an expansion in the supply of government debt in Appendix C. Increased government debt raises the return on financial assets and crowds out private liquidity supplied through firm equity and housing. In turn, aggregate effective liquidity, measured by the sum of households' wealth weighted by asset acceptability, falls, which reduces aggregate demand. The fall in aggregate demand combined with the fall in the financial discount factor leads to an increase in unemployment. An expansion of government debt unequivocally reduces welfare for all households, but the effects are strongest for the low wealth and the unemployed.

A natural next step consists of introducing aggregate shocks on labor productivity so that the value of firms becomes stochastic, thereby affecting the liquidity services of stocks and the strength of the interest rate channel, and potentially the behavior of unemployment over the business cycle. It would also be natural to introduce consumer credit as an additional self-insurance tool for households. Finally, one could explore different forms of asset liquidity (e.g., adjustment costs versus partial acceptability or pledgeability) to see how they affect the functioning of the different channels we described in this paper. We leave these extensions for future work.

# References


Algan, Y., E. Challe, and X. Ragot (2011). Incomplete markets and the output-inflation tradeoff. *Economic Theory 46*, 55–84.

Anderson, R. and B. Jones (2011, September). A comprehensive revision of the u.s. monetary services (divisia) indexes. *Federal Reserve Bank of St. Louis Review September*, 325–360.

Auclert, A. (2019, June). Monetary polity and the redistribution channel. *American Economic Review 109*(6), 2333–67.

Benati, L. (2015). The long-run phillips curve: A structural var investigation. *Journal of Monetary Economics 76*, 15–28.

Berensten, A., G. Menzio, and R. Wright (2011). Inflation and unemployment in the long run. *American Economic Review 101*, 371–398.

Bethune, Z., G. Rocheteau, and P. Rupert (2015). Aggregate unemployment and household unsecured debt. *Review of Economic Dynamics 18*(1), 77–100.

Bewley, T. (1980). The optimum quantity of money. In J. Kereken and N. Wallace (Eds.), *Models of Monetary Economics*. Federal Reserve Bank of Minneapolis.

Bewley, T. (1983). A difficulty with the optimum quantity of money. *Econometrica 51*, 1485–1504.

Branch, W., N. Petrosky-Nadeau, and G. Rocheteau (2016, July). Financial frictions, the housing market, and unemployment. *Journal of Economic Theory 164*, 101–135.

Branch, W. and M. Silva (2020). Liquidity, unemployment, and the stock market. in mimeo.

Browning, M. and T. Crossley (2001). Unemployment insurance benefit levels and consumption changes. *Journal of Public Economics 80*(1), 1–23.

Chiu, J. and M. Molico (2010). Liquidity, redistribution, and the welfare cost of inflation. *Journal of Monetary Economics 57*(4), 428–438.

Chiu, J. and M. Molico (2011). Uncertainty, inflation, and welfare. *Journal of Money, Credit, and Banking 43*(2), 487–512.

Doepke, M. and M. Schneider (2006, December). Inflation and the redistribution of nominal wealth. *Journal of Political Economy 114*(6), 1069–1097.

Eeckhout, J. and A. Sepahsalari (2021). The effect of wealth on worker productivity. in mimeo.

Ganong, P. and P. Noel (2019). Consumer spending during unemployment: Positive and normative implications. *American Economic Review 109*(7), 2383–2424.

Graves, S. (2021). Does unemployment risk affect business cycle dynamics. in mimeo.

Green, E. and R. Zhou (1998, August). A rudimentary random-matching model with divisible money and prices. *Journal of Economic Theory 81*, 252–271.

Hall, R. and S. Schulhofer-Wohl (2018). Measuring job-finding rates and matching efficiency with heterogeneous job-seekers. *American Economic Journal: Macroeconomics 10*(1), 1–32.

Hansen, G. and A. Imrohoroglu (1992). The role of unemployment insurance in an economy with liquidity constraints and moral hazard. *Journal of Political Economy 100*(1), 118–142.

He, C., R. Wright, and Y. Zhu (2015). Housing and liquidity. *Review of Economic Dynamics 18*(3), 435–455.

Herrenbrueck, L. (2019, May). Frictional asset markets and the liquidity channel of monetary policy. *Journal of Economic Theory 181*, 82–120.

Hurd, M. and S. Rohwedder (2010). Effects of the financial crisis and great recession on american households. NBER Working Papers 16407.

Jappelli, T. and L. Pistaferri (2014). Fiscal policy and mpc heterogeneity. *American Economic Journal: Macroeconomics 6*(4), 107–136.

Jin, G. and T. Zhu (2019). Nonneutrality of money in dispersion: Hume revisited. *International Economic Review 60*(3), 1329–1353.

Jin, G. and T. Zhu (2020). Heterogeneity, decentralized trade, and the long-run real effects of inflation. MPRA Working Paper No. 101927.

Kam, T. and J. Lee (2021). Inflationary redistribution vs. trading opportunities. in mimeo.

Kaplan, G., G. Menzio, L. Rudanko, and N. Trachter (2019). Relative price dispersion: Evidence and theory. *American Economic Journal: Microeconomics 11*(2), 68–124.

Kaplan, G., B. Moll, and G. Violante (2018). Monetary policy according to hank. *American Economic Review 108*(3), 697–743.

Kekre, R. (2021). Unemployment insurance in macroeconomic stabilization. in mimeo.

Krusell, P., T. Mukoyama, and A. Sahin (2010). Labor-market matching with precautionary savings and aggregate fluctuations. *Review of Economic Studies 77*(4), 1477–1507.

Lagos, R., G. Rocheteau, and R. Wright (2017). Liquidity: A new monetarist perspective. *Journal of Economic Literature 55*(2), 371–440.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lagos, R. and S. Zhang (2020, June). Turnover liquidity and the transmission of monetary policy. *American Economic Review 110*(6), 1635–1672.

Lester, B., A. Postlewaite, and R. Wright (2012). Information, liquidity, asset prices, and monetary policy. *Review of Economic Studies 79*, 1209–1238.

Li, Y., G. Rocheteau, and P.-O. Weill (2012). Liquidity and the threat of fradulent assets. *Journal of Political Economy 120*, 815–846.

Lucas, R. (2000). Inflation and welfare. *Econometrica 68*(2), 247–274.

Menzio, G., S. Shi, and H. Sun (2013). A monetary theory with non-degenerate distributions. *Journal of Economic Theory 148*(6), 2266–2312.

Molico, M. (2006). The distribution of money and prices in search equilibrium. *International Economic Review 47*(3), 701–722.

Mortensen, D. and C. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies 61*, 397–415.

Parker, J., N. Souleles, D. Johnson, and R. McClelland (2013, October). Consumer spending and the economic stimulus payments of 2008. *American Economic Review 103*(6), 2530–2553.

Rocheteau, G. and A. Rodriguez-Lopez (2014). Liquidity provision, interest rates, and unemployment. *Journal of Monetary Economics 65*, 80–101.

Rocheteau, G., P. Rupert, and R. Wright (2007). Inflation and unemployment in general equilibrium. *Scandanavian Journal of Economics 109*, 837–855.

Rocheteau, G., P.-O. Weill, and T.-N. Wong (2021). An heterogeneous-agent new-monetarist model with an application to unemployment. *Journal of Monetary Economics 117*, 64–90.

Rocheteau, G. and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica 73*(1), 175–202.

Rocheteau, G., R. Wright, and C. Zhang (2018). Corporate finance and monetary policy. *American Economic Review 108*(4-5), 1147–1186.

Schaal, E. (2017). Uncertainty and unemployment. *Econometrica 85*(6), 1675–1721.

Shi, S. (1998). Search for a monetary propogation mechanism. *Journal of Economic Theory 81*, 314–352.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review 95*(1), 25–49.

Silva, J. I. and M. Toledo (2009). Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics 13*, 76–96.

Sommer, K., P. Sullivan, and R. Verbrugge (2013). The equilibrium effect of fundamentals on house prices and rents. *Journal of Monetary Economics 60*, 854–870.

Venkateswaran, V. and R. Wright (2013). Pledgability and liquidity: A new monetarist model of financial and macroeconomic activity. *NBER Macroeconomics Annual*.

Williamson, S. (2012). Liquidity, monetary policy, and the financial crisis: A new monetarist approach. *American Economic Review 102*(6), 2570–2605.

# 8 Appendix A: Proofs of Propositions

**Proof of Proposition 1.** We apply standard contraction-mapping arguments to the Bellman equation (4),

$$W_e(a) = \max_{c,h,\mathbf{y},\hat{\mathbf{a}}} \left\{ U(c,h,e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega \left[ v(y_{\omega e'}) + W_{e'}\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}\right)\right] + (1-\alpha) W_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right\} \right\} \tag{42}$$
$$\text{s.t. } c + p^h h = a + w_e - \mathbf{R}^{-1}.\hat{\mathbf{a}} \text{ and } p^y y_{\omega e'} \leq \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}, \;\; e' \in \{0,1\}.$$

Consider the space $C(\{0,1\} \times \mathbb{R}_+)$ of bounded and continuous functions from $\{0,1\} \times \mathbb{R}_+$ into $\mathbb{R}$, equipped with the sup norm. By Theorem 3.1 in Stokey, Lucas, and Edward Prescott (1989, henceforth SLP), this is a complete metric space. Now, for any $f \in C(\{0,1\} \times \mathbb{R}_+)$, consider the Bellman operator:

$$T[f]_e(a) = \max \left\{ U(c,h,e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega \left[ v(y_{\omega e'}) + f_{e'}\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}\right)\right] + (1-\alpha) f_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right\} \right\}$$

with respect to $c \geq 0$, $h \geq 0$, $y_{\omega e'} \leq \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}/p^y$, and $\mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e - c - p^h h$. It is straightforward to verify that $T$ satisfies the Blackwell sufficient condition for a contraction (Theorem 3.3 in SLP). Moreover, the constraint set is non-empty, compact valued, and continuous. Hence, by the Theorem of the Maximum (Theorem 3.6 in SLP), we obtain that $T[f]$ is continuous. It is bounded since all the functions on the right side of the Bellman equation, including $U$ and $v$, are bounded. Note as well that if $f$ is concave, then $T[f]$ is also concave since the objective is concave (because $U$ and $v$ are concave by assumption) and the constraint correspondence has a convex graph. An application of the Contraction Mapping Theorem (Theorem 3.2 in SLP) implies that the fixed point problem $f = T[f]$ has a unique bounded solution, $W_e(a)$, and that this solution is continuous and concave. From the assumptions that $U(c,h,e)$ and $v(y)$ are increasing in $(c,h)$ and $y$, respectively, it follows that $W_e(a)$ is increasing. Given a fixed point $W_e(a)$ of the Bellman operator $T$, we can define $V_e(\hat{\mathbf{a}})$ as in equation (3). By identical arguments as above, one sees that $V_e(\hat{\mathbf{a}})$ is bounded, continuous, concave, and strictly increasing.

The indirect utility of the household in the second stage corresponds to the following Lagrangian:

$$\Omega_{\omega e}(\hat{\mathbf{a}}) = \max_y \left\{ v(y) + W_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y) + \lambda_{\omega e}\left(\boldsymbol{\chi}_\omega.\hat{\mathbf{a}} - p^y y\right) \right\}, \tag{43}$$

where $\lambda_{\omega e} \geq 0$ denotes the Lagrange multiplier corresponding to the constraint $y \leq \boldsymbol{\chi}_\omega.\hat{\mathbf{a}}/p^y$. The objective is strictly concave and the constraint is linear. By Corollary 1 in Marimon and Werner (2015) the value function, $\Omega_{\omega e}(\hat{\mathbf{a}})$, is differentiable with

$$\frac{\partial \Omega_{\omega e}(\hat{\mathbf{a}})}{\partial a^j} = W_e'(\mathbf{1}.\hat{\mathbf{a}} - p^y y) + \chi_\omega^j \lambda_{\omega e}.$$

Moreover, the first-order condition for $y$ gives:

$$v'(y_e) = p^y W_e'(\mathbf{1}.\hat{\mathbf{a}} - p^y y_e) + p^y \lambda_{\omega e}.$$

Hence, the partial derivatives of $\Omega_{\omega e}(\hat{\mathbf{a}})$ are also given by

$$\frac{\partial\Omega_{\omega e}(\hat{\mathbf{a}})}{\partial\hat{a}^j} = \chi_\omega^j\frac{v'(y_{\omega e})}{p^y} + (1-\chi_\omega^j)W_e'(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}).$$

Substituting $\Omega_{\omega e}(\hat{\mathbf{a}})$ from (43) into (42), the Bellman equation can be rewritten as:

$$W_e(a) = \max_{h,\hat{\mathbf{a}}}\left\{U\left(a + w_e - p^h h - \mathbf{R}^{-1}.\hat{\mathbf{a}}, h, e\right) + \beta\mathbb{E}_e\left\{\sum_{\omega\in\Omega_{\mathbb{A}}}\alpha_\omega\Omega_{\omega e'}(\hat{\mathbf{a}}) + (1-\alpha)W_{e'}(\mathbf{1}.\hat{\mathbf{a}})\right\}\right\}. \quad (44)$$

By the same Corollary 1 in Marimon and Werner (2015) the value function $W_e(a)$ is differentiable for all $a > 0$ with $W_e'(a) = U_c\left[c_e(a), h_e(a), e\right]$. From (3), the value function in the second stage can be reexpressed as

$$V_e(\hat{\mathbf{a}}) = \sum_{\omega\in\Omega_{\mathbb{A}}}\alpha_\omega\Omega_e(\hat{\mathbf{a}}) + (1-\alpha)W_e(\mathbf{1}.\hat{\mathbf{a}}).$$

The partial derivatives are

$$\frac{\partial V_e(\hat{\mathbf{a}})}{\partial\hat{a}^j} = \sum_{\omega\in\Omega_{\mathbb{A}}}\alpha_\omega\left[\chi_\omega^j\frac{v'(y_{\omega e})}{p^y} + (1-\chi_\omega^j)W_e'(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e})\right] + (1-\alpha)W_e'(\mathbf{1}.\hat{\mathbf{a}}).$$

■

**Proof of Proposition 2.** The condition for liquidity to be abundant is $a \geq \mu y_\mu^\star$, which from (31) gives (36). The result $r = \rho$ follows directly from (28). Let us turn to equilibria with scare liquidity. By market clearing households' asset holdings are $a = (1+r)\theta k^f/\left[\delta^f + \lambda(\theta)\right] + A^g$. From (29) we can reexpress the first term on the right side as

$$\frac{\theta k^f}{\delta^f + \lambda(\theta)} = \frac{\frac{\lambda(\theta)}{\delta^f+\lambda(\theta)}(\bar{q} - w_1) + \alpha\left(\frac{\mu-1}{\mu}\right)a}{r + \delta^f}.$$

Hence, $a$ is a solution to

$$a = \frac{(1+r)\left[\frac{\lambda(\theta)}{\delta^f+\lambda(\theta)}(\bar{q} - w_1) + \alpha\left(\frac{\mu-1}{\mu}\right)a\right]}{(r+\delta^f)} + A^g. \quad (45)$$

Under the assumption $\bar{q} - w_1 > 0$, (45) admits a finite positive solution if $(1+r)\alpha\left(\frac{\mu-1}{\mu}\right)/(r+\delta^f) < 1$. Under this condition, the solution $a(r,\theta,A^g)$ is decreasing in $r$, and increasing in $\theta$ and $A^g$. We can reduce (28)-(31) to

$$\frac{\rho - r}{1+r} = \alpha\left\{\mu^{-1}v'\left[\frac{a(r,\theta,A^g)}{\mu}\right] - 1\right\} \quad (46)$$

$$\left\{\left[\frac{\mu(1-\alpha)+\alpha}{\mu}\right]r + \delta^f - \alpha\left(\frac{\mu-1}{\mu}\right)\right\}\frac{\theta k^f}{\lambda(\theta)} = \bar{q} - w_1 + \alpha\left(\frac{\mu-1}{\mu}\right)\left(1 + \frac{\delta^f}{\lambda(\theta)}\right)A^g. \quad (47)$$

Note that the first term between brackets on the left side of (47) is positive provided that the solution $a$ to (45) exists. The asset market clearing condition, (46), gives a positive relationship between $r$ and $\theta$. Assuming $\bar{q} - w_1 > 0$, the job creation condition, (47), gives a negative relationship between $\theta$ and $r$. In the space $(\theta, r)$ an increase in $A^g$ shifts the asset market clearing condition upward and the job creation condition to the right. Hence, $\partial r/\partial A^g > 0$ but $\partial\theta/\partial A^g \lesseqgtr 0$. ■

# Appendix B. A simple linear model

We describe a simple version of our model that has no housing and linear preferences both in the last stage, $U(c_t, e_t) = c_t + (1 - e_t)\ell$, as in the MP model, and in the early stage, $v(y) = \bar{v}y$ with $\bar{v} > 1$.[25] In the presence of liquidity constraints, this linear specification does not make the distribution of asset holdings degenerate, but it renders distributional effects inoperative, thereby allowing us to focus on the interest rate and aggregate demand channels. We assume that all assets are equally pledgeable, $\chi = 1$, so that the two forms of public liquidity, government bonds and money, are perfect substitutes. We focus on equilibria with bonds only and denote $a^g = A^g/n$ the supply of one-period real bonds per employed household. We set $\kappa(y) = y$, which means that the production $\bar{q} = \bar{y}$ can be stored across stages with no additional transformation cost to sell to early consumers. With no loss in generality (because of linear preferences and a balanced budget of the government), we set $\bar{w}_0 = \tau_0 = 0$.

**Price of early consumption goods** The price of early consumption must satisfy $p^y = p = 1$. Indeed, if $p < 1$ firms sell all their output to late consumers, which is inconsistent with market clearing for early consumption. If $p > 1$, then all the output is sold to early consumers and there is no output left to finance the entry costs of new firms.

**The consumption/saving decision** Let $V' \equiv V'(a)$ denote the expected discounted utility of one unit of asset at the beginning of a period. It solves

$$V' = \alpha\bar{v} + (1 - \alpha)\beta R V' \Longrightarrow V' = \frac{\alpha\bar{v}}{1 - (1 - \alpha)\beta R}. \tag{48}$$

With probability $\alpha$ the asset is traded for one unit of early consumption, which generates a utility $\bar{v}$. With complement probability, $1 - \alpha$, the asset is saved for the following period (which is always weakly optimal by market clearing), which generates the expected discounted utility $\beta R V'$. The consumption/saving decision in the last stage is given by:

$$\max_{\hat{a} \geq 0} \left( -\frac{\hat{a}}{R} + \beta V' \hat{a} \right) \text{ s.t. } \hat{a} \leq R\left(a + \bar{w}_e + \tau_e\right).$$

The demand for assets is positive only if $\beta R V' \geq 1$, which can be reexpressed as:

$$R \geq \underline{R} \equiv \frac{1}{\beta\left(\alpha\bar{v} + 1 - \alpha\right)}. \tag{49}$$

The lower bound for the real interest rate is less than $\rho$ and it decreases with the frequency, $\alpha$, and value, $\bar{v}$, of early-consumption opportunities. If $R = \underline{R}$, then households are just indifferent between saving and consuming late. If $R > \underline{R}$, then households save their full income, $\hat{a} = R\left(a + \bar{w}_e + \tau_e\right)$.

[25] Such linear specification is used in the context of New-Monetarist models in the example in Section 6.2 of Rocheteau et al. (2018) and in Herrenbrueck (2019). It is also used in the context of an over-the-counter market with liquidity constraints by Lagos and Zhang (2020).

**The demand for private assets** Let $\bar{\Omega}$ denote aggregate wealth at the beginning of the second stage if $R > \underline{R}$. It satisfies:

$$\bar{\Omega}_{t+1} = R\left[(1-\alpha)\bar{\Omega}_t + n_t(\bar{w}_1 + \tau_1)\right].$$

Aggregate wealth in period $t+1$ is equal to the wealth of the $1-\alpha$ households who did not spend it on early consumption, plus total labor income and transfers, everything capitalized at rate $R$. From the budget constraint of the government, $\tau_1 = (1-R)\,a^g/R$. The stationary solution is

$$\bar{\Omega}(R) = \frac{n\left[R\bar{w}_1 + (1-R)\,a^g\right]}{1-R(1-\alpha)} \text{ if } R < (1-\alpha)^{-1}. \tag{50}$$

If $R(1-\alpha) > 1$, then the dynamics of wealth accumulation are explosive, $\bar{\Omega} = +\infty$. We define by $\bar{\omega}(R) \equiv \left[\bar{\Omega}(R) - A^g\right]/n$ the maximum holdings of private assets per employed households. From (50):

$$\bar{\omega}(R) = \frac{R\left(\bar{w}_1 - \alpha a^g\right)}{1-R(1-\alpha)} \text{ if } R < (1-\alpha)^{-1}. \tag{51}$$

Note that $\bar{w}_1 > \alpha a^g$ is necessary for households to accumulate private assets. Under that condition, $\bar{\omega}(R)$ is increasing in $R$.

**Job creations and the supply of private assets** From (18), assuming the labor market is active, $\theta$ solves

$$\frac{\theta k^f}{\lambda(\theta)} = \frac{\bar{q} - \bar{w}_1}{r + \delta^f}. \tag{52}$$

Using that $\lim_{\theta \to 0} \lambda(\theta)/\theta = \lambda'(0) = 1$, $\theta > 0$ if $R < \bar{R} \equiv \left[\bar{q} - \bar{w}_1 + (1-\delta^f)k^f\right]/k^f$. Hence, for an active equilibrium to exist, $[\underline{R}, \bar{R})$ must be nonempty, i.e.,

$$\underline{R} < \bar{R} \Leftrightarrow \frac{\left[\rho + \delta^f - \alpha(\bar{v}-1)(1-\delta^f)\right]}{\alpha\bar{v} + 1 - \alpha} k^f < \bar{q} - \bar{w}_1. \tag{53}$$

We assume in the following that (53) holds, i.e., entry costs are sufficiently low to generate firm entry. The supply of private assets per employed worker is $a^p(R) \equiv \phi^f(R)$ where, from (13),

$$a^p(R) = \frac{R\left(\bar{q} - \bar{w}_1\right)}{R - (1-\delta^f)}, \ \forall R \in \left(1-\delta^f, \bar{R}\right). \tag{54}$$

It is decreasing in $R$ with $\lim_{R \searrow 1-\delta^f} a^p(R) = +\infty$.

**Determination of the real interest rate** The asset market clearing condition can be expressed as

$$\bar{\omega}(R) \geq a^p(R), \quad " = " \text{ if } R > \underline{R}. \tag{55}$$

If $\bar{\omega}$ is larger than the supply of private assets – a savings glut – then households do not save their full income, which requires $R = \underline{R}$.[26] An equilibrium is a list $(n, \theta, R)$ that solves (19), (52), and (55). The equilibrium condition (55) is represented graphically in Figure 15. The following proposition characterizes equilibria in closed form.

[26]By Walras's Law the clearing condition of the asset market, (55), and the clearing condition of the goods market are redundant. Hence, in the following we focus on the former.

**Proposition 3** ***(Linear model.)*** *Suppose $U(c,e) = c$, $v(y) = \bar{v}y$ with $\bar{v} > 1$, and (53) holds. Assume $\bar{w}_1 > \alpha a^g$. There are two regimes with an active labor market ($\theta > 0$).*

***(i) Savings glut.*** *If the following conditions hold,*

$$\bar{w}_1 - \alpha a^g \geq \frac{[\alpha\bar{v} - \rho(1-\alpha)]}{\rho + \delta^f - (1-\delta^f)\alpha(\bar{v}-1)}(\bar{q} - \bar{w}_1) \tag{56}$$

$$\rho + \delta^f > (1-\delta^f)\alpha(\bar{v}-1), \tag{57}$$

*then $r$ and $\theta$ are independent of $a^g$ and solve:*

$$r = \frac{\rho - \alpha(\bar{v}-1)}{1+\alpha(\bar{v}-1)} \tag{58}$$

$$\frac{\theta k^f}{\lambda(\theta)} = \frac{[1+\alpha(\bar{v}-1)](\bar{q}-\bar{w}_1)}{\rho+\delta^f-(1-\delta^f)\alpha(\bar{v}-1)}. \tag{59}$$

***(ii) Abundant asset supply.*** *If (56)-(57) do not hold and*

$$a^g < \frac{\alpha\bar{w}_1 + (1-\alpha)\bar{q} - \left[\alpha + \delta^f(1-\alpha)\right]k^f}{\alpha}, \tag{60}$$

*then $r$ and $\theta$ solve:*

$$r = \frac{\alpha(\bar{q}-\bar{w}_1) - \delta^f(\bar{w}_1 - \alpha a^g)}{(\bar{w}_1 - \alpha a^g) + (1-\alpha)(\bar{q}-\bar{w}_1)} \tag{61}$$

$$\frac{\theta k^f}{\lambda(\theta)} = \frac{\alpha(\bar{w}_1 - a^g) + (1-\alpha)\bar{q}}{\alpha + \delta^f(1-\alpha)}. \tag{62}$$

*Moreover, $\partial r/\partial a^g > 0$, $\partial\theta/\partial a^g < 0$, $\partial r/\partial\bar{w}_1 < 0$, $\partial\theta/\partial\bar{w}_1 > 0$, and $\partial n/\partial\bar{w}_1 > 0$.*

**Proof.** (i) The savings glut regime is defined by $R = \underline{R}$. From (49) and (52) $r$ and $\theta$ solve (58) and (59). A necessary condition for (55) to hold at $R = \bar{R}$ is $\bar{R} > 1 - \delta^f$, i.e., $\rho + \delta^f > (1-\delta^f)\alpha(\bar{v}-1)$. A sufficient condition for (55) to hold at $R = \bar{R}$ is

$$\alpha\bar{v} - (1-\alpha)\rho \leq 0, \tag{63}$$

in which case $\bar{\omega}(\bar{R}) = +\infty$. If $\rho < \alpha\bar{v}/(1-\alpha)$, then $\bar{\omega}(\bar{R}) \geq a^p(\bar{R})$ can be reexpressed as

$$\bar{w}_1 - \alpha a^g \geq \frac{[\alpha\bar{v} - \rho(1-\alpha)]}{\rho+\delta^f-(1-\delta^f)\alpha(\bar{v}-1)}(\bar{q}-\bar{w}_1). \tag{64}$$

Given $\bar{w}_1 - \alpha a^g > 0$, (63) implies (64). (ii) The second regime is such that $R \in (\underline{R}, \bar{R})$. The endogenous variables, $r$ and $\theta$, solve (61) and (62). It is easy to check that $R > \underline{R}$ is equivalent to (56) does not hold. Let's consider the comparative statics with respect to $w_1$. From (61),

$$\frac{\partial r}{\partial\bar{w}_1} = \frac{-\left(\alpha R + \delta^f\right)}{D}$$

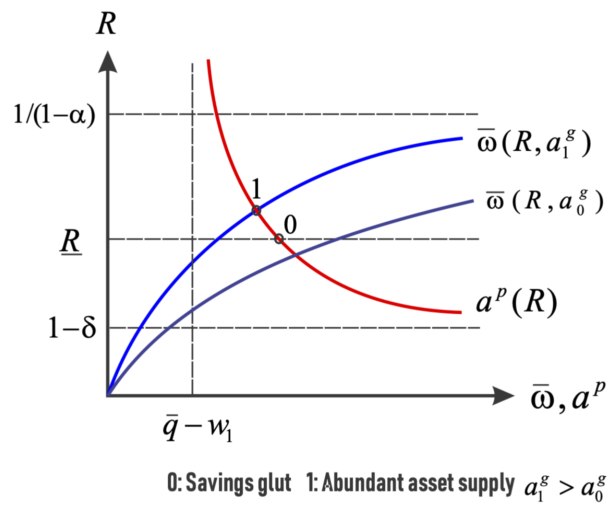


Figure 15: Equilibrium in simple linear model

where

$$D \equiv \bar{w}_1 - \alpha a^g + (1-\alpha)(\bar{q} - \bar{w}_1).$$

From (62), $\theta > 0$ implies $D > 0$. Hence, $\partial r/\partial \bar{w}_1 < 0$ since $R > 0$. The result $\partial \theta/\partial \bar{w}_1 > 0$ follows directly from (62). Let's consider next comparative statics with respect to $a^g$. From (61),

$$\frac{\partial r}{\partial a^g} = \frac{\alpha\left(\delta^f + r\right)}{D}.$$

From (52), $r > -\delta^f$. Hence, $\partial r/\partial a^g > 0$. The result $\partial \theta/\partial a^g < 0$ follows directly from (62).

■

In the first regime, the supply of assets is scarce relative to the potential wealth that households can accumulate, which drives the (gross) real interest to its lower bounds, $\underline{R}$. In Figure ?? we indicate such an equilibrium where $\bar{\omega}(\underline{R}) > a^p(\underline{R})$ by the marker "0". The supply of public liquidity, $a^g$, has no effect on $R$, and $\theta$. Indeed, if $a^g$ increases, then households ramp up their asset holdings without asking for a higher interest rate. The fact that households raise their early consumption has not effect on firms' profits since early consumption and late consumption are sold at the same price. The condition for a savings glut, (56), holds if $a^g$ is small, if $\bar{w}_1$ is large, or if $\alpha$ is small.

In the second regime the supply of assets is sufficiently abundant to drive the real interest rate above its lower bound. In Figure 15 we indicate such an equilibrium where $\bar{\omega}(R) = a^p(R)$ by the marker "1". Households save their full income in order to spend their wealth on early consumption opportunities. When $a^g$ increases, the supply of assets becomes larger than the maximum wealth households can accumulate given their income. As a result, $r$ increases, which reduces the supply of private assets, $\partial r/\partial a^g > 0$, $\partial \theta/\partial a^g < 0$, and $\partial n/\partial a^g < 0$. This effect is the interest channel of public liquidity.

## Adding a markup

In order to allow the composition of sales to early and late consumers to matter for firms' profits, suppose now that early consumption is sold at a markup $\mu > 1$ over the opportunity cost of selling late. We treat this markup as exogenous in this simple version of the model but it arises endogenously in the general version when $\kappa'' > 0$. Analogous to the assumption of random matching in search models, the demand for early consumption is divided evenly among the $n$ active firms in the market for early consumption.

Households' marginal value of assets solves (48) where $\bar{v}$ is replaced with $\bar{v}/\mu$. The lower bound for the real interest rate is $\underline{R} \equiv (1+\rho) / [\alpha(\bar{v}/\mu) + 1 - \alpha]$. The average sales of a firm in terms of the numeraire are now

$$q = \bar{q} + \alpha \frac{\mu - 1}{\mu} \left( a^g + \phi^f \right). \tag{65}$$

The second term on the right side of (65) corresponds to the additional profits received by a firm from selling to early consumers. Each unit of asset spent on early consumption generates a profit equal to $(1/\mu) - 1$ in terms of the numeraire, and the demand per firm is $\alpha a$ where, by market clearing, $a = a^g + \phi^f$. This second term creates a link between firms' average revenue and households' wealth. From (18) market tightness solves

$$\frac{\theta k^f}{\lambda(\theta)} = \frac{\mu\left(\bar{q} - \bar{w}_1\right) + \alpha\left(\mu - 1\right) a^g}{\delta^f \mu + [\alpha + (1-\alpha)\mu]\, r - \alpha\left(\mu - 1\right)}. \tag{66}$$

The provision of public liquidity has now a direct effect on market tightness. For given $r$, $\partial\theta/\partial a^g > 0$ if $\mu > 1$ because firms raise their profits by selling to early consumers. The upper bound for $R$ above which the labor market shuts down is

$$\bar{R} \equiv \frac{\mu\left(\bar{q} - \bar{w}_1\right) + \alpha\left(\mu - 1\right) a^g + (1-\delta^f)\mu k^f}{[\alpha + (1-\alpha)\mu]\, k^f}.$$

We impose $\underline{R} < \bar{R}$. The supply of private assets per employed worker as a function of the gross real interest rate as

$$a^p(R) = \frac{R\left[\bar{q} - \bar{w}_1 + \alpha(1-\mu^{-1})a^g\right]}{R - (1-\delta^f) - \alpha\left(1 - \mu^{-1}\right)}, \ \forall R \in \left(1 - \delta^f + \alpha\left(1 - \mu^{-1}\right), \bar{R}\right). \tag{67}$$

The maximum wealth per employed worker, $\bar{\omega}(R)$, is still given by (51) and the market-clearing condition is given by (55). The outcome of the asset market is represented graphically in Figure **??**. A key difference is that the curve $a^p(R)$ is now parameterized by $a^g$.

**Proposition 4** ***(Linear model with markup.)*** *Suppose $U(c,e) = c$ and $v(y) = \bar{v}y$ with $\bar{v} > 1$. Moreover, early consumption is sold at a markup $\mu > 1$. Assume $\bar{w}_1 > \alpha a^g$. There are two regimes with an active labor market ($\theta > 0$).*

***(i) Savings glut.*** *If the following condition holds,*

$$\begin{aligned}
\bar{w}_1 - \alpha a^g &\geq \frac{\left[\alpha \bar{v}\mu^{-1} - \rho(1-\alpha)\right]\left[\bar{q} - \bar{w}_1 + \alpha(1-\mu^{-1})a^g\right]}{1+\rho - \left[\alpha\left(\bar{v}\mu^{-1}-1\right)+1\right]\left[1-\delta^f + \alpha\left(1-\mu^{-1}\right)\right]} && (68)\\
1+\rho &> \left[\alpha(\bar{v}\mu^{-1}-1)+1\right]\left[1-\delta^f+\alpha\left(1-\mu^{-1}\right)\right] && (69)
\end{aligned}$$

*then the real interest rate and market tightness are given by:*

$$\begin{aligned}
r &= \frac{\rho - \alpha\left(\bar{v}/\mu - 1\right)}{\alpha\bar{v}/\mu + 1 - \alpha} && (70)\\
\frac{\theta k^f}{\lambda(\theta)} &= \frac{\left[1+\alpha\left(\frac{\bar{v}}{\mu}-1\right)\right]\left[\mu\left(\bar{q}-\bar{w}_1\right)+\alpha\left(\mu-1\right)a^g\right]}{\mu\left(\delta^f+\rho\right) - \alpha(1+\rho)\left(\mu-1\right) - (1-\delta^f)\mu\alpha\left(\frac{\bar{v}}{\mu}-1\right)}. && (71)
\end{aligned}$$

*Moreover,* $\partial r/\partial a^g = 0$ *and* $\partial\theta/\partial a^g > 0$.

***(ii) Abundant asset supply.*** *If (68) and (69) do not hold and*

$$a^g < \frac{\alpha(\bar{w}_1 - k^f) + (1-\alpha)\mu\left(\bar{q} - \delta^f k^f\right)}{\alpha}, \quad (72)$$

*then the real interest rate and market tightness are given by*

$$\begin{aligned}
r &= \frac{\alpha\left(\mu\bar{q}-\bar{w}_1\right) - \delta^f\mu\left(\bar{w}_1 - \alpha a^g\right)}{\alpha(\bar{w}_1 - a^g) + (1-\alpha)\mu\bar{q}} && (73)\\
\frac{\theta k^f}{\lambda(\theta)} &= \frac{\alpha(\bar{w}_1 - a^g) + (1-\alpha)\mu\bar{q}}{\alpha + \mu(1-\alpha)\delta^f}. && (74)
\end{aligned}$$

*Moreover,* $\partial r/\partial a^g > 0$ *and* $\partial\theta/\partial a^g < 0$.

In a savings glut, an increase in $a^g$ does not affect the real interest rate but it raises firms' profits, market tightness, and employment. By raising the amount of wealth that households can accumulate, an increase in $a^g$ raises the consumption of early consumers which is sold at a markup. This effect is the aggregate demand channel of public liquidity. Graphically, in Figure **??**, a small increase in $a^g$ shifts the curve $\bar{\omega}$ upward but its intersection with the curve $a^p$, which also shifts upward, is still located below $\underline{R}$. In the case of abundant asset supply, the increase in $a^g$ crowds out private assets by raising $r$ – the interest rate channel of public liquidity. In that case, market tightness decreases and employment decreases.

# 9 Appendix C: The effects of an expansion in the supply of government debt

So far we have studied public liquidity management in the form of "helicopter drops", i.e., money creation financed by lump-sum transfers to households. We now consider a policy that changes the interest rate on financial assets by changing the real supply of bonds held by the public, e.g., through quantitative easing. Figure 17, illustrates the effects of permanently increasing $A^g$ on steady-state equilibrium prices/returns (left panel) and quantities (right panel) for

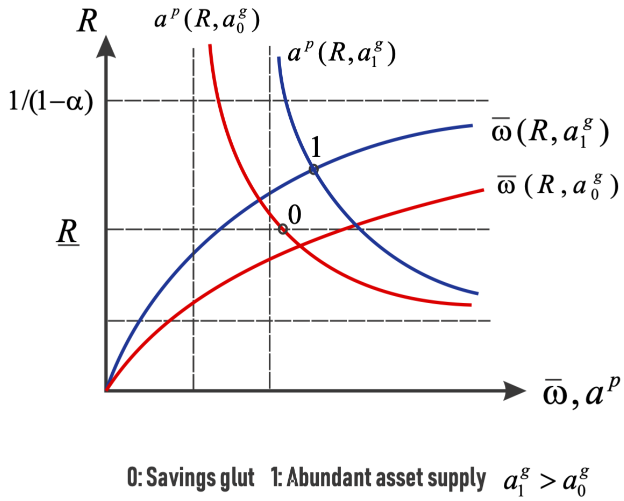


Figure 16: Equilibrium in simple linear model with a markup

assets and early consumption, leaving all other parameters unchanged. A higher $A^g$ leads to an increase in the rate of return on financial assets, $R^f$, but crowds out the supply of private financial assets in firm equity, $A^f$. As the return on financial wealth increases, households rebalance their portfolios by substituting away from holding wealth in real money balances, $A^m$, and housing, $A^h$. The fall in demand for housing increases its return, $R^h$. Taken together, the effective liquidity of households, defined as $\sum_{\omega \in \Omega_{\mathbb{A}}} \alpha_\omega \boldsymbol{\chi}_\omega \mathbf{a}$, shrinks since money and houses are more liquid than financial wealth. This causes a decrease in the demand for early consumption. The supply of early consumption also decreases since the financial discount factor decreases. Quantitatively, the demand effect is stronger and the price of early consumption, $p^y$, falls modestly. The expansion of government debt increases the unemployment rate.

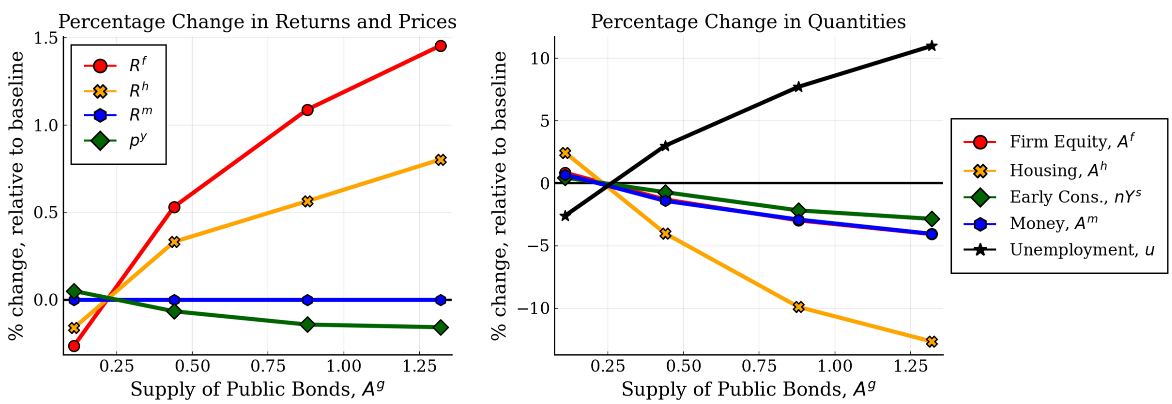


Figure 17: The effect of the supply of government bonds on prices and quantities

## 9.1 Partially-liquid government debt and aggregate welfare

Figure 18 shows the aggregate welfare cost of expanding government debt in steady state, ignoring transitional dynamics. The left panel computes the steady-state, consumption-equivalent welfare cost of changing $A^g$ from the level in the baseline calibration $A^g = 0.22$ to a new level given on the x-axis. We find that increasing government debt reduces aggregate welfare. For instance, households are willing to pay 1.1% of total consumption in order to avoid doubling the supply of government debt from the baseline level. The right panel repeats the decomposition exercise from Figure 10 in which we recompute the welfare cost associated with the change in $\{R^f, R^h, p^y\}$, leaving the other two prices constant, but allowing wages, labor market tightness, and transfers to change.

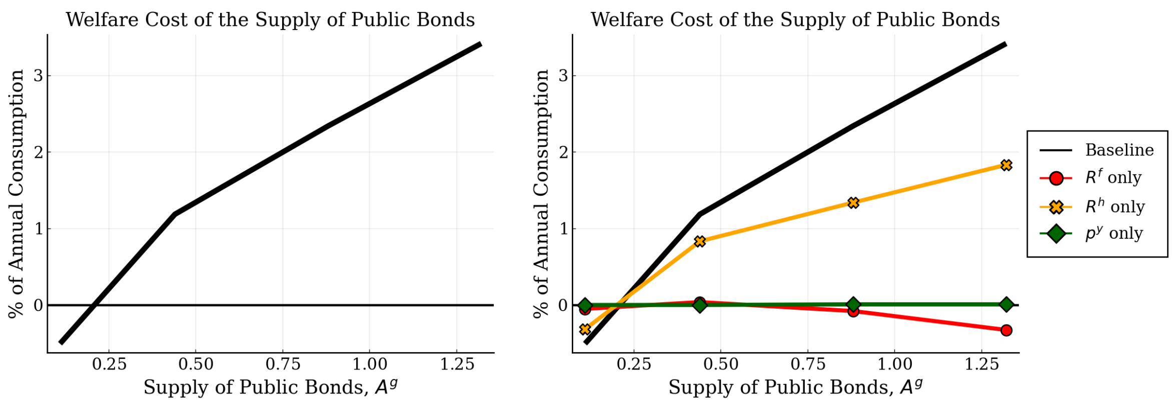


Figure 18: The welfare cost of changing the supply of government debt

The red-circled line shows the welfare cost associated with the change in the return on financial wealth, $R^f$. Consider an increase in $A^g$ that raises the return. On one hand, aggregate welfare improves because households are able to better self-insure against unemployment risk using financial wealth. On the other hand, aggregate welfare worsens since the increase in $R^f$ decreases financial discount rates, discourages firm entry, reduces labor market tightness, and increases average unemployment duration. Additionally, households' tax burden increases as a result of larger interest payments on the stock of debt. Quantitatively, these effects tend to cancel each other out in the aggregate. For low levels of government debt, the second effect dominates leading to a small welfare cost while for larger levels of government debt the benefits start to outweigh the costs and there is a modest, positive welfare gain.

The orange-crossed line shows the welfare cost associated with the response in $R^h$. As $A^g$ increases, $R^h$ increases which improves welfare by providing households with better means of self-insurance. However, the fall in the value of housing causes supply to decrease, which leads to an increase in the rental price of housing. We find the second effect dominates for all the levels of government debt we analyze. The green-diamonded line illustrates the welfare cost associated with the fall in the price of early consumption. The welfare effects of this channel are negligible because the benefit of cheaper early consumption cancels out the costs associated with lower aggregate demand and lower labor market tightness.

## 9.2 Government debt and the composition of wealth

Figure 19 shows the effects of doubling the supply of government debt from the baseline level on asset portfolio shares, by total wealth and income. Qualitatively, the effects are similar to the response of household portfolios to inflation. Poor households, around the 10th percentile of total wealth, respond the most strongly to the increase in $A^g$ by substituting towards financial wealth and away from liquid wealth and housing. The very poorest households continue to exclusively hold liquid wealth while middle-wealth and high-wealth households substitute more moderately.

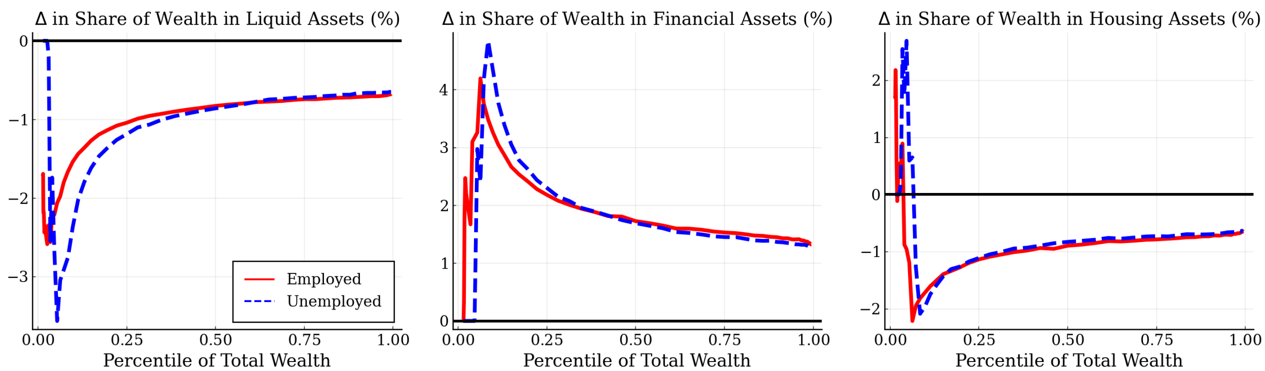


Figure 19: The effect of doubling the supply of government debt on portfolio shares, by wealth and income

Figure 20 illustrates the welfare cost of doubling the supply of government debt in the cross section. The expansion of government debt harms all households. However, the largest burden falls on households with the lowest wealth and income. Households in the bottom of the wealth distribution would be willing to pay 1.47% of consumption to avoid doubling the supply of government debt, while those in the top of the distribution would be willing to pay 1.32%.

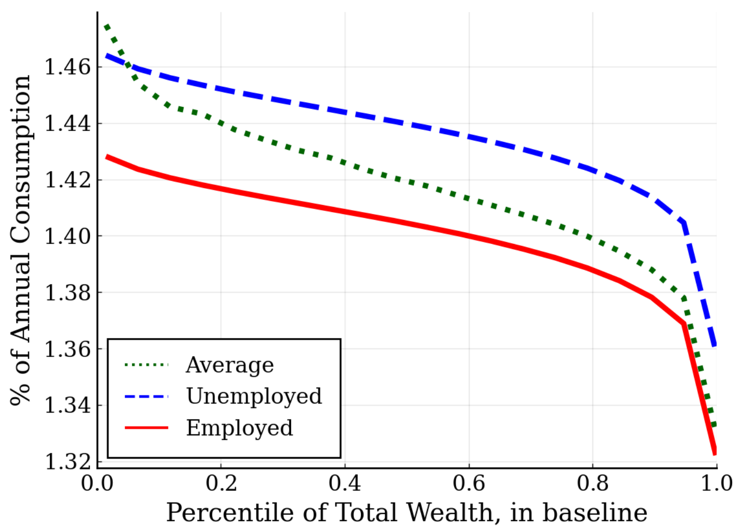


Figure 20: The welfare cost of doubling the supply of government bonds, by wealth and income