Unemployment and the Distribution of Liquidity*

Zachary Bethune
Rice University

Guillaume Rocheteau
University of California, Irvine

July 2023

Abstract

We study the long-run effects of money creation and inflation in a New-Monetarist model of unemployment in which distributional considerations matter. Households face employment and expenditure risk and self-insure by accumulating assets with different liquidity and returns. Inflation affects unemployment primarily through two channels: an aggregate-demand channel through which inflation reduces households' liquid wealth and firms' expected revenue, and an interest-rate channel through which inflation affects firms' financial discount rate. Quantitatively, the aggregate-demand channel dominates and the long-run Phillips curve has a positive slope - inflation increases unemployment - although inflation can have large redistributive effects and increase aggregate welfare.

JEL Classification Numbers: D83, E24, E40, E50

Keywords: unemployment, money creation, inflation, distributional effects.

*Contacts: Zachary Bethune: bethune@rice.edu, Guillaume Rocheteau: grochete@uci.edu. We are thankful for useful suggestions and comments from conference and seminar participants at Penn State, UC - Irvine, Virginia, Arizona State, Rice, the Society of Economic Dynamics meeting in Minneapolis, the Money, Payments, Banking and Finance Workshop, the RISE Inflation Conference, and Econometric Society Winter Meetings, the virtual Search and Matching Congress, the Southern Economics Association conference in Houston, and the virtual COCONUTS workshop.

1 Introduction

How does money creation affect equilibrium unemployment? The answer to this core question in macroeconomics has proven elusive empirically.[1] The question is also challenging theoretically as it requires a model with frictions in both goods and labor markets so as to make money essential and to generate unemployment in equilibrium. In the confines of pure currency economies, **?** – BMW thereafter – constructs such a model and shows that a higher rate of money creation lowers the rate of return on currency, thereby reducing consumers' holdings of liquid assets, firms' revenue, and job openings, i.e., the long-run Phillips curve is positively sloped.[2] For tractability, however, BMW omits the distributional effects of monetary policy – there is evidence that such effects are quantitatively important (e.g., **?**, and **?**) – and assumes that households are neutral to unemployment risk. Moreover, BMW assumes that the ownership of firms is distributed evenly across consumers and cannot be traded, thereby shutting down a main channel from incomplete-market models, namely, that the rate at which firms discount future profits is endogenous and depends on both public and private liquidity. The objective of this paper is to construct and calibrate a framework that unharnesses the ex-post heterogeneity resulting from both idiosyncratic expenditure and employment risks and that allows households to self-insure with both public and private liquidity in order to tease out and quantify the mechanisms through which money creation affects unemployment and welfare.

Our model is a two-good version of a **??** economy with multiple assets where risk-averse households are unable to commit and hence cannot borrow. Following the banking literature, we label the two consumption goods as *early* (because consumption takes place before labor income and asset returns are paid) and *late*, where preferences over the goods are subject to idiosyncratic shocks. The distinction between early and late consumption has two purposes. First, the endogenous relative price between the two goods provides a channel through which the distribution of households' liquid wealth affects firms' revenue and job creation decisions. Second, it allows us to differentiate assets (money, government bonds, stocks) according to their degree of liquidity. Specifically, while all assets can be liquidated in the late stage of each period, assets differ according to the ease with which they can be liquidated in the early stage, which we take as our notion of asset liquidity. In terms of market structures, goods markets are competitive and open in sequence. The labor market is frictional, with workers and jobs being matched bilaterally according to a time-consuming process, which creates an idiosyncratic unemployment risk.

[1] **?**, using both classical and Bayesian structural VARs, shows that the data cannot reject positively or negatively sloped long-run Phillips curves.

[2] A related model was first proposed by **?** where large households are composed of a continuum of members who pool their money and share their resources. Our model will be closer to the version in BMW.

We start by studying a simplified version of our model that is analytically tractable and directly comparable to BMW. Relative to BMW, liquid wealth in our model is composed of both publicly-supplied liquidity (money and government bonds) and privately-supplied liquidity (stock mutual funds). We identify two main channels through which changes in the money growth rate affect unemployment. As in BMW, there is an aggregate demand channel according to which the revenue of the firm increases with the liquid wealth of consumers. Because we have a broader notion of liquid wealth, this channel is magnified in our model: an increase in public liquidity has a multiplier effect on aggregate liquidity through the valuation of stocks. There is a separate interest-rate channel according to which the discount rate of firms is endogenous and depends on the supply of liquidity. In the simplest version of our model with quasi-linear preferences, the two channels work in opposite directions: an increase in the money growth rate or, equivalently, a decrease in the supply of private liquidity, raises unemployment according to the aggregate demand channel but reduces it according to the interest rate channel. When preferences are strictly concave, the interest rate channel can be positive or negative due to competing substitution and wealth effects.

The second part of the paper explores the quantitative implications of our full model by calibrating it to match standard labor market moments, the distribution of household liquid wealth and portfolio shares from the Survey of Consumer Finances, and the liquidity premium. Targeting the cross-section of liquid wealth disciplines the frequency and size of expenditure risk that households face, and in turn, their demand for liquidity. Targeting the distribution of portfolio shares helps discipline the extent to which households are exposed to the inflation tax. Targeting the liquidity premium disciplines the degree to which non-monetary wealth can be liquidated on short notice. The calibration implies that the primary driver of households' demand for liquidity is for precautionary savings to finance large, but relatively infrequent expenditures. The size and frequency of expenditure risk in the model is consistent with evidence about common, unplanned households expenditures, such as vehicle and home repairs and medical expenses. The model performs well when compared to untargeted moments of the cross-section of marginal propensities to consume and consumption responses after job loss, both measures of how well households are insured against income and expenditure risk.

Using the calibrated model, we illustrate the effects of changes in the rate of return of liquid assets through money growth. When money creation is used to finance unproductive government expenditures, inflation reduces aggregate demand, increases unemployment, and leads to a decline in output – the long-run Phillips curve is positively-sloped. The primary contributor to the relationship between inflation and unemployment is the aggregate demand channel, which explains 94% of the slope of the long-run Phillips curve. The interest rate channel explains only 6%. While there are strong substitution effects that lead households to increase their demand for illiquid wealth when inflation rises, thereby increasing illiquid asset prices, we find that negative wealth and distributional effects cancel this out, leading inflation to have small effects on the long-run financial discount rate, and in turn unemployment, in equilibrium. The Phillips curve becomes more vertical (but still positively-sloped) when money creation is distributed to households lump-sum. The interest rate channel becomes stronger and the aggregate demand channel becomes weaker. In a counter-factual experiment, we show that if non-monetary wealth becomes more liquid over time, the long-run Phillips curve could invert and become negatively sloped.

The welfare effects of inflation and the optimal inflation rate rely on how money creation is used. If money creation is wasted, inflation always reduces aggregate welfare, e.g., the cost of transitioning from 0% to 10% inflation reduces aggregate welfare by 2% of annual consumption. By disaggregating the effects of inflation across households at different points of the wealth and income distribution, we show that the welfare cost is larger for wealthier and higher income households, ranging from 5% of annual consumption for the highest wealth and income household to 0.4% for the lowest wealth and income household. If money creation is instead distributed lump sum, then increasing inflation from zero is welfare improving. Transitioning from 0% to 6% inflation is optimal and increases aggregate welfare by 0.1%. At the individual level, welfare costs are still increasing in wealth and income. Transitioning from 0% to 10% inflation improves the welfare of lowest wealth and income households by 2.2% while it decreases the welfare of the highest income and wealth households by 3.3%.

1.1 Literature

Our model has a similar structure as in BMW that extends the quasi-linear environment of **?** to include a frictional labor market.[3] Relative to BMW, goods markets are competitive, as in **?**; we generalize preferences to make households risk averse so that unemployment risk is relevant; claims on firms' profits are tradable and their rate of return is endogenous; the set of assets is broader and includes partially liquid government bonds and housing. These elements have been studied individually in models of unemployment with degenerate wealth distributions. Liquid claims have been incorporated by **?** and **?**. Our description of housing is similar to **?** and **?**.

New-Monetarist models with non-degenerate distributions of money holdings include **?**, **?**, **??**, **?**, and

[3] In **?** only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. The BMW model has been extended to incorporate credit and various forms of liquidity, e.g., **?** and **?**.

?, among others.[4] Our approach is closer to **?**, which includes both expenditure and unemployment risks. However, our model is more general in terms of preferences and asset structure.

Our model where goods markets are competitive and households are price takers can be interpreted as a two-sector **??** economy with multiple assets. Related Bewley economies include **?** who study optimal unemployment insurance and **?** who study temporary and permanent changes in money growth. Frictional labor markets have been added to incomplete-market models by **?** and **?**, among others. A key difference in our approach is the distinction between early and late consumption that allows us to differentiate assets according to their liquidity following **?**, **?**, and many other contributions to the New Monetarist literature surveyed in **?**.

The recent Heterogeneous Agent New Keynesian (HANK) literature pioneered by **?** also includes assets with different degrees of liquidity. In their model, the lack of liquidity of an asset is formalized through transaction costs to deposit or withdraw from an illiquid account. **?** and **?** study business cycles in HANK models with frictional unemployment and two assets (bonds and capital). **?** focuses the role of time-varying unemployment benefits as macroeconomic stabilization while **?** studies how the composition of assets is important for the amplification of shocks. In both environments, financial discount rates do not affect firm entry because either firms are owned by risk-neutral entrepreneurs instead of risk-averse households or because monetary policy fixes the real return of capital. Our focus on the long-run implications of inflation also differs from theirs.

2 Environment

Time is discrete and is indexed by $t \in \mathbb{N}$. The economy is populated by a unit measure of infinitely-lived households. Each period of time is divided into three stages. The first stage is a frictional labor market. The second and third stages have markets for goods and assets opening sequentially. There are two perishable goods: an early-consumption good produced in the second stage and a late-consumption good produced in the last stage. We take the late-consumption good as the numeraire.

The lifetime expected utility function of a household is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t v(y_t) + U(c_t, e_t) \right], \tag{1}$$

where $\beta = (1+\rho)^{-1} \in (0,1)$, $y_t \in \mathbb{R}_+$ is early (second-stage) consumption, $\varepsilon_t \in \{0,1\}$ is a preference shock for early consumption, $c_t \in \mathbb{R}_+$ is late (third-stage) consumption, and $e_t \in \{0,1\}$ is the worker's em-

[4] See also **??**, **??** and **?**.

ployment status. The utility functions, $v(y_t)$ and $U(c_t, e_t)$, are bounded, twice continuously differentiable, strictly increasing, and concave in (c, h) and y, respectively. We adopt the normalization $v(0) = 0$. The utility functions satisfy the following Inada conditions: $U_c(0, e_t) = +\infty$, $v'(0) = +\infty$, and $v'(\infty) = 0$. Preferences shocks, $\{\varepsilon_t\}_{t=0}^{+\infty}$, are i.i.d. across agents and time with $\Pr[\varepsilon_t = 1] = \alpha$ and $\Pr[\varepsilon_t = 0] = 1 - \alpha$. So a household wishes to consume early with probability α. The price of early-consumption in terms of the numeraire is p. Households are ex ante heterogenous in terms of their productivity and hence their income. We index workers' productivity by $z \in \mathcal{Z}$. The share of type-z households is $\varpi(z)$.

A firm is a technology to produce the second-stage good (early consumption) and the numeraire with one unit of labor as the only input. This technology depends on worker's productivity, z, and is represented by the production-possibility frontier, $zQ(y/z)$, that specifies the amount of numeraire a firm can produce if it has already produced y units of the second-stage good. The production-possibility frontier satisfies $Q(0) = \bar{q} > 0$, $Q(\bar{y}) = 0$ for some $\bar{y} > 0$, $Q' < 0$, and $Q'' < 0$, $Q'(0) = 0$, and $Q'(\bar{y}) = -\infty$. So a firm matched with a worker with productivity z can produce up to $z\bar{q}$ of the numeraire good and up to $z\bar{y}$ of the second-stage good. An example of a production possibility frontier satisfying the assumptions above is

$$Q(y) = \left(1 - y^{\frac{1}{a}}\right)^{a}, \quad a \in (0, 1). \tag{2}$$

For this specification, $\bar{q} = \bar{y} = 1$.[5] In Figure **??**, we draw two production possibility frontiers, one for high-productivity workers, $z = z_h$, and one for low-productivity workers, $z = z_\ell < z_h$. We see that an increase in productivity shifts the frontier outward. We define the opportunity cost of producing y as the difference between $zQ(0)$ and $zQ(y/z)$, which we express as $z\kappa(y/z)$ where $\kappa(\tilde{y}) \equiv Q(0) - Q(\tilde{y})$. Hence, $\kappa(0) = 0$, $\kappa(\bar{y}) = \bar{q}$, $\kappa'(y) > 0$, $\kappa''(y) > 0$, $\kappa'(0) = 0$, and $\kappa'(\bar{y}) = +\infty$. If the production possibility frontier is given by (**??**), then $\kappa(y) = 1 - \left(1 - y^{\frac{1}{a}}\right)^{a}$.

The labor market is segmented by productivity type, i.e., a firm can target its search toward a given type of worker. In order to create a job in the market for type-z workers in period t, firms must open a vacant position, which costs $zk > 0$ in terms of the numeraire in $t - 1$. The assumption that the vacancy posting cost is proportional to productivity is made for tractability so as to allow the job finding and vacancy filling probabilities to be independent of z and equal across all labor markets. The measure of matches

[5] The foundations for the production possibility frontier in (**??**) are as follows. Suppose the production function for each good takes the form $f(h) = (h)^a$ where h represents worker's time. The endowment of time of each worker is $h_q + h_y = 1$. Then,

$$Q(y) = \max_{h_q, h_y} f(h_q) \text{ s.t. } f(h_y) = y \text{ and } h_q + h_y = 1.$$

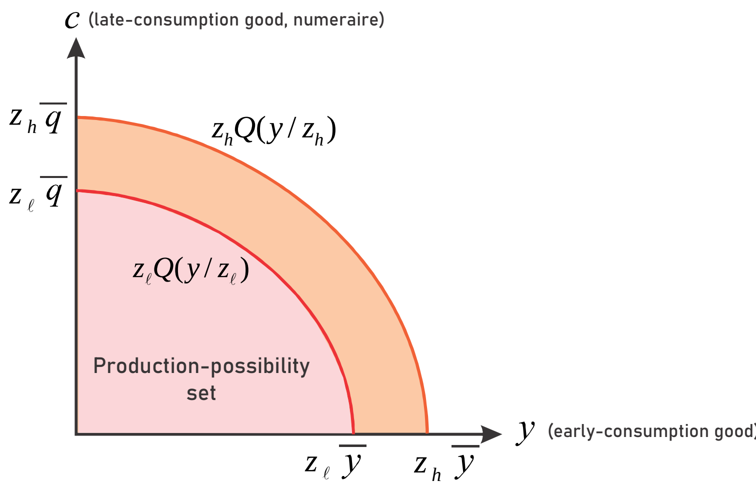

Figure 1: Production possibility frontier between early and late consumption for high- (z_h) and low-productivity (z_ℓ) workers.

between vacant jobs and unemployed workers in period t is given by $\mathcal{M}(s_t, o_t)$, where s_t is the measure of job seekers and o_t is the measure of job openings. The matching function, $\mathcal{M}$, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $\mathcal{M}(0, o_t) = \mathcal{M}(s_t, 0) = 0$ and $\mathcal{M}(s_t, o_t) \leq \min(s_t, o_t)$. The job finding probability of a worker is $\lambda_t = \mathcal{M}(s_t, o_t)/s_t = \mathcal{M}(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. The vacancy filling probability for a job is $\mathcal{M}(s_t, o_t)/o_t = \mathcal{M}(1/\theta_t, 1) = \lambda_t/\theta_t$. An existing match is destroyed at the beginning of a period with probability δ. A worker who loses his job in period t is unemployed in period t and becomes a job seeker in period $t+1$. Therefore, $s_{t+1} = u_t$. The measure of employed workers of type-z (measured after the matching phase at the beginning of the second stage) is denoted $n_t(z)$ and the measure of unemployed workers is $u_t(z)$. Therefore, $u_t(z) + n_t(z) = \varpi(z)$.

A household's disposable labor income, $w_e(z)$, is a function of its employment status and its productivity. It is decomposed into two components: a transfer (or tax) from the government, $\tau_e(z)$, and a non-transfer income, $z\bar{w}_e$, that we assume to be proportional to the household's productivity. Hence, $z\bar{w}_1$ is the wage in terms of the numeraire good paid in the last stage. We either take $\bar{w}_1$ as exogenous or we adopt some ad hoc wage determination rule. The unemployed household receives an endowment in terms of the numeraire good equal to $z\bar{w}_0$. In the calibrated version of the model, we interpret $z\bar{w}_0$ as unemployment benefits without formalizing explicitly the financing of the unemployment insurance scheme.

Households are anonymous (i.e., their employment and trading histories are private) and cannot commit to honor future obligations. Hence, idiosyncratic expenditure and employment shocks are uninsurable through credit, which creates a need for precautionary savings. There are three types of assets, indexed in $\mathbb{A} \equiv \{m, g, f\}$. Fiat money is perfectly divisible, storable, and non-counterfeitable. Its supply, M_t, grows as the constant rate $\pi > \beta - 1$. The price of money in terms of the numeraire is ϕ^m. There is a fixed supply of one-period real government bonds, A^g. Each bond issued in the third stage is a claim to one unit of numeraire in the third stage in the following period and is priced at ϕ^g. The third type of asset corresponds to shares in fully diversified investment fund that mutualizes claims to firms' profits. The supply of these claims is endogenous and equal to the market capitalization of all firms. The price of a claim is given by ϕ^f.

We divide assets according to their resalability, $\mathbb{A} = \mathbb{A}^m \cup \mathbb{A}^\iota$ where $\mathbb{A}^m \equiv \{m\}$ is the set of liquid assets and $\mathbb{A}^\iota \equiv \{g, f\}$ is the set of partially illiquid assets. Money is the ultimate liquid asset that can always serve as means of payments in the second stage. All other assets are partially illiquid in that they can serve as means of payment with probability less than one. Formally, conditional on $\varepsilon = 1$, only fiat money is acceptable to finance early consumption with probability α_0/α while all assets are acceptable with probability α_1/α where $\alpha_0 + \alpha_1 = \alpha$.[6] In the last stage, all assets are equally acceptable. We denote $R^m = 1 + r^m$ the gross real rate of return of fiat money and $R^\iota = 1 + r^\iota$ the gross real rate of return of partially illiquid assets (government bonds and claims on firms' profits).

3 Equilibrium

We characterize steady-state equilibria where the distribution of asset portfolios, the rates of return of assets, and the relative price of consumption goods and services are constant over time.

3.1 Households

We first describe the household's consumption and asset portfolio problem taking the price of early consumption, p^y, and the gross rates of return of assets, R^m and R^ι, as given. The state of a household when entering the last stage is composed of its productivity, z, its employment status, $e \in \{0, 1\}$, and its total wealth expressed in the numeraire, $a = a^m + a^\iota$, where a^m denotes real money balances and a^ι represents holdings of partially illiquid assets (bonds and shares of mutual funds). The household's value function is given by:

$$W_e(a; z) = \max_{c, \hat{\mathbf{a}}} \{U(c, e) + \beta \mathbb{E}_e V_{e'}(\hat{\mathbf{a}}; z)\} \quad \text{s.t.} \quad c + \mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e(z), \tag{3}$$

where all control variables are subject to nonnegativity constraints, $V_{e'}$ is the value function of the household in the employment state $e' \in \{0, 1\}$ at the start of the second stage, and $\mathbb{E}_e$ is the expectation operator with

[6] This idea is formalized, e.g., in **?** and **?**.

respect to e' conditional on its current employment state, e. The transition from e to e' occurs in the first stage. According to (**??**), the household chooses its current consumption, c, and next-period's portfolio, $\hat{\mathbf{a}} = (\hat{a}^m, \hat{a}^\iota)^\intercal$, in order to maximize its current utility plus its discounted continuation value in the following period. The budget identity specifies that total consumption of goods and housing services and the next-period discounted asset portfolio are equal to current income and wealth. The vector of discount factors for the different types of assets is denoted $\mathbf{R}^{-1} = (1/R^m, 1/R^\iota)$ and $\mathbf{R}^{-1}.\hat{\mathbf{a}} = \hat{a}^m/R^m + \hat{a}^\iota/R^\iota$.

The value function at the beginning of the second stage solves:

$$V_e(\hat{\mathbf{a}}; z) = \sum_{\omega \in \{0,1\}} \alpha_\omega \max_{y_{\omega e}} \left[v(y_{\omega e}) + W_e \left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}; z \right) \right] + (1-\alpha) W_e(\mathbf{1}.\hat{\mathbf{a}}; z) \text{ s.t. } p^y y_{\omega e} \in \left[0, \hat{a}^m + \omega \hat{a}^\iota\right]. \tag{4}$$

The variable $\omega \in \{0, 1\}$ indicates whether or not the household can finance its early consumption with partially illiquid assets. Formally, the total expenditure, $p^y y$, cannot exceed the household's resalable wealth, $\hat{a}^m + \omega \hat{a}^\iota$. With probability, $1 - \alpha$, the household does not wish to consume early and enters the third stage with total wealth $\mathbf{1}.\hat{\mathbf{a}} = \hat{a}^m + \hat{a}^\iota$. We combine (**??**) and (**??**) to obtain a single Bellman equation:

$$W_e(a; z) = \max_{c, \mathbf{y}, \hat{\mathbf{a}}} \left\{ U(c, e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \{0,1\}} \alpha_\omega \left[v(y_{\omega e'}) + W_{e'} \left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}; z \right) \right] + (1-\alpha) W_{e'}(\mathbf{1}.\hat{\mathbf{a}}; z) \right\} \right\} \tag{5}$$

$$\text{s.t. } c + \mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e(z) \text{ and } p^y y_{\omega e'} \leq \hat{a}^m + \omega \hat{a}^\iota, \;\; e' \in \{0, 1\}.$$

The household makes plans for its next-period early-consumption contingent on its future employment status and asset resalability, $\mathbf{y} = (y_{\omega e})$. In the last stage, all assets are perfectly fungible in total wealth, which is represented by $a \in \mathbb{R}_+$. In the second stage, assets differ in their acceptability and, hence, the asset portfolio is represented by a vector, $\hat{\mathbf{a}} \in \mathbb{R}^2_+$, of liquid and illiquid assets. In the following, we omit the permanent type, z, from the value functions. Proposition **??** guarantees there is a unique solution to (**??**)-(**??**). All proofs are provided in Appendix A in Section **??**.

Proposition 1 ***(Households' Value Functions.)*** *There is a unique pair of value functions, (W_e, V_e), solutions to (**??**)-(**??**) in the space of continuous and bounded functions. Moreover, W_e and V_e are increasing, concave, and continuously differentiable with $W'_e(a) = U_c\left[c_e(a), e\right]$ and*

$$\frac{\partial V_e(\hat{\mathbf{a}})}{\partial \hat{a}^j} = \sum_{\omega \in \{0,1\}} \alpha_\omega \left\{ \chi^j_\omega \frac{v'\left[y_{\omega e}(\hat{\mathbf{a}})\right]}{p^y} + (1 - \chi^j_\omega) W'_e\left[\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}})\right] \right\} + (1-\alpha) W'_e(\mathbf{1}.\hat{\mathbf{a}}), \tag{6}$$

for all $j \in \{m, \iota\}$, where $\chi^j_\omega \equiv \mathbb{I}_{\{j=m\}} + \mathbb{I}_{\{j=\iota,\omega=1\}}$ is an indicator variable equal to one when asset j is liquid (i.e., it is of type m or of type ι and $\omega = 1$), where $c_e(a)$ is the last-stage policy function that *specifies late-consumption as a function of total wealth and employment status; and $y_{\omega e}(\hat{\mathbf{a}})$ is the second-stage policy function that specifies early-consumption as a function of the portfolio at the start of the second stage, $\hat{\mathbf{a}} = (\hat{a}^m, \hat{a}^\iota)^\intercal$, employment status, and resalability event.*

The optimal portfolio choice in the last stage obeys Euler equations obtained by substituting $c = a + w_e(z) - \mathbf{R}^{-1}.\hat{\mathbf{a}}$ into (**??**) and taking first-order conditions:

$$- U_c(c,e) + R^j \beta \mathbb{E}_e \frac{\partial V_{e'}(\hat{\mathbf{a}})}{\partial a^j} \leq 0, \text{ `` = '' if } \hat{a}^j > 0, \text{for all } j \in \{m, \iota\}. \tag{7}$$

According to (**??**), the marginal utility of late consumption, $U_c(c,e)$, is equalized to the discounted marginal benefit of asset j in the following period. In order to understand the role of the resalability coefficients for asset pricing, it is instructive to substitute $\partial V_{e'}(\hat{\mathbf{a}})/\partial a^j$ by its expression given by (**??**), i.e.,

$$\begin{aligned} &-U_c(c,e) + R^j \beta \mathbb{E}_e \left[\sum_{\omega \in \{0,1\}} \alpha_\omega W'_{e'} \left[\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}(\hat{\mathbf{a}}) \right] + (1-\alpha) W'_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right] \\ &+ R^j \beta \mathbb{E}_e \left[\sum_{\omega \in \{0,1\}} \alpha_\omega \chi^j_\omega \left\{ \frac{v' \left[y_{\omega e'}(\hat{\mathbf{a}}) \right]}{p^y} - W'_{e'} \left[\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}(\hat{\mathbf{a}}) \right] \right\} \right] \leq 0, \end{aligned} \tag{8}$$

with an equality if $\hat{a}^j > 0$. The only term that can account for differences in rates of return is the last term on the left side that depends on the acceptability of asset j. It can be interpreted as the expected non-pecuniary return from investing in an additional unit of asset j that can serve as means of payment for early consumption for all resalability events ω such that asset j is accepted, $\chi^j_\omega = 1$.

We now turn to the optimality conditions for goods and services. The choice of early consumption is obtained from (**??**) and the associated first-order condition:

$$v'(y_{\omega e}) \geq p^y W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}) \text{ `` = '' if } p^y y_{\omega e} < \hat{a}^m + \omega \hat{a}^\iota. \tag{9}$$

From (**??**) the optimal early-consumption choice is based on the comparison of the household's marginal utility from spending a unit of pledgeable wealth in the second stage, $v'(y)/p^y$, and the marginal value of wealth in the last stage, $W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y)$. The two terms are equal provided that $p^y y \leq \hat{a}^m + \omega \hat{a}^\iota$ does not bind. If it binds, then the household spends all its resalable wealth. Hence, even wealthy households can face binding liquidity constraints if the amount they invested in resalable assets is low.

3.2 Distribution of asset holdings

We characterize the steady-state distributions of asset holdings across households recursively following the logic of the Bellman equations (**??**) and (**??**). We denote $G_e(a; z)$ the measure of households of type z in state $e \in \{0,1\}$ holding at most a units of wealth at the start of the last stage (before late consumption) in period t. It solves:

$$
\begin{aligned}
G_e(a;z) &= \int \left[\sum_{\omega \in \{0,1\}} \alpha_\omega \mathbb{I}_{\{\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}};z) \le a\}} + (1-\alpha)\mathbb{I}_{\{\mathbf{1}.\hat{\mathbf{a}} \le a\}} \right] dF_e(\hat{\mathbf{a}};z) \qquad (10) \\
G(a;z) &= G_0(a;z) + G_1(a;z), \qquad (11)
\end{aligned}
$$

where $\int \mathbb{I}_{\mathcal{A}} dF_e(\hat{\mathbf{a}};z)$ is the measure of households of type z in employment state e with portfolio $\hat{\mathbf{a}} \in \mathcal{A} \in \mathcal{B}(\mathbb{R}^2_+)$ at the start of the second stage, and where $\mathcal{B}$ is the Borel algebra on $\mathbb{R}^2_+$. According to the integrand on the right side of (**??**), the measure of households who hold at most a in the third stage is equal to the measure of agents who hold a portfolio $\hat{\mathbf{a}}$ worth less than a in the second stage and do not consume, with probability $1-\alpha$, plus the measure of households who had an opportunity to consume early and whose post-trade wealth, $\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}(\hat{\mathbf{a}};z)$, is less than a.

The distribution of asset portfolios at the start of the second stage, $F_{e'}(\hat{\mathbf{a}};z)$, is obtained recursively from G_e as follows:

$$
\int \mathbb{I}_{\mathcal{A}} dF_{e'}(\hat{\mathbf{a}};z) = \sum_{e \in \{0,1\}} \gamma_{e,e'} \int \mathbb{I}_{\{(x,e):\hat{\mathbf{a}}(x,e;z) \in \mathcal{A}\}} dG_e(x;z) \text{ for all } \mathcal{A} \in \mathcal{B}(\mathbb{R}^2_+) \qquad (12)
$$

where $\gamma_{e,e'}$ is the transition probability from e to e', e.g., $\gamma_{0,1} = \lambda$ and $\gamma_{1,0} = \delta$, and $\hat{\mathbf{a}}(x,e;z)$ is the portfolio choice conditional on holding x units of wealth in employment state e. The measure of households of type z with a portfolio in the second stage belonging to set $\mathcal{A}$ and an employment status e' is equal to the measure of households of the same type z whose wealth, x, and employment status, e, in the last stage is such that the optimal portfolio choice is $\hat{\mathbf{a}}(x,e;z) \in \mathcal{A}$ and who transitioned to employment status e' in the first stage. The marginal cumulative distributions for each asset are $F^j_e(x;z) = \int \mathbb{I}_{\hat{a}^j \in [0,x]} dF_e(\hat{\mathbf{a}};z)$. Moreover, $F^j(x;z) = F^j_0(x;z) + F^j_1(x;z)$ and $\int_x dF^j(x;z) = \int_x dG(x;z) = \varpi(z)$ for all $z \in \mathcal{Z}$.

3.3 Pricing jobs and Lucas trees

Creation and value of jobs The value of a filled job of productivity z at the beginning of the second stage solves:

$$
\phi^f(z) = zq(p^y) - z\bar{\omega}_1 + (1-\delta)\frac{\phi^f(z)}{R^\iota}. \qquad (13)
$$

It is equal to the expected revenue from early and late sales expressed in terms of the numeraire, zq, net of the wage, $z\bar{\omega}_1$, plus the expected discounted profits of the job if it is not destroyed, with probability $1-\delta$, where the gross discount rate, R^ι, corresponds to the gross real rate of return on partially illiquid assets. The productivity-adjusted revenue of a job expressed in terms of the numeraire, q, is given by:

$$q(p^y) = \max_{\tilde{y}\in[0,\bar{y}]} \{p^y \tilde{y} + Q(\tilde{y})\} = \bar{q} + \max_{\tilde{y}\in[0,\bar{y}]} \{p^y \tilde{y} - \kappa(\tilde{y})\}, \tag{14}$$

where the second equality is obtained by using that $\kappa(\tilde{y}) = \bar{q} - Q(\tilde{y})$. The first term on the right side is the firm's total output in terms of numeraire. The second term represents the firm's profits from selling to early consumers. The optimal supply of goods in the retail market is $z\tilde{y}_t^s$ where

$$\tilde{y}_t^s = \kappa'^{-1}(p^y). \tag{15}$$

Given the assumptions on Q, the solution is interior. The price of early consumption is equal to the firm's marginal cost from producing early. It follows that $p^y = \kappa'(\tilde{y}_t^s)$ and q can be re-expressed as:

$$q = \bar{q} + [\mu(\tilde{y}^s) - 1]\,\kappa(\tilde{y}^s), \tag{16}$$

where $\mu(\tilde{y}^s) \equiv \kappa'(\tilde{y}^s)\tilde{y}^s/\kappa(\tilde{y}^s) \geq 1$. When the cost function is strictly convex, μ is akin to a gross markup over average cost.

From (**??**), we can rewrite the value of a firm as $\phi^f(z) = z\tilde{\phi}^f$. The investment fund that mutualizes claims on firms' profits finances the entry of new firms into the type-z labor market as long as the following condition holds:

$$-k + \frac{\lambda(\theta)}{\theta}\frac{\tilde{\phi}^f}{R^\iota} \leq 0, \;\; "=" \text{ if } \theta > 0, \tag{17}$$

where we have used that the recruiting cost of a firm, zk, is proportional to its worker's productivity. The first term on the right side of (**??**) is the cost of opening a vacancy in terms of numeraire while the second term is the expected discounted value of a job, where the vacant job is filled with probability λ/θ. Substituting $\tilde{\phi}^f = \phi^f(z)/z$ from (**??**), market tightness solves:

$$-\frac{\theta k}{\lambda(\theta)} + \frac{q - \varpi_1}{r^\iota + \delta} \leq 0, \;\; "=" \;\text{ if } \theta > 0, \tag{18}$$

where $r^\iota = R^\iota - 1$. So market tightness does not depend on workers' productivity. We take ϖ_1 as exogenous but we impose the participation constraints $W_1(a;z) \geq W_0(a;z)$ and $\varpi_1 < q$. The employment rate of type-z workers at the steady state is

$$n = \frac{m(1,\theta)}{\delta + m(1,\theta)}. \tag{19}$$

The employment rate is also independent of worker's productivity, z.

3.4 Market clearing and steady-state equilibrium

The market clearing conditions are:

$$n\sum_{z\in\mathcal{Z}}\varpi(z)z\tilde{y}^{s} = \sum_{(z,\omega,e)\in\mathcal{Z}\times\{0,1\}^2}\varpi(z)\alpha_{\omega}\int y_{\omega e}(\hat{\mathbf{a}};z)dF_e(\hat{\mathbf{a}};z) \tag{20}$$

$$A^m = \sum_{z\in\mathcal{Z}}\varpi(z)\int xdF^m(x;z) \tag{21}$$

$$A^g + A^f = \sum_{z\in\mathcal{Z}}\varpi(z)\int xdF^{\iota}(x;z), \tag{22}$$

where

$$A^m = \phi_t^m M_t \text{ and } A^f = n\sum_{z\in\mathcal{Z}}\varpi(z)z\tilde{\phi}^f. \tag{23}$$

Equation (**??**) is the market clearing condition for early consumption. The left side is the aggregate supply from a measure n of firms where each firm matched with a type-z worker produces $z\tilde{y}^s$. The right side is the aggregate demand arising from the measure, α households with a preference for early consumption. Equation (**??**) is the market clearing condition for money. Using that A^m is constant in a steady-state equilibrium, $R^m = M_t/M_{t+1} = (1+\pi)^{-1}$. Equation (**??**) is the market-clearing condition for partially illiquid assets. The left side of (**??**) is the aggregate supply of illiquid assets while the right side is the aggregate demand.

Finally, taxes and transfers must be such that the budget constraint of the government is satisfied, which requires:

$$n\sum_{z\in\mathcal{Z}}\varpi(z)\tau_1(z) + (1-n)\sum_{z\in\mathcal{Z}}\varpi(z)\tau_0(z) + g = \pi\phi_t^m M_t + \left(\frac{1}{R^{\iota}} - 1\right)A^g, \tag{24}$$

where g is (wasteful) government spending. So net transfers to households are financed with money creation and the issue of new bonds net of the redemption of old ones. We now have the different components to define an equilibrium.

Definition 1 *A steady-state monetary equilibrium is composed of: (i) Value functions, W_e and V_e, and policy functions satisfying (**??**) and (**??**); (ii) Distributions of asset holdings, (G_e, F_e), satisfying (**??**)-(**??**) and (**??**); (iii) Market tightness, θ, satisfying (**??**); (iv) Prices of early consumption, p^y satisfying (**??**); (v) Asset prices, $\{\phi_t^m\}$, ϕ^f and $\{R^j\}$, that satisfy market-clearing conditions, (**??**) and (**??**) and asset pricing condition (**??**); (vi) Transfers that satisfy the government budget constraint (**??**).*

4 A simplified model

In this section, we describe the main channels – an interest rate channel, an aggregate demand channel, and a distributional channel – through which policy affects equilibrium outcomes in a simplified, slightly-modified, version of our model that is easily comparable to the literature. Preferences are given by $U(c_t, e_t) = c_t + (1 - e_t)\ell$. We assume there is only one productivity type $z = 1$ and set $\kappa(y) = y$, which means that the production $\bar{q} = \bar{y}$ is perfectly storable across stages. We assume that early consumption is sold at a markup $\mu > 1$ over the opportunity cost of selling late, i.e., the price of early consumption is $p^y = \mu$. We replace the market-clearing condition, (**??**), with a feasibility condition

$$\alpha \sum_{e \in \{0,1\}} \int y_e(\hat{a}) dF_e(\hat{a}) \leq n\bar{y}. \tag{25}$$

This condition requires that aggregate early consumption is no greater than the total output produced by firms. We assume that the early consumption is divided evenly across firms and the wage $\overline{w}_1$ is a constant. Finally, all assets are equally acceptable, so that the two forms of public liquidity, government bonds and money, are perfect substitutes. We focus on equilibria with bonds only and denote $a^g = A^g/n$ the supply of one-period real bonds per employed household.

4.1 Equilibria with degenerate distributions

We start with equilibria where the constraint $c_t \geq 0$ does not bind and the distribution of wealth is degenerate. Relative to BMW, the real interest is endogenous and depends on public liquidity. From Proposition **??**, $W_e'(a) = 1$. Combining (**??**) and (**??**) with $p = \mu$, the household's choice of asset holdings solves:

$$\frac{\alpha \left[v'(a/\mu) - \mu\right]^+}{\mu} = \frac{\rho - r}{1 + r}, \tag{26}$$

where $[x]^+ = \max\{x, 0\}$. The left side is the marginal benefit of liquid wealth to finance early consumption while the right side is the cost of holding assets. Equation (**??**) specifies the household's optimal asset holdings, $a^\star(r)$, where $a^\star$ is independent of e, increases with r and α, but decreases with μ. The constraint $c \geq 0$ does not bind if the household with no assets and no job can accumulate the optimal wealth target in a single period, $Rw_0 \geq a^\star$.

From (**??**), assuming an interior solution, market tightness solves

$$\frac{(r + \delta)\theta k}{\lambda(\theta)} = \bar{q} + \alpha \left(\frac{\mu - 1}{\mu}\right) \frac{\min\{a, \mu y_\mu^\star\}}{n} - \bar{w}_1, \tag{27}$$

where $y_\mu^\star$ solves $v'(y_\mu^\star) = \mu$ and n is a function of θ given by (**??**). The aggregate demand channel is captured by the second term on the right side of (**??**). The term $\alpha \min\{a, \mu y_\mu^\star\}$ is the amount of assets spent by the measure α of households with an opportunity for early consumption. If firms have market power, $\mu > 1$, then their revenue increases (weakly) with the amount of liquid assets held by households, $\partial\theta/\partial a \geq 0$. We denote $\theta^\star$ the solution to (**??**) when households' liquidity needs are satiated, in which case $r = \rho$ and $\min\{a, \mu y_\mu^\star\} = \mu y_\mu^\star$, i.e.,

$$\frac{(\rho+\delta)\theta^\star k}{\lambda(\theta^\star)} = \bar{q} - \bar{w}_1 + \alpha\,(\mu-1)\,y_\mu^\star\left[1+\frac{\delta}{\lambda(\theta^\star)}\right]. \tag{28}$$

We assume $(\rho+\delta)k < \bar{q} - \bar{w}_1$ so that $\theta^\star > 0$ for all μ.

The real interest rate adjusts to clear the asset market:

$$a^\star(r) = \frac{(1+r)\theta(r)k}{\delta+\lambda\left[\theta(r)\right]} + A^g, \tag{29}$$

where $\theta(r)$ is defined implicitly by (**??**). The left side is households' asset demand. The right side is the supply of assets in the form of shares of mutual funds (first term) and government bonds (second term). From (**??**) and (**??**), the market capitalization of the mutual funds is $n\phi^f = R\theta k/\left[\delta+\lambda(\theta)\right]$. The interest rate channel, captured by (**??**), specifies that the interest rate at which firms' profits are discounted, r, depends on the supply of private and public liquidity.

An equilibrium is a triple, (a, θ, r), that solves (**??**), (**??**), and (**??**) with $a^\star(r) = a$. The textbook MP model corresponds to the special case when $\alpha = 0$, i.e., there is no demand for early consumption. In that case, from (**??**), $r = \rho$. From (**??**), market tightness solves $(r+\delta)\theta^\star k/\lambda(\theta^\star) = \bar{q} - \bar{w}_1$. In the MP model, the aggregate-demand and interest-rate channels are inoperative.

Abundant liquidity

In the class of equilibria with degenerate distributions, one can distinguish two subclasses. The first subclass is when liquidity needs are satiated, $a \geq \mu y_\mu^\star$. From (**??**) assets have no liquidity value and $r = \rho$. Given r, market tightness is uniquely pinned down by (**??**). The occurrence of this regime necessitates that the supply of private and public assets, $n\phi^f + A^g$, is larger than households' demand for assets, $a^\star$. If this condition holds, a change in A^g has no effect on r and θ.

Scarce liquidity

The second subclass of equilibria is when liquidity is scarce, $a < \mu y^\star$. In such equilibria, asset prices exhibit a liquidity premium, $r < \rho$. From the job creation condition, (**??**), the aggregate supply of private assets is equal to

$$n\phi^f = \frac{(1+r)\theta k}{\delta+\lambda(\theta)} = \frac{1+r}{r+\delta}\left[\frac{\lambda(\theta)}{\delta+\lambda(\theta)}\left(\bar{q}-\bar{w}_1\right) + \alpha\left(\frac{\mu-1}{\mu}\right)a\right]. \tag{30}$$

The middle term is obtained from the left term by using that $n = \lambda(\theta) / [\delta + \lambda(\theta)]$ and $\phi^f = R\theta k / \lambda(\theta)$. The right side corresponds to the discounted sum of profits from all firms. It has two components: the profits if all the output was sold to late consumers and the profits arising from early sales at a markup. Substituting $n\phi^f$ by its expression given by (**??**) into the total asset supply, $a = n\phi^f + A^g$, and solving for a, we obtain $a = \bar{A}(\theta, r, a^g)$, where

$$\bar{A}(\theta, r, a^g) = \frac{\lambda(\theta)}{\delta + \lambda(\theta)} \frac{[(1+r)(\bar{q} - \overline{w}_1) + (r+\delta) a^g]}{r + \delta - (1+r)\alpha\mu^{-1}(\mu - 1)}. \tag{31}$$

There is a unique $\bar{A}(\theta, r, a^g) \in (0, +\infty)$ provided that $(1+r)\alpha\mu^{-1}(\mu - 1) / (r + \delta) > 0$. If $\mu > 1$, then

$$\frac{\partial \bar{A}}{\partial A^g} = \frac{r + \delta}{r + \delta - (1+r)\alpha\mu^{-1}(\mu - 1)} > 1,$$

i.e., there is a multiplicative effect of public liquidity on aggregate liquidity. If public liquidity increases, then the value of firms increases through the aggregate demand channel, which raises private liquidity and hence the overall liquidity. The asset market clearing condition, (**??**), becomes

$$a^\star(r) = \bar{A}(\theta, r, a^g). \tag{32}$$

Since $\bar{A}(\theta, r, a^g)$ is increasing in θ but decreasing in r, (**??**) gives a positive relationship between r and θ. As θ increases, the supply of assets increases, which requires a higher real interest rate to clear asset markets.

We now turn to the determination of θ. The wealth dedicated to the accumulation of private assets is $\bar{A}(\theta, r, a^g) - na^g$. Hence, $\bar{A}(\theta, r, a^g)/n - a^g = \phi^f = R\theta k / \lambda(\theta)$, which can be re-expressed as

$$\frac{\bar{q} - \bar{w}_1 + \alpha\mu^{-1}(\mu - 1) a^g}{r + \delta - (1+r)\alpha\mu^{-1}(\mu - 1)} = \frac{\theta k}{\lambda(\theta)}. \tag{33}$$

This condition gives a negative relationship between θ and r. An equilibrium with scarce liquidity can now be reduced to a pair, (θ, r), solution to (**??**) and (**??**). In Figure **??**, the upward-sloping curve representing (**??**) is labelled EE (for Euler Equation) while the downward-sloping curve representing (**??**) is labelled FE. There is a unique equilibrium at the intersection of the two curves.

Channels of monetary policy

The channels of monetary policy can be illustrated graphically. First, an increase in a^g raises the supply of liquidity, which, for given θ, raises r. Graphically, the EE curve moves upward. Second, an increase in a^g raises households' expenditure on early consumption, which raises firms' profits when $\mu > 1$, and induces more job creation. Graphically, the JC curve moves to the right. The overall effect is positive on r but ambiguous on θ. We summarize our results so far in the following proposition.

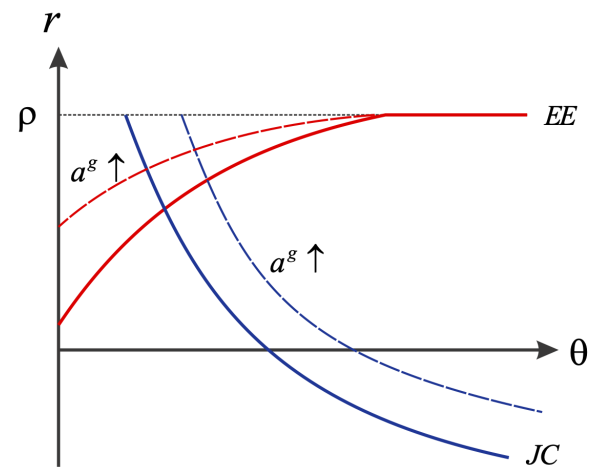

Figure 2: Equilibrium with degenerate distributions.

Proposition 2 *Assume* $U_{e_t}(c_t) = c_t + (1 - e_t)\ell$.

1. ***Abundant liquidity.*** *If*
$$\frac{(1+\rho)\theta^{\star}k}{\delta + \lambda(\theta^{\star})} + A^g \geq \mu y_{\mu}^{\star}, \tag{34}$$
then $y = y_{\mu}^{\star}$ *and* $r = \rho$. *Moreover,* $\partial\theta/\partial A^g = \partial r/\partial A^g = 0$.

2. ***Scarce liquidity.*** *If (**??**) does not hold, then* $y < y_{\mu}^{\star}$ *and* $r < \rho$. *Moreover,* $\partial r/\partial A^g \geq 0$ *but* $\partial\theta/\partial A^g \lesseqgtr 0$.

We describe two special cases where each case has a single channel operative. Suppose first that the early consumption is priced competitively, $\mu = 1$.[7] Equations (**??**) and (**??**) become:

$$\begin{aligned} a^{\star}(r) &= \frac{(1+r)\,\theta k + \lambda(\theta)a^g}{\delta + \lambda(\theta)} \\ \frac{\theta k}{\lambda(\theta)} &= \frac{\bar{q} - \bar{w}_1}{r + \delta}. \end{aligned}$$

Only the interest rate channel is operative and $\partial\theta/\partial A^g < 0$. In that case public liquidity crowds out private liquidity, which generates an increase in the real interest rate.

The second special case is when the utility for early consumption is linear, $v(y) = \bar{v}y$ where $\bar{v}$ is constant. We consider equilibria where $c \geq 0$ does not bind. (We provide a full characterization of the model with linear utility in Appendix B.) From (**??**)

$$r = \frac{\rho\mu - \alpha\,(\bar{v} - \mu)}{\alpha\,(\bar{v} - \mu) + \mu}.$$

[7]This first case is analogous to **?**.

The interest rate channel is inoperative and only the aggregate demand channel prevails when $\mu > 1$, $\partial\theta/\partial A^g > 0$. An increase in public liquidity raises the wealth in households' hands, which raises firms' revenue from their sales to early consumers.

4.2 Equilibria with distributional effects

We now consider equilibria where $c \geq 0$ binds, i.e., households only consume early, and the distribution of asset holdings is nondegenerate. We focus on such equilibria where, in the event of an expenditure shock for early consumption, households deplete their asset holdings in full, i.e., $y_e(a) = a$ for all a in the support of F_e.[8] From (**??**) this is the case when $v'(a/\mu) \geq \mu W_e'(0)$, i.e., the marginal utility of consumption is larger than the marginal of wealth when assets are depleted. Households' target for asset holdings is $a^\star$, defined as a solution to (**??**).

Two-point distribution

The simplest equilibrium with a non-degenerate distribution has two mass points. Employed workers accumulate $a^\star$ in a single period, which requires $(1+r)w_1 > a^\star$. In contrast, it takes two periods for unemployed workers with depleted asset holdings to reach their target. This requires $R < a^\star/w_0 < R\,(1+R)$. In a steady state, a measure αu of households own $(1+r)w_0$ assets, those households that are unemployed and received an expenditure shock in the previous period, while the remaining $1-\alpha u$ households own $a^\star$. Hence, the distribution of asset holdings is

$$F(a) = \alpha u \mathbb{I}_{\{a \geq (1+r)w_0\}} + (1-\alpha u)\mathbb{I}_{\{a \geq a^\star\}}. \tag{35}$$

The distribution of asset holdings depends directly on both expenditure and unemployment risk through α and u. It also depends on the income of the unemployed, w_0. Hence, money creation implemented through transfers to households will affect the distribution through both r and τ_0. The relation between the supply of public liquidity and transfers is given by the budget constraint of the government, (**??**),

$$(1+r)\,(n\tau_1 + u\tau_0) = -rA^g. \tag{36}$$

The asset market clearing condition, (**??**), becomes:

$$(1-\alpha u)\,a^\star(\overset{+}{r}) + \alpha u(1+r)w_0 = \bar{A}(\overset{+}{\theta}, \overset{-}{r}, \overset{+}{a^g}), \tag{37}$$

[8] Similar equilibria have been studied in detail in **?**. Relative to this paper, we endogenize the real interest rate, r.

where $\bar{A}(\theta, r, a^g)$ is given by (**??**) and the signs above the variables represent partial derivatives. An equilibrium is a pair, (θ, r), that solves (**??**) and (**??**).

As before, a change in a^g triggers the aggregate demand and interest channels. In addition, via the government budget constraint (**??**), it is associated to a change in transfers. Consider an increase in τ_0 in isolation (independently from A^g). From (**??**) it generates a fall in r for given θ. Indeed, if unemployed households receive a higher income, then they can accumulate assets faster and so aggregate asset holdings are larger. This lowers the real interest rate and induces firms to open more jobs. In general equilibrium, there is an amplification mechanism as the decrease in u raises r further.

Three-point distribution

Consider another class of tractable equilibria where both employed and unemployed workers need two periods to reach their targeted real balances, $Rw_1 < a^\star$ and $w_0 R(1+R) > a^\star$. Households have incentives to deplete their asset holdings in full provided that Rw_e is sufficiently close to $a^\star$. There are now αn employed and αu unemployed households who save their full income in order to self-insure against the expenditure risk. The distribution of asset holdings is

$$F(a) = \alpha u \mathbb{I}_{\{a \geq (1+r)w_0\}} + \alpha n \mathbb{I}_{\{a \geq (1+r)w_1\}} + (1-\alpha)\mathbb{I}_{\{a \geq a^\star\}}. \tag{38}$$

The distribution of asset holdings depends on both w_0 and w_1, and hence on τ_0 and τ_1. Asset market clearing is given by

$$\alpha u R w_0 + \alpha n R w_1 + (1-\alpha)a^\star = \bar{A}(\overset{+}{\theta}, \overset{-}{r}, \overset{+}{a^g}). \tag{39}$$

The real interest rate is a decreasing function of both w_0 and w_1. If we combine (**??**) with the budget constraint of the government, (**??**), i.e., $\alpha R\left(n\tau_1 + u\tau_0\right) = -\alpha r A^g$, then the asset market clearing condition becomes

$$\alpha u R \bar{w}_0 + \alpha n R \bar{w}_1 + (1-\alpha)a^\star = \bar{A}(\theta, r, a^g) + \alpha r n a^g.$$

The transfers reinforce the effects of a^g on r.

While our simplified model allows us to identify different channels of monetary policy, we had to impose several restrictive assumptions to achieve some amount of tractability: households are risk neutral relative to late consumption, all assets are equally liquid, and we could only study the class of equilibria where households deplete all their wealth in the event of an expenditure shock. The next section will relax all these restrictions and will quantify the effects at work.

5 Calibration

We now study the quantitative implications of money growth and inflation in a calibrated version of our model. We consider two cases for how the proceeds from money creation are spent. As our baseline, money creation only finances wasteful government purchases, $g = \pi\phi_t^m M_t$, whereas interest payments on debt are financed with lump-sum taxes, $\tau = (1/R^\iota - 1)A^g$. For our second case, we consider 'helicopter money', following **?**, in which money creation is distributed lump-sum to all households.[9] Studying these two cases underscores the importance of the fiscal side of money creation in environments that depart from quasi-linear preferences. We calibrate both versions of the model using the same procedure, but focus our discussion in this section on the baseline economy. We report the case with helicopter money in Appendix C.

We choose the length of a time period to be one month. Preferences are given by $v(y) = \Psi(y^{1-\psi} - 1)/(1-\psi)$, where $\Phi, \phi > 0$, and $U(c,e) = (c^{1-\gamma} - 1)/(1-\gamma)$ with $\gamma = 1.5$. Relative to Section **??**, the function U is not bounded and we set $\ell = 0$. The matching function takes the form $\mathcal{M}(s,o) = so/(s^\nu + o^\nu)^{1/\nu}$. Following **?**, we set $\nu = 1.6$ to fit the U.S. Beveridge curve. The separation rate is $\delta = 0.035$, implying a quarterly job destruction rate of 10%. Vacancy costs, k, are set to imply an average monthly job finding rate of 30%, consistent with evidence in **?**.[10] The production possibility frontier is $Q(y) = \left(1 - y^{\frac{1}{a}}\right)^a$. We set a to target a ratio of price over average cost of 30% in the market for early consumption, an estimate of markups in the retail trade sector.[11]

We assume there are three levels productivity $z \in \{z_\ell, z_m, z_h\}$ distributed according to $\varpi(z)$. We interpret the low-productivity type, z_ℓ, as representing workers with an associate's degree or less and set $\varpi(z_\ell) = 0.62$ to match the fraction of working age adults reported in **?**, Table 1. The high-productivity type, z_h, represents workers in the top-10% share of labor income, $\varpi(z_h) = 0.10$. We normalize the average productivity to one, $\bar{z} = \sum_{i\in\{\ell,m,h\}} z_i\varpi(z_i) = 1$. Given this, we set z_ℓ and z_h to target a college wage premium of 1.7, consistent with evidence in **?** and **?**, and a share of top-10% income share of 0.3, given evidence in **?**.[12] Wages per efficiency unit z_i are set proportional to a firm's productivity. Let $\bar{w}_1 = \mu q(p)$, where q is output per efficiency unit given by (**??**). We choose $\mu = 0.7$ to target a labor share of 70%.

[9] In Friedman's (1969) words: "Let us suppose now that one day a helicopter flies over this community and drops an additional \$1,000 in bills from the sky, which is, of course, hastily collected by members of the community."

[10] Average unemployment duration in the model is 3.3 months or 14.4 weeks, which is slightly below the average in the decade before the Great Recession between 1999 and 2008 of 16.5 weeks.

[11] See www.census.gov/econ/retail for more information. If one interprets early and late consumption goods as the same physical good sold to consumers with different valuations, our model generates a standard deviation of prices of 11.3%, which is in line with evidence of retail price dispersion in **?**.

[12] In our model, the college wage premium is given by $[\varpi(z_m)z_m + \varphi(z_h)z_h]/[(\varphi(z_m) + \varphi(z_h)z_\ell]$ and the top-10% income share is given by $\varpi(z_h)z_h/\bar{z}$.

Income of the unemployed is set based on a replacement rate of 40% following **?**, $\bar{w}_0 = .4\bar{w}_1$. The decline in income upon job loss generates an average fall in consumption of 6.7%, slightly below the evidence in **?** of 16%, **?** of 11%, and **?** of 9%.

There are 6 remaining parameters to calibrate: the discount factor, β, preference parameters for early consumption, (Ψ, ψ), expenditure risk, α, the acceptability of illiquid wealth, α_1, and the stock of government debt, A^g. We set these to target moments related to the household wealth and asset liquidity. We map the components of wealth in the model to data on portfolio holdings in the 1998-2013 waves of the Survey of Consumer Finances (SCF). We interpret liquid assets as zero maturity wealth held by households, which predominately corresponds to transaction accounts (checking, saving, money market, and brokerage cash). We set the annual real rate of return on liquid balances, R^m, to -1.5%, equal to the average real rate of return of zero maturity assets (MZM) between 1999 and 2008 reported by the Federal Reserve Bank of St. Louis.[13] Illiquid wealth in the model captures the remainder and corresponds to households' direct and indirect holdings of publicly-traded stocks and bonds, private business equity, and net worth directly held in residential structures and consumer durables, including vehicles. We set α_1 to target the liquidity premium. Since our model does not feature aggregate risk, we measure the liquidity premium using the real user cost of holding MZM given by **?**.[14] The real user cost equals the interest rate spread between the real return on MZM and the maximum real return on a set of short-term money market accounts and the Baa corporate bond yield. The average use cost over 1998-2013 was 6.2%. This leads to a probability that illiquid assets are accepted, conditional on an expenditure shock, of $\alpha_1/\alpha = 0.06$. We set A^g to target the fraction of aggregate household wealth held either directly or indirectly in bonds of 13% in the SCF.

The remaining parameters, β, Ψ, ψ, and α, are chosen to target moments from the distribution of liquid wealth to annual income and the distribution of the share of total wealth held in liquid assets – the 'liquid share'. These distributions are important measures of the extent to which households are directly exposed to changes in long-run inflation, R^m, via their portfolios. Instead of choosing ad-hoc, cross-sectional moments to target, we minimize the sum of squared residuals over the entire distribution. Figure **??** illustrates the fit between model (solid-blue curves) and the average of the cross-sectional distributions in the 1998 to 2013 waves of the SCF (dashed-green curves). The left panel shows the distribution of liquid wealth to annual income while the right panel shows the distribution of the liquid share.

The model matches the empirical distribution of liquid wealth to income well. The median is 0.57 in the model and 0.43 in the data, and the inter-quartile range is 0.18 in the model and 0.12 in the data. In terms

[13]We compute the real return using the nominal return on MZM, FRED series MZMOWN, and the consumer price deflator).

[14]See FRED series OCMZMA.

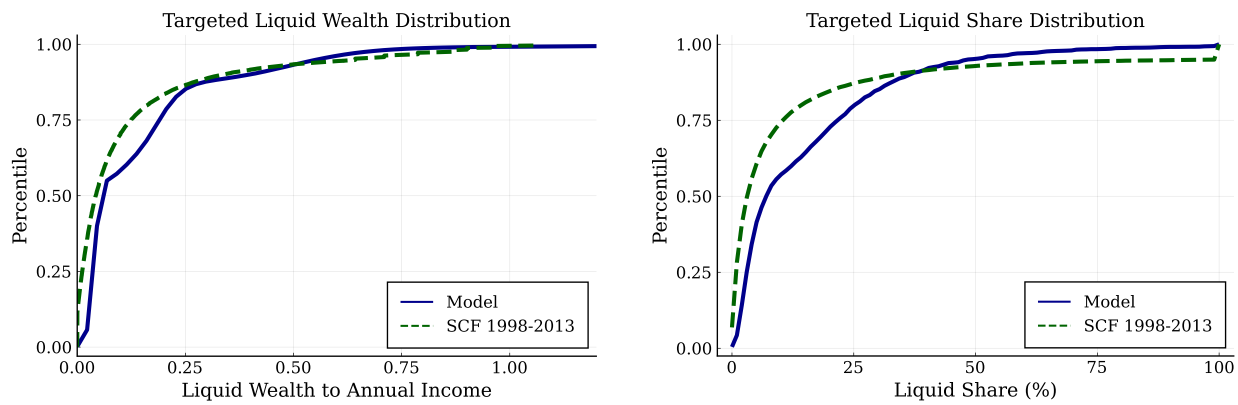

Figure 3: The distribution of liquid wealth to income (left) and the distribution of the share of wealth held in liquid assets (right), in model versus data.

of the share of wealth held in liquid assets, the model produces a slightly more dispersed distribution and right-skewed distribution compared to the data. The median liquid share in the model is 6.3% while in the data it is 3.8%, and the inter-quartile range in the model is 17 percentage points while in the data it is 10 percentage points. In terms of total wealth, the model produces lower median and average wealth to income ratios than in the SCF data (median of 1.1 versus 1.6 and average of 1.5 versus 3.8) and slightly lower Gini coefficient (0.55 versus 0.63).[15]

The calibration procedure above leads to an annual discount rate of $1/\beta^{annual} - 1 = 5.49\%$ and an annual return on illiquid wealth of $R^{\iota,annual} = 4.7\%$. The calibrated probability of a preference shock is $\alpha = 0.075$, implying households receive an average of 0.9 shocks per year. The level and curvature of the utility of early consumption are $\Psi = 2.2$ and $\psi = 0.28$, respectively, which imply an average increase in monthly consumption of 26% in the event of a shock. Hence, preference shocks in the model are best interpreted as relatively large but infrequent, unplanned household expenditures.

In the SCF, expenditure shocks are reported as one of the most common reasons households choose to save.[16] While standard surveys on household consumption do not measure unplanned expenditures, the 2015 Survey of American Family Finances (SAFF), a nationally-representative survey administered by Pew Charitable Trusts, asked respondents if in the past 12 months their family had experienced one of 6 types

[15] This is to be expected given the ex-ante heterogeneity is limited to three productivity levels and households only face a two-point distribution of employment and expenditure risk. In Appendix E, we report the results in a model in which we add idiosyncratic risk to labor productivity.

[16] For instance in the 2013 SCF, 28% of households reported their most important reason for saving was for emergencies, unexpected events, or illness and medical/dental expenses.

Parameter	Value	Moment	Data	Model
Parameters Set Directly				
rate of return on liquid wealth, $R^{m^{12}}$	0.985	annual real return of MZM	-1.5	-1.5
replacement rate, $\bar{w}_0/\bar{w}_1$	0.400	40% replacement rate	0.40	0.40
worker share of revenue, μ	0.700	labor share of 70%	0.70	0.70
job depreciation rate, δ	0.035	quarterly separation rate, BEA	10%	10%
matching curvature, ν	1.600	**?**		
Permanent Labor Productivity				
labor productivity dist., $\varpi(z)$	$[0.62, 0.28, 0.10]$	see discussion in Section **??**		
labor productivity, z_ℓ	$[0.58, 0.98, 2.15]$			
Parameters Calibrated Jointly - outer loop				
bond supply, A^g	0.366	bond share of wealth	13%	5.2%
entry cost, k	6.460	monthly job finding rate	30%	30%
production cost curvature, a	0.751	average retail markup	30%	32%
acceptability of illiquid, α_1/α	0.060	liquidity premium	6.2%	6.2%
Parameters Calibrated Jointly - inner loop				
discount rate, β^{12}	0.948	see discussion in Section **??** and Figure **??**		
early consumption - curvature, ψ	0.282			
early consumption - level, Ψ	2.225			
preference shock, α	0.075			

Table 1: Calibration - parameters and target moments

of unexpected expenses, including medical expenditures, divorce, and vehicle repairs or replacement.[17] Survey respondents reported receiving an average of 1.2 shocks per year and a median expenditure amount of $2,000, or about 60% of average monthly consumption.[18]. The frequency and size of shocks in the model are in line with the data. Table **??** provides a summary of the calibrated parameters and targeted moments in the model and the data.

[17]See here for background and methodology of the survey and here for a summary and data tables.

[18]Using average per-capita annual consumption expenditures of $38,000 in 2015 from the Bureau of Economic Analysis (FRED series A794RC0A052NBEA).

5.1 Untargeted heterogeneity in MPCs, responses to job loss, and savings

The model produces heterogeneity along several dimensions that are not directly targeted in the calibration but are consistent with empirical evidence. Figure **??** illustrates the marginal propensity to consume out of liquid wealth (left panel) and the consumption and savings responses to a job loss (right panel), across the household wealth distribution.

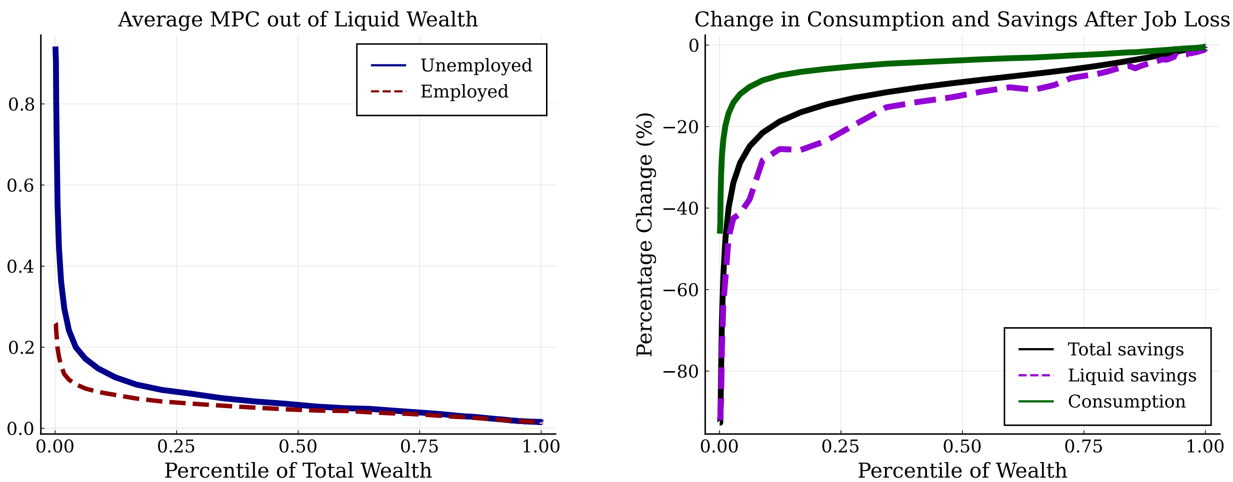

Figure 4: Model cross-section of MPCs and consumption and savings responses to job loss.

We measure the three-month marginal propensity to consume (MPC) from a one-time unanticipated transfer of liquid wealth of \$500.[19] This experiment is comparable to evidence in **?** from spending responses to one-time stimulus payments. The average MPC in the model is 8.3%, just below the range of responses of 12% to 30% found in **?** on non-durable spending. Along the cross section, the model matches the pattern of spending responses by wealth and income from the data. MPCs fall in total wealth, ranging from 85% for the poorest, unemployed households to 5% for the wealthiest, employed households. The larger MPC responses are in line with evidence in the 2010 Italian Survey of Household Income and Wealth (SHIW), reported in **?**. The lowest percentile of wealth reports an average MPC of around 70%. However, higher wealth households have larger MPCs in the data of around 35%.[20] In terms of employment status, the model yields an average MPC of 7.8% for employed households and 16.2% for unemployed households, a

[19]Computationally, we calculate the change in total consumption, between a household that received the unanticipated transfer versus the same household type that did not, as a fraction of the size of the transfer. We start the experiment at the beginning of the period in the steady-state equilibrium. For each household type, $(e, \mathbf{a}, z)$ we simulate their sequence of expense and employment shocks over the three periods using the equilibrium stochastic processes.

[20]See Figure 2 in **?**. They report percentiles of "cash-on-hand" equal to household total disposable income plus financial wealth, net of consumer debt.

difference of 10.7 percentage points. **?** reports a larger MPC for unemployed households using the 2010 SHIW.

The right panel of Figure **??** illustrates the consumption and savings responses to job loss. The green-solid curve shows the percentage change in average total consumption (early and late) in the first month after a job loss for a household with a given percentile of total wealth. The consumption response is a measure of how well households are insured against employment risk. The average consumption decline is 6.7%, but the response varies strongly with wealth. For the wealthiest agents, consumption remains unchanged while for the poorest agents consumption falls by 45%.[21] The solid-black and dashed-violet lines show the change in average total and liquid savings, respectively, in the first month after a job loss. As usual in incomplete-markets models, savings responses are larger than consumption responses, for any given total wealth level. Liquid savings is more responsive to current income than illiquid savings, a fact in line with empirical evidence in **?**. This behavior is also in line with models in which illiquid assets are subject to adjustment costs, such as **?**.

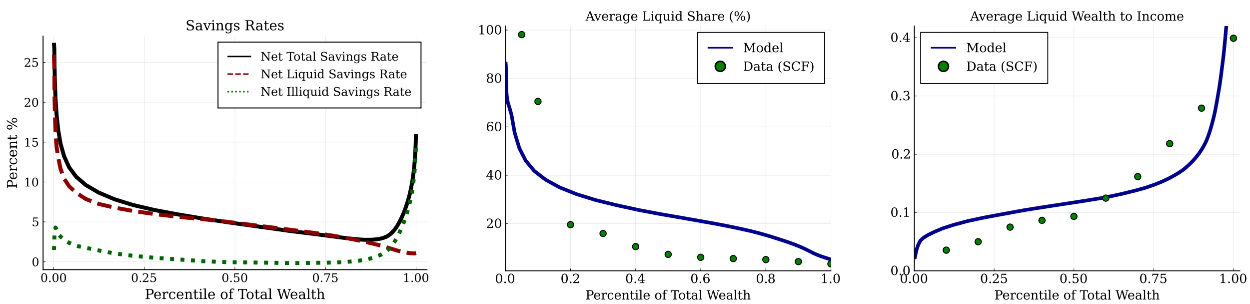

Figure 5: Net savings, liquid share, and liquid wealth to income, by total wealth

The left panel of Figure **??** illustrates average net savings relative to disposable income as a function of a household's total wealth. We define disposable income as labor earnings net of transfers plus the net return on wealth. The net savings rate measures the change in wealth from the beginning of one period to the next. We also decompose the total savings rate into liquid and illiquid components. The total savings rate is U-shaped, decreasing initially for low wealth levels and increasing for high wealth levels. Savings rates are relatively flat across most of the wealth distribution, a fact consistent with evidence from Norway in **?**, however, there are meaningful compositional differences between liquid and illiquid wealth. High savings rates for poor households are predominately in liquid wealth while higher savings rates for wealthy

[21]Unfortunately, there is limited empirical evidence on the effect of wealth on the consumption response to job loss because surveys typically do not feature rich enough data on consumption, labor market outcomes, and wealth.

households are predominately in illiquid wealth. The middle and right panels of Figure **??** confirm that a household's share of wealth held in liquid assets falls with total wealth (left panel), a fact we do not target in the calibration but is consistent with the SCF evidence, while the level of liquid wealth to income rises with total wealth (right panel).

6 Decomposing the effects of money growth on unemployment

In this section, we explore the effects of a change in the rate of money creation, π, on unemployment, asset returns, and prices. Our objective is to quantify the importance of the channels we discussed in previous sections, in particular the interest rate and aggregate demand channels, in determining the slope of the Phillips curve in the long-run. In Figure **??**, we illustrate the model-implied relationship between inflation and unemployment (solid-green line), as well as the raw data with a linear fit (blue dots and solid-blue line, respectively), in the model with wasteful government purchases.[22] Unemployment is shown as percentage point deviations from a 2% inflation steady state. Consistent with the data, the model predicts a positively-sloped, but nearly vertical long-run Phillips curve.

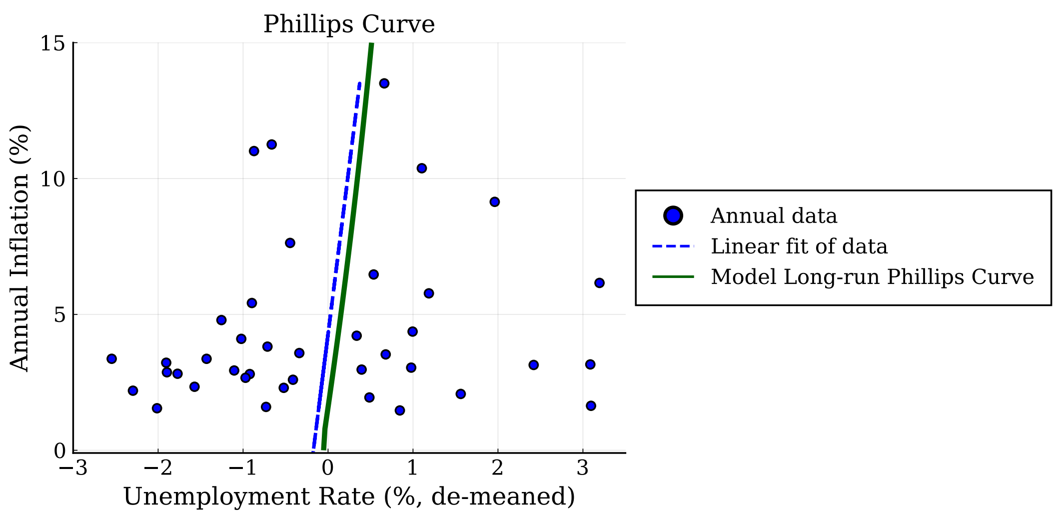

Figure 6: Inflation and unemployment: model vs. data.

[22]We choose to compare the model against the raw, un-filtered data instead of taking a stand on the correct way to estimate the long-run trend component of each series. However, this makes little difference in practice; regardless of the filter chosen the relationship between the long-run trend components remains weakly positive. The unemployment rate in the data is given by the Bureau of Labor Statistics U-3 measure of unemployment, FRED series UNRATE.

6.1 Quantifying the interest-rate and aggregate-demand channels

The general equilibrium relationship between inflation and unemployment nets out the interest-rate and aggregate-demand channels. In the context of our model, we separate the two as follows. From the free-entry condition, (**??**), and the Beveridge curve, (**??**), unemployment can be written as a function of two prices: (i) the price of illiquid financial wealth, $1/R^{\iota}$, and (ii) the price of early consumption, p^y. The changes in the unemployment rate that result from changes in R^{ι} will be attributed to the interest rate channel. Changes in unemployment that result from changes in p^y will correspond to the aggregate demand channel. We illustrate these prices in the left and middle panels of Figure **??**, respectively.

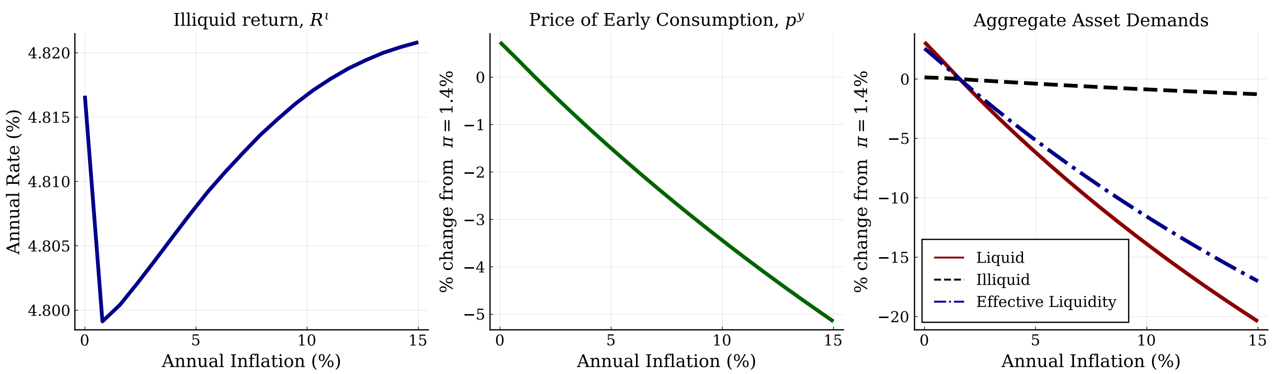

Figure 7: Inflation versus asset returns, prices, and aggregate wealth.

First, consider the interest rate channel. There are direct and indirect effects through which inflation alters the demand and supply of illiquid assets, and hence R^{ι}. First, the inflation tax creates a substitution effect that causes households to shift their savings away from liquid assets towards illiquid assets. This substitution effect increases the price of illiquid assets, decreases their return, R^{ι}, and increases firm entry, since firm profits are discounted at a lower rate. However, inflation also causes a negative wealth effect that lowers households' demand for all assets, including illiquid ones. Further, as wages and labor income risk (captured through changes in tightness, θ) adjust, households' asset demands also respond. The left panel of Figure **??** illustrates the general equilibrium response in R^{ι} to inflation. We find a u-shaped response – the illiquid return falls with inflation for inflation rates below 1%, and rises thereafter. Quantitatively, the substitution and income effects tend to cancel each other, leading inflation to only cause only small movements in R^{ι}. As a result, the interest rate channel contributes approximately 6% to the slope of the Phillips curve. The left panel of Figure **??** shows the contribution of R^{ι} to unemployment as the dash-dotted red line.

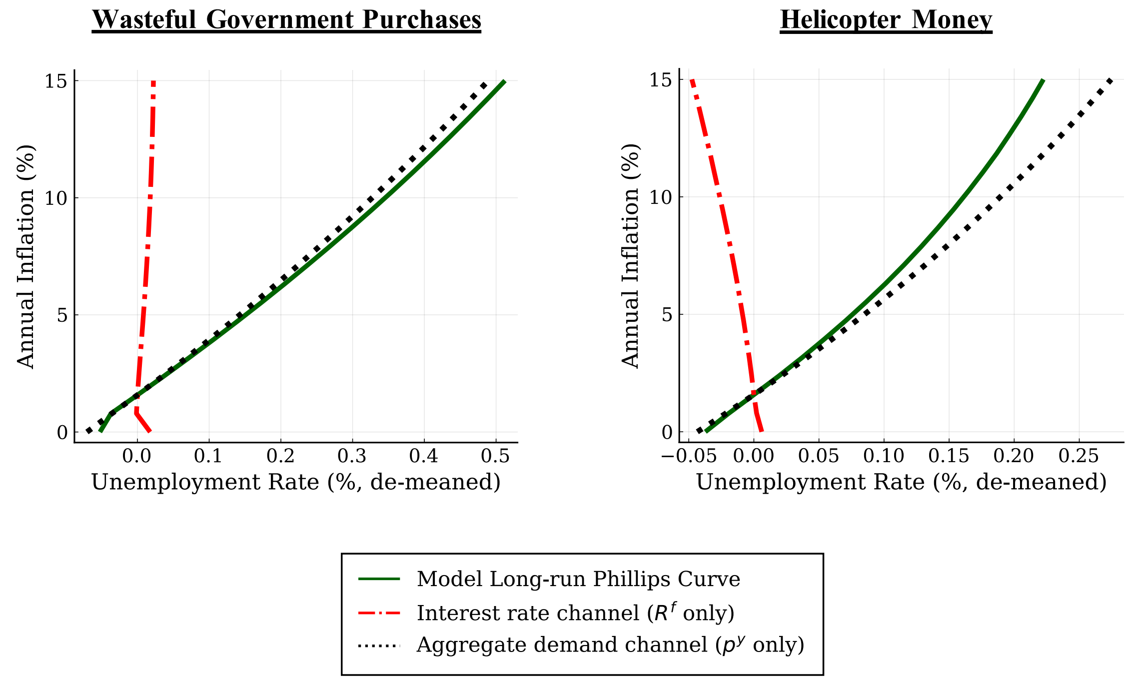

Figure 8: Aggregate demand and interest channels of the long-run Phillips curve when money creation finances wasteful government spending (left) or is given lump-sum to households (right).

The aggregate-demand channel is the primary contributor to the long-run Phillips curve, explaining the remaining 94% of the slope (illustrated as the dotted black line in Figure **??**). Inflation reduces aggregate effective liquidity (market capitalization of each asset weighted by its acceptability), tightens households' liquidity constraints, lowers the demand for early consumption, and decreases p^y. Raising steady-state inflation from 2% to 10% decreases the price of early consumption by 3.5% and decreases aggregate effective liquidity by 12%, as show in the middle and right panels of Figure **??**, respectively. The fall in p^y, lowers firms' expected revenue and profits, reduces entry, and increases unemployment.

Allowing the proceeds from money creation to be rebated lump sum to all households instead of being wasted does not significantly alter the shape of the long-run Phillips curve, as shown in the right panel of Figure **??**. The interest-rate channel changes sign and the aggregate demand channel is slightly weaker in magnitude, resulting is a slightly more vertical Phillips curve. This change is primarily driven by the difference between the calibrated liquidity of non-monetary wealth, α_1, between the two economies. We find $\alpha_1/\alpha = 0.06$ in the baseline economy whereas $\alpha_1/\alpha = 0.36$ in the economy with lump-sum transfers. This leads to a stronger substitution channel in which inflation always lowers the return on non-monetary wealth, R^ι.

6.2 Interest-rate and aggregate-demand channels in the cross section

A novel feature of our model, relative to BMW and others in the New-Monetarist literature, is that households are risk averse and have a motive to self-insure against income and expenditure risk across periods. This departure from quasi-linear preferences creates a non-degenerate distribution of asset holdings and leads to heterogenous responses to inflation in household savings and consumption. Studying these responses helps to clarify how the aggregate demand and interest rate channels are formed in general equilibrium.

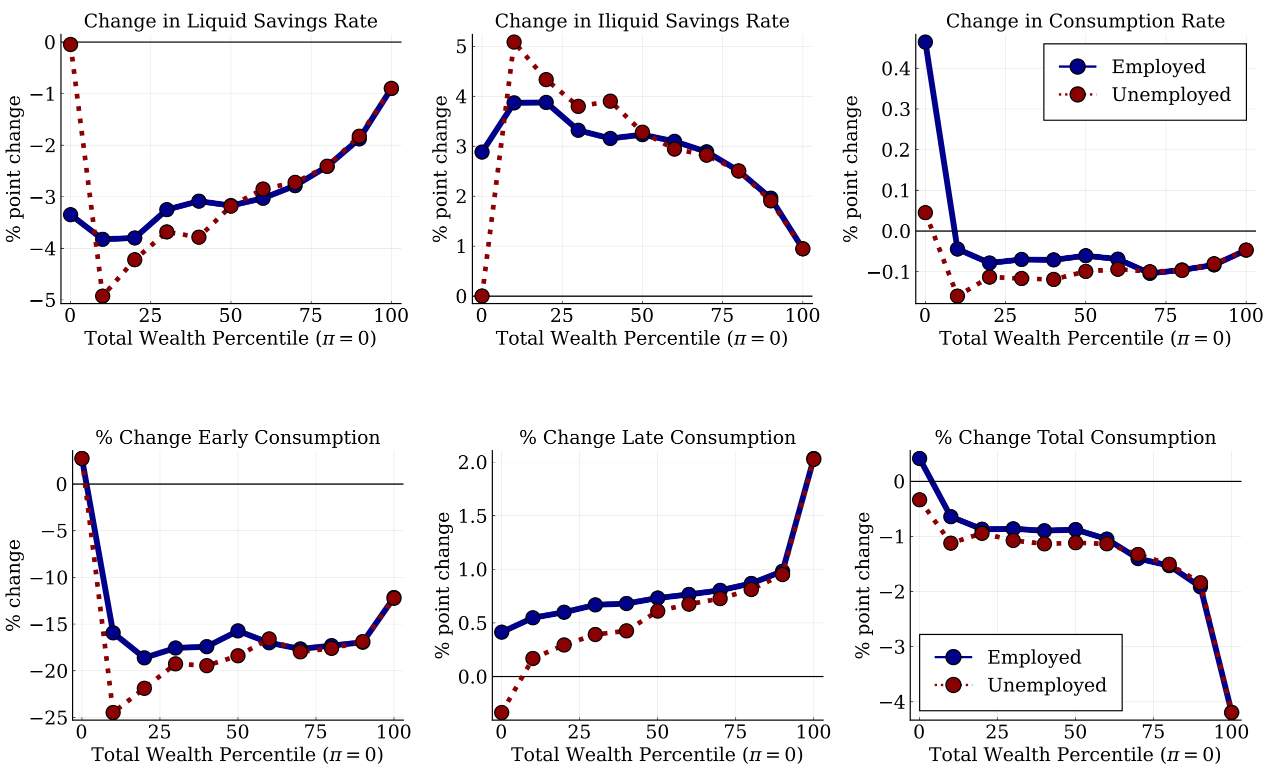

Figure 9: The effect of increasing π from 0% to 10% on savings and consumption, by wealth and employment.

Figure **??** illustrates the change in household savings and consumption induced by increasing inflation from 0% to 10%, in the cross-section of household wealth and employment. First, consider the effects of inflation on household savings rates, shown in the top row. The top-left panel shows the effect on the liquid savings rate, the top-middle panel shows the effect on the illiquid savings rate, and the top-right panel shows the effect on the total consumption rate, or one minus the total savings rate. We illustrate the change by the percentile of total wealth in the $\pi = 0\%$ steady state, separately by employment status.

Nearly all households decrease their rate of liquid savings, increase their rate of illiquid savings, and increase their total savings rate in response to an increase in inflation. The most responsive are unemployed households (illustrated by the dashed-red line) with low (but not the lowest) wealth. As wealth increases from the 10th percentile, households' savings rates tend to be less responsive to inflation. For wealth above the median, employment status makes little difference in savings. Keeping the distribution of wealth and income fixed, the demand for illiquid assets increases. In general equilibrium, however, we find that the demand for illiquid wealth falls (right panel of Figure **??**). The difference is entirely driven by the distributional channel between employed and unemployed households. The unemployed have lower savings rates than the employed, for all levels of wealth. Since inflation increases unemployment, the distribution of wealth shifts towards households with a lower demand to save. This causes the demand for illiquid wealth to fall, and the return R^ι to rise in general equilibrium.

The bottom panels in Figure **??** show the response in the level of early consumption (bottom-left panel), late consumption (bottom-middle panel), and the total consumption (bottom-right panel) by wealth and employment status. The aggregate demand channel is formed by quantitatively-large decreases in early consumption across the distribution of wealth and income. Most households respond to the rise of inflation by decreasing their early consumption (induced by the fall in liquid savings) and by increasing their late consumption. The lowest-wealth households increase their early consumption despite holding less liquid wealth, which illustrates a 'hot potato' effect of inflation. Total consumption decreases across the wealth distribution except for employed households at the bottom of the wealth distribution. The consumption response is larger for households with higher wealth.

7 Inflation and welfare

We now turn to the normative implications of money creation. The left panel of Figure **??** plots the welfare cost of inflation following the methodology in **?**. It is measured as the percentage change in consumption that agents would accept, on average, to avoid moving from a steady state with constant inflation rate, π, to a new steady state with constant inflation rate, $\hat{\pi}$. Given the distribution of wealth is a slow-moving, aggregate state variable, our welfare measure accounts for non-trivial transitional dynamics from the initial steady state to the terminal one that includes perfect-foresight equilibrium paths of all prices, decision rules, and distributions of wealth and employment.

As in Section **??**, we take as our baseline the case in which money creation finances wasteful government purchases. Given any perfect-foresight equilibrium time paths of value functions, decision rules, and prices associated with a sequence of money growth rates $\boldsymbol{\pi} \equiv \{\pi_t\}_{t=0}^{\infty}$, we define aggregate welfare in period t as $\mathcal{W}_t(\boldsymbol{\pi}, \Delta) = \sum_{e\in\{0,1\}, z\in\mathcal{Z}} \int W_{e,t}(a,\Delta) dF_{e,t}(a)$, where $W_{e,t}(a,\Delta)$ is defined recursively by

$$W_{e,t}(a,\Delta) = U(\Delta c_t, e) + \beta \mathbb{E}_{e,t}\left\{ \sum_{\omega\in\{0,1\}} \alpha_\omega \left[v(\Delta y_{\omega e',t+1}) + W_{e',t+1}\left(\mathbf{1}.\hat{\mathbf{a}}_t - p_{t+1} y_{\omega e',t+1}, \Delta\right) \right] \right. \tag{40}$$

$$\left. + (1-\alpha) W_{e',t+1}(\mathbf{1}.\hat{\mathbf{a}}_t, \Delta) \right\},$$

and $F_{e,t}(a)$ are distributions induced by the sequence of decision rules, with initial conditions $F_{e,0}(a)$ and M_0. Our consumption equivalent welfare measure includes both early and late consumption. The welfare cost at time-0 of a one-time unanticipated, permanent change in the constant money growth rate from π to $\hat{\pi}$ is given by $1-\Delta$, where Δ is the solution to $\mathcal{W}_0(\boldsymbol{\pi}, \Delta) = \mathcal{W}_0(\hat{\boldsymbol{\pi}}, 1)$. The left-panel of Figure **??** also illustrates the welfare cost across steady states, not taking into consideration the transition. The welfare cost is monotonically increasing in inflation between 0% and 15%. Moving from steady-state inflation of 0% to 10% harms aggregate welfare by 2% of consumption.

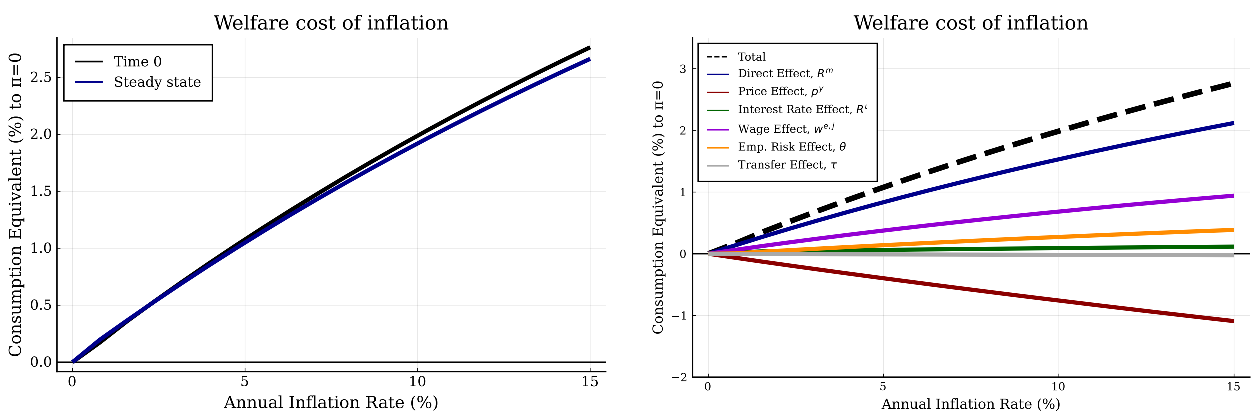

Figure 10: The aggregate welfare cost of inflation (left) and its decomposition (right).

7.1 Quantifying the contributors to the aggregate welfare cost

The right panel of Figure **??** decomposes the welfare cost into components associated with the different channels affecting households. Specifically, we take the equilibrium timepaths of prices, transfers, and labor market tightness, $\mathbf{X} \equiv \{R_t^m, R_t^\iota, p_t^y, w_{e,z,t}, \tau_t, \theta_t\}_{t=0}^{\infty}$, induced by the transition from $\pi = 0\%$ to $\hat{\pi} > \pi$. We then compute the amount by which agents would decrease their consumption in the steady state with $\pi = 0\%$ to avoid the transition of a single element of $\mathbf{X}$, keeping the other elements at their steady-state values.

Consider the welfare cost associated with the return on liquid wealth, R^m, illustrated by the blue line. This captures the direct effect of the inflation tax on households. Inflation reduces the return on liquid wealth, which unequivocally lowers welfare. The aggregate welfare cost of 10% inflation relative to 0%, due only to this direct effect is 1.5% of consumption.

The red line plots the welfare cost associated with the change in the price of early consumption, p. Inflation reduces p, which improves welfare as early consumption becomes cheaper. This mechanism raises welfare by 0.8% in consumption-equivalent terms. However, lower prices for early consumption also harm welfare by reducing workers' effective labor productivity - output per efficiency unit - and in turn their wages. This is illustrated by the violet line. In aggregate, this creates a welfare loss of 0.7% of consumption.

The green and grey lines represent the welfare costs due to changes in the return on illiquid wealth R^ι and the associated change in lump-sum taxes driven by the change in R^ι. Since the return on illiquid wealth adjusts only slightly as a result of inflation, as shown in Figure **??**, the welfare costs and benefits of these two channels are small in the aggregate. Finally, the orange line represents the welfare cost due to changes in labor market risk, driven by θ. Inflation reduces labor market entry on net and leads to an additional welfare cost.

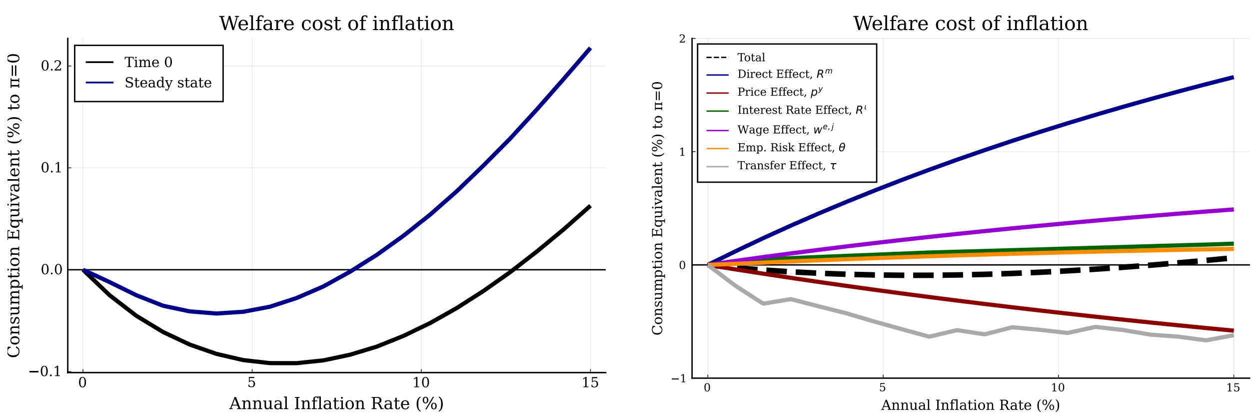

Figure 11: The aggregate welfare cost of inflation (left) and its decomposition (right) under 'helicopter drops'.

Figure **??** illustrates the welfare cost under 'helicopter money'. Moderate inflation is welfare improving, whether or not we take into consideration the transition to a new steady state in the welfare calculation. The optimal inflation rate is 4% without considering the transition and 5.8% including the transition. The right panel of Figure **??** shows this welfare gain is almost entirely driven by the risk-sharing benefits of the transfer. This channel is large in magnitude, which underscores the role of distributional policies in an economy with ex-ante and ex-post heterogeneity.

7.2 Inflation and welfare in cross-section

Figure **??** reports the welfare cost of increasing steady-state inflation from 0% to 10%, by wealth, employment status, and ex ante productivity, when money creation is wasteful (left panel) and when the proceeds of money creation are distributed lump sum to all households (right panel). As in Section **??**, we compute the consumption-equivalent welfare measure that includes transitional dynamics to the new steady state. The solid lines represent unemployed households while the dashed lines represent employed households.

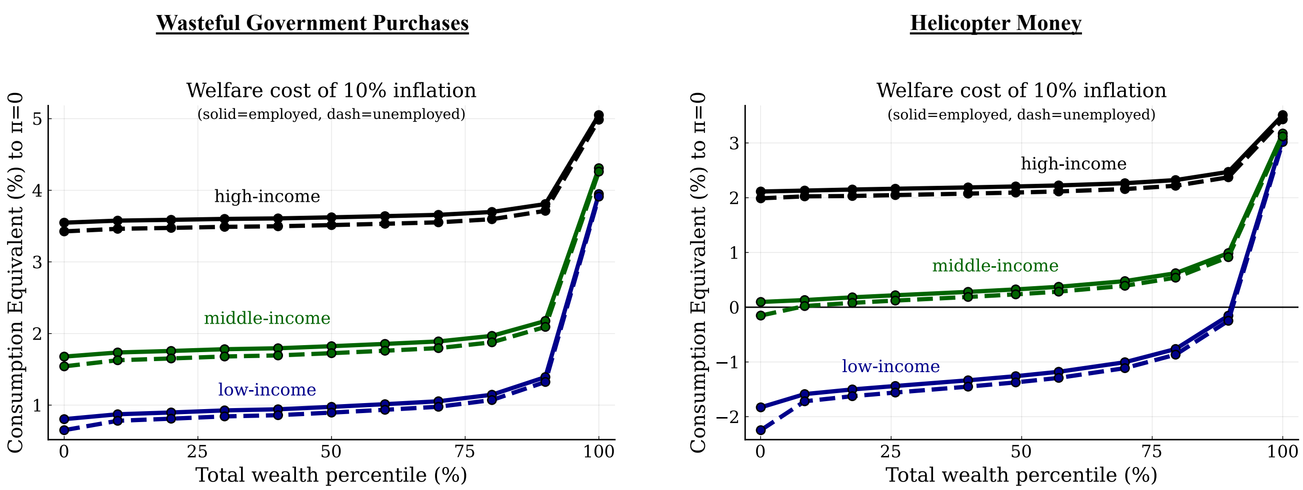

Figure 12: The welfare cost of increasing inflation from 0% to 10%, by wealth and income.

When money creation is wasted, the aggregate welfare cost is 2% of consumption. However individual welfare costs range from 0.76% to 5.0% of consumption. Inflation is the least costly for low-productivity, unemployed households at the bottom of the wealth distribution and most costly for high-productivity, employed households at the top of the wealth distribution. For a given level of income, the welfare cost of inflation is increasing in a household's total wealth. The welfare cost is lower for unemployed households, conditional on wealth and productivity. Finally, households with larger ex ante productivity face a larger welfare cost.

When money creation is distributed lump sum, the qualitative patterns of the welfare cost of inflation across wealth and income remain unchanged; inflation is most costly for high-wealth, high-income households and is least costly for low-wealth, low-income households. However, lump-sum transfers magnify the redistributive nature of money creation and reduce the welfare cost for all. Most low-productivity households benefit from the transfers financed with money creation, except for those above 90th-percentile of wealth. Middle- and high-productivity households are mostly harmed, except for the poorest, unemployed, middle-productivity ones.

In order to understand the drivers of these cross-sectional welfare results, Figure **??** decomposes the welfare cost into the six different channels discussed at the beginning of Section **??**. The strongest driver is the direct inflation tax effect (top-right panel) that leads to many of the comparative statics in the total welfare cost. Wealthy and high-income households in the model hold the most liquid wealth (even though it represents a smaller share of their total wealth). Hence the inflation tax affects them strongly.

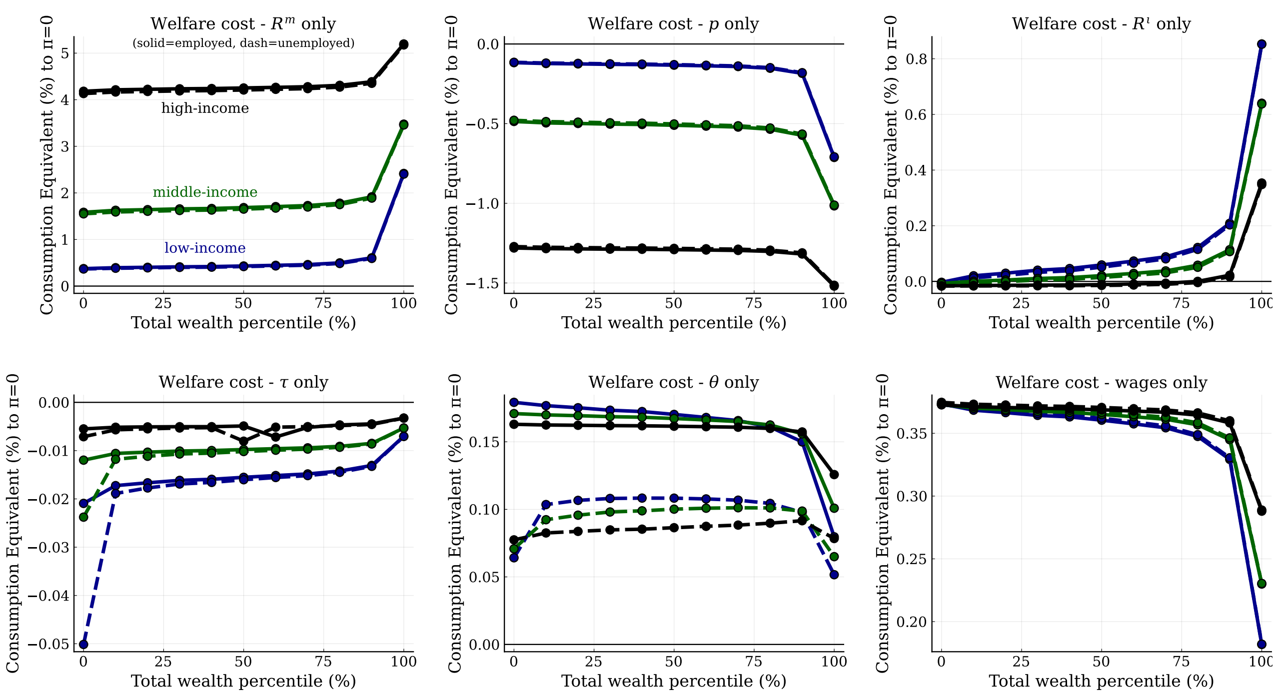

Figure 13: The channels of the welfare cost of increasing inflation from 0% to 10%, by wealth and income, when money creation is wasted.

By inducing a lower p^y, inflation improves the welfare of all households (top-middle panel). Wealthier and higher-productivity households gain more due to this channel. The top-right panel illustrates the welfare effects of inflation due to the real return on illiquid wealth. In steady-state, R^ι increases slightly with higher inflation (see Figure **??**), however the unanticipated increase in money growth generates a temporary increase in asset prices and a temporary decline in the real return. The latter effect dominates and generates a positive welfare cost. Since high-wealth households tend to hold the most illiquid wealth, they face a larger welfare cost. The temporary increase in asset prices induces a temporary, positive increase in transfers (since the interest expense on government debt declines), which in turn induces a positive welfare effect of inflation (bottom-left panel). The rise in asset prices also creates a temporary increase in firm entry and job finding rates that generates a positive welfare benefit. However, job finding rates are lower in the higher-inflation steady-state, which creates a negative welfare effect (bottom-middle panel). Finally, the fall in the price of early consumption lowers wages for all households, thereby generating a welfare cost (bottom-right panel).

8 Conclusion

We constructed a New-Monetarist model with competitive goods and asset markets opening sequentially and a frictional labor market described as in **?**. Households, who are risk averse, face two types of uninsurable idiosyncratic risk, an endogenous employment risk and an expenditure risk. They self-insure against these risks by accumulating a portfolio of multiple assets (money, bonds, and stocks) with various degrees of liquidity. We applied our model to the study of money creation, either wasteful or distributed as lump-sum transfers to households, a.k.a., "helicopter money", and its effects on unemployment, households' asset portfolios and rates of return. We showed that money creation affects the economy through a variety of channels that we identified and quantified. First, there is an aggregate demand channel according to which the consumption of households who receive expenditure shocks increases with aggregate liquidity – a weighted-average of the market capitalization of all assets. Second, there is an interest-rate channel according to which anticipated inflation causes a rebalance of household portfolios and alters the return on financial assets.

Our calibrated model shows that anticipated inflation has a positive effect on equilibrium unemployment – the long-run Phillips curve is positively sloped. The primary contributor to the long-run relationship is the aggregate demand channel. Inflation has a strong negative effect on firm's expected revenue per period, but only modest effects on financial discount rates. This result holds regardless of how money creation is used, either spent on wasteful government purchases or distributed to households lump-sum. We show, however, that if stocks and bonds become more liquid over time the Phillips curve relationship could invert and become negatively sloped.

The welfare cost of moving from 0% to 10% inflation is 2.0% of annual consumption if money creation is wasted, but -0.6% if given lump-sum, suggesting there are large welfare benefits of improved risk-sharing despite the inflation tax on liquid balances. By disaggregating the effects of inflation across households at different points of the wealth distribution, we showed that transfers financed with money creation raises the welfare of all households except the most wealthy.

A natural next step consists of introducing aggregate shocks on labor productivity so that the value of firms becomes stochastic, thereby affecting the liquidity services of stocks and the strength of the interest rate channel, and potentially the behavior of unemployment over the business cycle. It would also be natural to introduce consumer credit as an additional self-insurance tool for households. Finally, one could explore different forms of asset liquidity (e.g., adjustment costs versus partial acceptability or pledgeability) to see how they affect the functioning of the different channels we described in this paper. We leave these extensions for future work.

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Appendix A: Proofs of Propositions

Proof of Proposition ??. We apply standard contraction-mapping arguments to the Bellman equation (**??**),

$$W_e(a;z) = \max_{c,\mathbf{y},\hat{\mathbf{a}}} \left\{ U(c,e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \{0,1\}} \alpha_\omega \left[v(y_{\omega e'}) + W_{e'}\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}; z\right)\right] + (1-\alpha) W_{e'}(\mathbf{1}.\hat{\mathbf{a}}; z) \right\} \right\}$$

$$\text{s.t. } c + \mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e(z) \text{ and } p^y y_{\omega e'} \leq \hat{a}^m + \omega \hat{a}^\iota, \;\; e' \in \{0,1\}. \tag{41}$$

To reduce notation, from here we suppress the dependence on the permanent productivity type, z, . Consider the space $C\left(\{0,1\} \times \mathbb{R}_+\right)$ of bounded and continuous functions from $\{0,1\} \times \mathbb{R}_+$ into $\mathbb{R}$, equipped with the sup norm. By Theorem 3.1 in Stokey, Lucas, and Edward Prescott (1989, henceforth SLP), this is a complete metric space. Now, for any $f \in C\left(\{0,1\} \times \mathbb{R}_+\right)$, consider the Bellman operator:

$$T\left[f\right]_e(a) = \max \left\{ U(c,e) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \{0,1\}} \alpha_\omega \left[v(y_{\omega e'}) + f_{e'}\left(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e'}\right)\right] + (1-\alpha) f_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right\} \right\}$$

with respect to $c \geq 0$, $p^y y_{\omega e'} \leq \hat{a}^m + \omega \hat{a}^\iota$, and $\mathbf{R}^{-1}.\hat{\mathbf{a}} = a + w_e - c$. It is straightforward to verify that T satisfies the Blackwell sufficient condition for a contraction (Theorem 3.3 in SLP). Moreover, the constraint set is non-empty, compact valued, and continuous. Hence, by the Theorem of the Maximum (Theorem 3.6 in SLP), we obtain that $T[f]$ is continuous. It is bounded since all the functions on the right side of the Bellman equation, including U and v, are bounded. Note as well that if f is concave, then $T[f]$ is also concave since the objective is concave (because U and v are concave by assumption) and the constraint correspondence has a convex graph. An application of the Contraction Mapping Theorem (Theorem 3.2 in SLP) implies that the fixed point problem $f = T[f]$ has a unique bounded solution, $W_e(a)$, and that this solution is continuous and concave. From the assumptions that $U(c,e)$ and $v(y)$ are increasing in c and y, respectively, it follows that $W_e(a)$ is increasing. Given a fixed point $W_e(a)$ of the Bellman operator T, we can define $V_e(\hat{\mathbf{a}})$ as in equation (**??**). By identical arguments as above, one sees that $V_e(\hat{\mathbf{a}})$ is bounded, continuous, concave, and strictly increasing.

The indirect utility of the household in the second stage corresponds to the following Lagrangian:

$$\Omega_{\omega e}(\hat{\mathbf{a}}) = \max_y \left\{ v(y) + W_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y) + \lambda_{\omega e}\left(\hat{a}_m + \omega \hat{a}^\iota - p^y y\right)\right\}, \tag{42}$$

where $\lambda_{\omega e} \geq 0$ denotes the Lagrange multiplier corresponding to the constraint $p^y y_{\omega e} \leq \hat{a}^m + \omega \hat{a}^\iota$. The objective is strictly concave and the constraint is linear. By Corollary 1 in Marimon and Werner (2015) the value function, $\Omega_{\omega e}(\hat{\mathbf{a}})$, is differentiable with

$$\frac{\partial \Omega_{\omega e}(\hat{\mathbf{a}})}{\partial a^j} = W_e'(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}) + \chi_\omega^j \lambda_{\omega e},$$

where we recall that $\chi^j_\omega \equiv \mathbb{I}_{\{j=m\}} + \mathbb{I}_{\{j=\iota,\omega=1\}}$. Moreover, the first-order condition for y gives:

$$v'(y_{\omega e}) = p^y W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}) + p^y \lambda_{\omega e}.$$

Hence, the partial derivatives of $\Omega_{\omega e}(\hat{\mathbf{a}})$ are also given by

$$\frac{\partial \Omega_{\omega e}(\hat{\mathbf{a}})}{\partial \hat{a}^j} = \chi^j_\omega \frac{v'(y_{\omega e})}{p^y} + (1-\chi^j_\omega) W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}).$$

Substituting $\Omega_{\omega e}(\hat{\mathbf{a}})$ from (**??**) into (**??**), the Bellman equation can be rewritten as:

$$W_e(a) = \max_{\hat{\mathbf{a}}} \left\{ U\left(a + w_e - \mathbf{R}^{-1}.\hat{\mathbf{a}}, e\right) + \beta \mathbb{E}_e \left\{ \sum_{\omega \in \{0,1\}} \alpha_\omega \Omega_{\omega e'}(\hat{\mathbf{a}}) + (1-\alpha) W_{e'}(\mathbf{1}.\hat{\mathbf{a}}) \right\} \right\}. \tag{43}$$

By the same Corollary 1 in Marimon and Werner (2015) the value function $W_e(a)$ is differentiable for all $a > 0$ with $W'_e(a) = U_c\left[c_e(a), e\right]$. From (**??**), the value function in the second stage can be reexpressed as

$$V_e(\hat{\mathbf{a}}) = \sum_{\omega \in \{0,1\}} \alpha_\omega \Omega_e(\hat{\mathbf{a}}) + (1-\alpha) W_e(\mathbf{1}.\hat{\mathbf{a}}).$$

The partial derivatives are

$$\frac{\partial V_e(\hat{\mathbf{a}})}{\partial \hat{a}^j} = \sum_{\omega \in \{0,1\}} \alpha_\omega \left[\chi^j_\omega \frac{v'(y_{\omega e})}{p} + (1-\chi^j_\omega) W'_e(\mathbf{1}.\hat{\mathbf{a}} - p^y y_{\omega e}) \right] + (1-\alpha) W'_e(\mathbf{1}.\hat{\mathbf{a}}).$$

■

Proof of Proposition ??. The condition for liquidity to be abundant is $a \geq \mu y^\star_\mu$, which from (**??**) gives (**??**). The result $r = \rho$ follows directly from (**??**). Let us turn to equilibria with scare liquidity. By market clearing households' asset holdings are $a = (1+r)\theta k / \left[\delta + \lambda(\theta)\right] + A^g$. From (**??**) we can reexpress the first term on the right side as

$$\frac{\theta k}{\delta + \lambda(\theta)} = \frac{\frac{\lambda(\theta)}{\delta + \lambda(\theta)} \left(\bar{q} - w_1\right) + \alpha \left(\frac{\mu - 1}{\mu}\right) a}{r + \delta}.$$

Hence, a is a solution to

$$a = \frac{(1+r)\left[\frac{\lambda(\theta)}{\delta + \lambda(\theta)} \left(\bar{q} - w_1\right) + \alpha \left(\frac{\mu-1}{\mu}\right) a\right]}{(r+\delta)} + A^g. \tag{44}$$

Under the assumption $\bar{q} - w_1 > 0$, (**??**) admits a finite positive solution if $(1+r)\alpha \left(\frac{\mu-1}{\mu}\right) / (r+\delta) < 1$. Under this condition, the solution $a(r, \theta, A^g)$ is decreasing in r, and increasing in θ and A^g. We can reduce

(**??**)-(**??**) to

$$\frac{\rho - r}{1+r} = \alpha \left\{ \mu^{-1} v' \left[\frac{a(r,\theta,A^g)}{\mu} \right] - 1 \right\} \tag{45}$$

$$\left\{ \left[\frac{\mu(1-\alpha)+\alpha}{\mu} \right] r + \delta - \alpha \left(\frac{\mu - 1}{\mu} \right) \right\} \frac{\theta k}{\lambda(\theta)} = \bar{q} - w_1 + \alpha \left(\frac{\mu - 1}{\mu} \right) \left(1 + \frac{\delta}{\lambda(\theta)} \right) A^g. \tag{46}$$

Note that the first term between brackets on the left side of (**??**) is positive provided that the solution a to (**??**) exists. The asset market clearing condition, (**??**), gives a positive relationship between r and θ. Assuming $\bar{q} - w_1 > 0$, the job creation condition, (**??**), gives a negative relationship between θ and r. In the space (θ, r) an increase in A^g shifts the asset market clearing condition upward and the job creation condition to the right. Hence, $\partial r / \partial A^g > 0$ but $\partial \theta / \partial A^g \lesseqgtr 0$. ■

Appendix B: Numerical procedure to compute transitional dynamics

This section describes the numerical procedure to compute transitional dynamics used in our welfare calculations. We compute perfect foresight transitions in response to a one-time, unanticipated, permanent monetary growth rate shock using a sequence space Jacobian method following **?**. We map the model to a directed acyclic graph (DAG), illustrated in Figure **??**, from the exogenous sequence of money supply, $\mathbf{M} \equiv \{M_t\}$, and endogenous prices, $\mathbf{X} \equiv \{R_t^m, R_t^\iota, p_t^y\}$ to market clearing conditions $\mathbf{H} \equiv \{H_{1,t}, H_{2,t}, H_{3,t}\}$. The first market clearing condition is excess supply of real money, $H_{1,t} = M_t \phi_t^m - A_t^m$, where $A_t^m = \sum_z \varpi z \int x dF^m(x;z)$ is the aggregate demand for money. The second market clearing condition is the excess supply for illiquid wealth, $H_{2,t} = A^g + n\bar{z}\tilde{\phi}_t^f - A_t^\iota$, where $\bar{z} = \sum_z \varphi(z) z$ is average labor productivity and $A_t^\iota = \sum_z \varpi(z) \int_z dF^\iota(x;z)$ is the aggregate demand for illiquid wealth. The last market clearing condition is the excess supply of early consumption, $H_{3,t} = n_t \bar{z} \tilde{y}_t^s - Y_t^d$, where $Y_t^d = \sum_{(z,\omega,e)\in\mathcal{Z}]\times\{0,1\}^2} \int y_{\omega e}(\hat{\mathbf{a}};z) dF_e(\hat{\mathbf{a}};z)$ is the aggregate demand for early consumption.

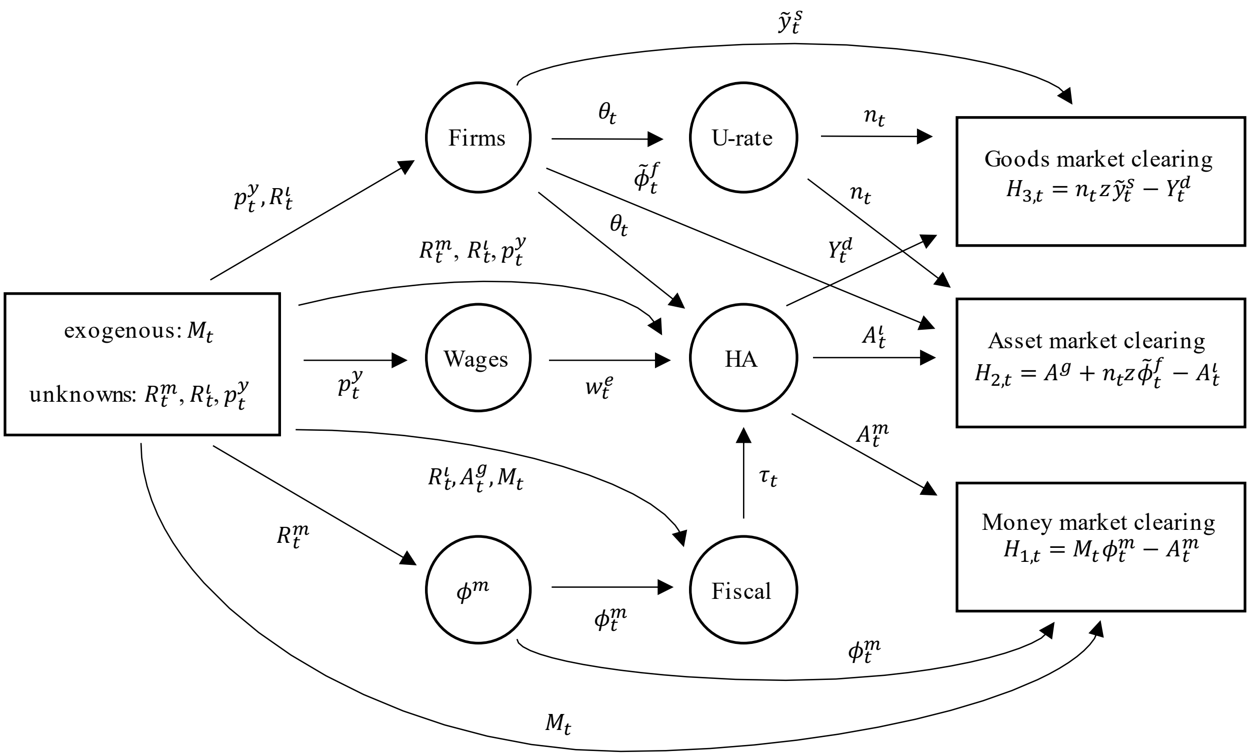

Figure 14: DAG representation of New-monetarist, heterogeneous-agents model with equilibrium unemployment

We set the number of time periods for the transition as T and truncate the dimension of $\mathbf{X}$ and $\mathbf{M}$ to T.

Then, the truncated equilibrium of the model is represented as the system of nonlinear equations given by $\mathbf{H}(\mathbf{X};\mathbf{M}) = 0$. To solve for the perfect-foresight transition after a one-time, unanticipated shock to $\mathbf{M}$, we numerically compute the Jacobian of the system $\mathbf{H}_{\mathbf{X}}(\mathbf{X}_{ss'};\mathbf{M}_{ss'})$, where $\mathbf{X}_{ss'}$ represents the system of prices at the new steady state and $\mathbf{M}_{ss'}$ represents the money supply sequence in the new steady state. We only consider steady states with a constant growth rate in the money supply, so this last term can be substituted for the growth rate.[23] The Jacobian $\mathbf{H}_{\mathbf{X}}$ represents the derivate of any market-clearing condition at any point in time with respect to any price, R^m_t, R^ι_t, p^y_t, at any point in time. We follow the "fake-news algorithm" in **?** to compute the Jacobian of the heterogeneous agents block ("HA" in Figure **??**) using two-sided numerical differentiation and an endogenous grid point method to compute household decision rules.

Our model has several dimensions new to the literature on transitional dynamics in heterogenous-agent models with incomplete markets, including our formalization of asset liquidity and early consumption in the second stage. These new dimensions, however, can be computed efficiently with a similar endogenous grid point method used to compute savings and consumption decisions in the last stage. By using the the first-order condition for early consumption, (**??**), with a non-binding liquidity constraint, we compute a household's desired level of early consumption, $y^*_{e,t}(a)$, using $v'(y^*_{e,t}) = p^y_t W'_{e,t}(a - p^y_t y^*_{e,t})$. Notice, in terms of a household's state, this only depends on total wealth, a and employment status e. Given a vector that represents $W'_{e,t}$, already computed in the endogenous grid point method, computing $y^*_{e,t}(a)$ can be done efficiently by inverting $W'_{e,t}$.[24] A household's decision rule for early consumption in the liquidity event ω is then given immediately by $y_{\omega e,t}(\mathbf{a}) = \min\{\mathbf{y^*_{e,t}}, (\mathbf{a^m} + \omega \mathbf{a}^\iota)/\mathbf{p^y_t}\}$.

We then use the Jacobian within a quasi-Newton method to solve for the dynamics of the equilibrium price system $\mathbf{X}$:

1. Guess an initial path for prices, $\mathbf{X}^0 = \mathbf{X}_{ss'}$.

2. Compute the perfect foresight equilibrium given $\mathbf{X}^0$.

3. Update the path for prices using $\mathbf{X}^{k+1} = \mathbf{X}^k - [\mathbf{H}_{\mathbf{X}}(\mathbf{X}_{ss'};\mathbf{M}_{ss'})]^{-1}\,\mathbf{H}\left(\mathbf{X}^k;\mathbf{M}\right)$

4. If $\max|\mathbf{X}^{k+1} - \mathbf{X}^k| < \epsilon$, for small ϵ, then stop. Otherwise set $\mathbf{X}^k = \mathbf{X}^{k+1}$ and return to step 3.

All other endogenous objects can be computed back backward induction (for decision rules) and forward induction (for distributions) given the equilibrium price paths, $\mathbf{X}$.

[23] In steady state, the level of the money supply does not affect equilibrium allocations. We normalize the level of the money supply in the period before the shock as $M_{t=-1} = 1$.

[24] Specifically, given a grid for $y^*_{e,t}$ you can compute the corresponding a for employment status e as $a = W'^{-1}_{e,t}(v'(y^*_{e,t})/p^y_t) + p^y_t y^*_{e,t}$. Then invert this function to get $y^*_{e,t}(a)$.

Appendix C: Additional numerical results

In this section, we report numerical results for two alternative versions of the model in which i) money creation is distributed to households lump sum or ii) when idiosyncratic labor productivity, z, is stochastic.

Lump-sum money creation with ex-ante, permanent heterogeneity in z We follow the same calibration procedure as outlined in Section **??**. The parameters set independently are identical to those in the baseline calibration, including labor productivity, z_ℓ and $\varpi(z)$, rate of return on liquid wealth, R^m, the replacement rate, $\bar{w}_0/\bar{w}_1$, the worker's share of revenue, μ, the job depreciation rate, δ, and the curvature of the matching function, ν. Table **??** reports how the jointly-calibrated parameters change from the baseline calibration to the new one (labeled as "lump-sum value"), as well as the fit of the model with lump-sum transfers.

Parameter	Baseline	Lump-sum	Moment	Data	Model
Parameters Calibrated Jointly - outer loop					
bond supply, A^g	0.366	2.565	bond share of wealth	13%	20%
entry cost, k	6.460	6.573	monthly job finding rate	30%	30%
production cost curvature, a	0.751	0.780	average retail markup	30%	29%
acceptability of illiquid, α_1/α	0.060	0.360	liquidity premium	6.2%	6.2%
Parameters Calibrated Jointly - inner loop					
discount rate, β^{12}	0.948	0.952	see discussion in Section **??** and Figure **??**		
early consumption - curvature, ψ	0.282	0.200			
early consumption - level, Ψ	2.225	2.182			
preference shock, α	0.075	0.110			

Table 2: Lump-sum money creation: jointly calibrated parameters

The fit of the distribution of liquid wealth to annual income and the distribution of the liquid share of wealth are shown in Figure **??**. Relative to the calibration of the baseline model with wasteful money creation, the calibration of the model with lump-sum money creation has a larger acceptability of illiquid wealth, $\alpha_1/\alpha = 0.36$ compared to $\alpha_1/\alpha = 0.06$, a larger rate of expenditure shocks, $\alpha = 0.110$ compared to $\alpha = 0.075$, and a smaller curvature of the utility of early consumption $\psi = 0.20$ relative to $\psi = 0.28$. The most significant difference is the acceptability of illiquid wealth that is six times larger in the model with lump-sum transfers. Transfers increase risk sharing and reduce the demand for pre-cautionary savings in illiquid wealth. In order to match the liquidity premium and other moments, the calibration prescribes a larger acceptability, α_1/α.

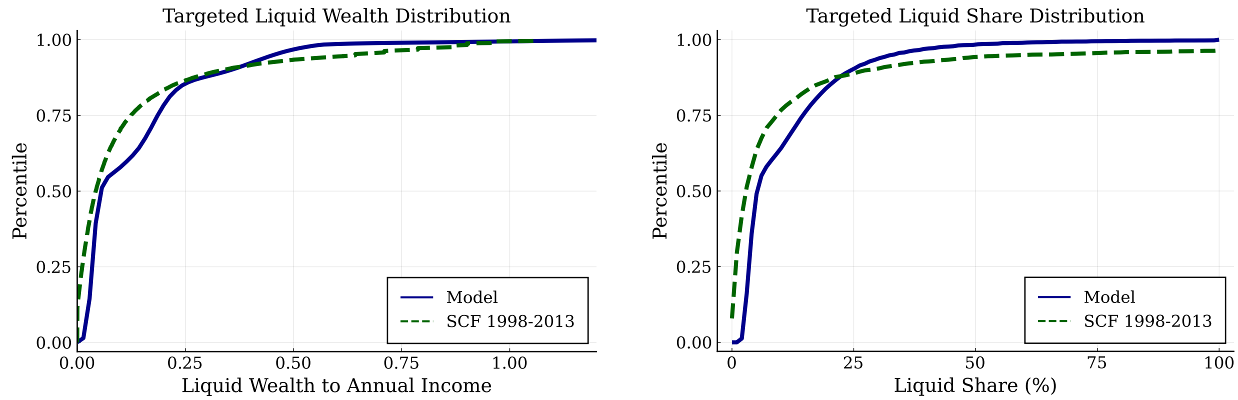

Figure 15: Lump-sum money creation: the distribution of liquid wealth to income (left) and the distribution of the share of wealth held in liquid assets (right), in model versus data.

The distribution of MPCs out of liquid wealth and the consumption and savings responses to job loss are little changed, relative to the baseline economy with wasteful money creation, as shown in Figure **??**. Both measures of household insurance deteriorate slightly, however the cross-sectional patterns are the same. The average MPC increases from 8.3% to 9.5% while the largest MPC for the poorest, unemployed household increases from 85% to 96%. The average change in consumption upon job loss falls from -6.7% to -7.4% and the most severe fall decreases from -45% to -48%.

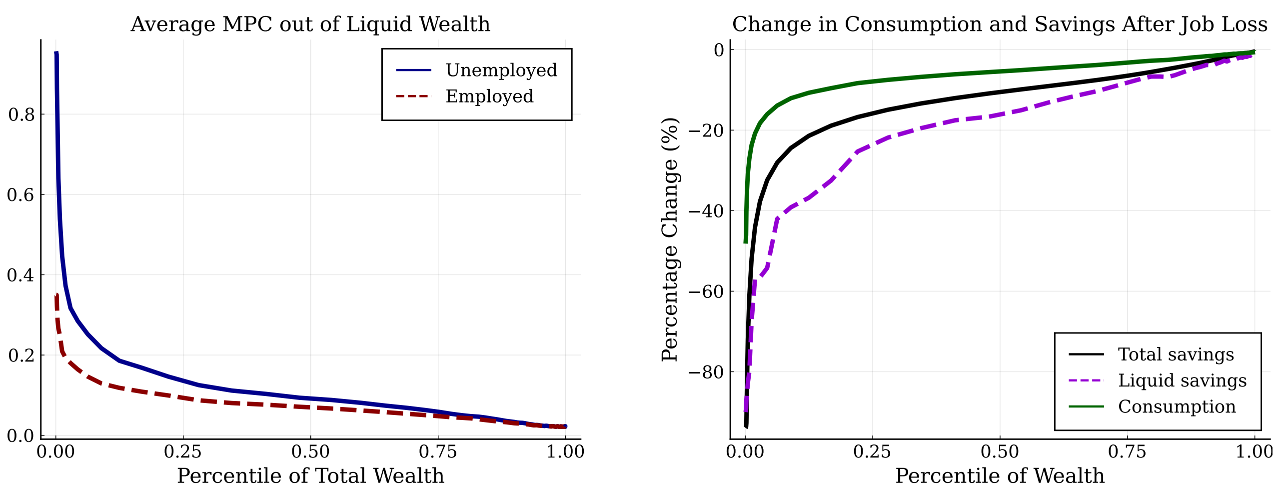

Figure 16: Lump-sum money creation: model cross-section of MPCs and consumption and savings responses to job loss.

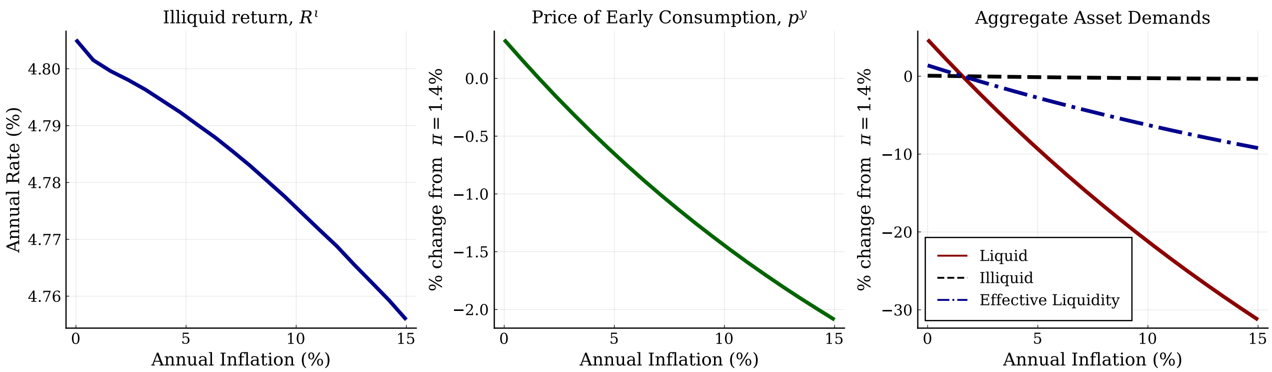

Figure 17: Lump-sum money creation: inflation versus asset returns, prices, and aggregate wealth.

Figure **??** illustrates the response of the financial discount rate, R^ι, the price of early consumption, p^y, and asset demands to inflation. In the baseline economy when money creation is wasteful, inflation leads to a u-shaped pattern in the financial discount rate. When money creation is distributed lump sum, we find the financial discount rate always declines with inflation. Since illiquid financial wealth is more acceptable, the asset substitution effect is stronger, leading to a rise in illiquid asset prices and a decline in the rate of return. Related, a household's effective liquidity, total wealth weighted by each asset's acceptability, is less responsive to inflation. This leads to a smaller effect on aggregate demand and a smaller, negative response in the price of early consumption.

Online Appendix: Not for publication

Appendix D. A simple linear model

We describe a simple version of our model that has no housing and linear preferences both in the last stage, $U(c_t, e_t) = c_t + (1 - e_t)\ell$, as in the MP model, and in the early stage, $v(y) = \bar{v}y$ with $\bar{v} > 1$.[25] In the presence of liquidity constraints, this linear specification does not make the distribution of asset holdings degenerate, but it renders distributional effects inoperative, thereby allowing us to focus on the interest rate and aggregate demand channels. We assume that all assets are equally pledgeable, $\chi = 1$, so that the two forms of public liquidity, government bonds and money, are perfect substitutes. We focus on equilibria with bonds only and denote $a^g = A^g/n$ the supply of one-period real bonds per employed household. We set $\kappa(y) = y$, which means that the production $\bar{q} = \bar{y}$ can be stored across stages with no additional transformation cost to sell to early consumers. With no loss in generality (because of linear preferences and a balanced budget of the government), we set $\bar{w}_0 = \tau_0 = 0$.

Price of early consumption goods The price of early consumption must satisfy $p = p = 1$. Indeed, if $p < 1$ firms sell all their output to late consumers, which is inconsistent with market clearing for early consumption. If $p > 1$, then all the output is sold to early consumers and there is no output left to finance the entry costs of new firms.

The consumption/saving decision Let $V' \equiv V'(a)$ denote the expected discounted utility of one unit of asset at the beginning of a period. It solves

$$V' = \alpha\bar{v} + (1-\alpha)\beta R V' \Longrightarrow V' = \frac{\alpha\bar{v}}{1-(1-\alpha)\beta R}. \tag{47}$$

With probability α the asset is traded for one unit of early consumption, which generates a utility $\bar{v}$. With complement probability, $1 - \alpha$, the asset is saved for the following period (which is always weakly optimal by market clearing), which generates the expected discounted utility $\beta R V'$. The consumption/saving decision in the last stage is given by:

$$\max_{\hat{a} \geq 0} \left(-\frac{\hat{a}}{R} + \beta V' \hat{a} \right) \quad \text{s.t. } \hat{a} \leq R\left(a + \bar{w}_e + \tau_e\right).$$

[25] Such linear specification is used in the context of New-Monetarist models in the example in Section 6.2 of **?** and in **?**. It is also used in the context of an over-the-counter market with liquidity constraints by **?**.

The demand for assets is positive only if $\beta RV' \geq 1$, which can be reexpressed as:

$$R \geq \underline{R} \equiv \frac{1}{\beta\left(\alpha\bar{v}+1-\alpha\right)}. \tag{48}$$

The lower bound for the real interest rate is less than ρ and it decreases with the frequency, α, and value, $\bar{v}$, of early-consumption opportunities. If $R = \underline{R}$, then households are just indifferent between saving and consuming late. If $R > \underline{R}$, then households save their full income, $\hat{a} = R\left(a + \bar{w}_e + \tau_e\right)$.

The demand for private assets Let $\bar{\Omega}$ denote aggregate wealth at the beginning of the second stage if $R > \underline{R}$. It satisfies:

$$\bar{\Omega}_{t+1} = R\left[(1-\alpha)\bar{\Omega}_t + n_t(\bar{w}_1+\tau_1)\right].$$

Aggregate wealth in period $t+1$ is equal to the wealth of the $1-\alpha$ households who did not spend it on early consumption, plus total labor income and transfers, everything capitalized at rate R. From the budget constraint of the government, $\tau_1 = (1-R)\,a^g/R$. The stationary solution is

$$\bar{\Omega}(R) = \frac{n\left[R\bar{w}_1 + (1-R)\,a^g\right]}{1-R(1-\alpha)} \;\text{ if }\; R < (1-\alpha)^{-1}. \tag{49}$$

If $R(1-\alpha) > 1$, then the dynamics of wealth accumulation are explosive, $\bar{\Omega} = +\infty$. We define by $\bar{\omega}(R) \equiv \left[\bar{\Omega}(R) - A^g\right]/n$ the maximum holdings of private assets per employed households. From (**??**):

$$\bar{\omega}(R) = \frac{R\left(\bar{w}_1 - \alpha a^g\right)}{1-R(1-\alpha)} \;\text{ if }\; R < (1-\alpha)^{-1}. \tag{50}$$

Note that $\bar{w}_1 > \alpha a^g$ is necessary for households to accumulate private assets. Under that condition, $\bar{\omega}(R)$ is increasing in R.

Job creations and the supply of private assets From (**??**), assuming the labor market is active, θ solves

$$\frac{\theta k}{\lambda(\theta)} = \frac{\bar{q}-\bar{w}_1}{r+\delta}. \tag{51}$$

Using that $\lim_{\theta\to 0}\lambda(\theta)/\theta = \lambda'(0) = 1$, $\theta > 0$ if $R < \bar{R} \equiv \left[\bar{q} - \bar{w}_1 + (1-\delta)k\right]/k$. Hence, for an active equilibrium to exist, $[\underline{R}, \bar{R})$ must be nonempty, i.e.,

$$\underline{R} < \bar{R} \Leftrightarrow \frac{\left[\rho+\delta-\alpha(\bar{v}-1)(1-\delta)\right]}{\alpha\bar{v}+1-\alpha}k < \bar{q}-\bar{w}_1. \tag{52}$$

We assume in the following that (**??**) holds, i.e., entry costs are sufficiently low to generate firm entry. The supply of private assets per employed worker is $a^p(R) \equiv \phi^f(R)$ where, from (**??**),

$$a^p(R) = \frac{R\left(\bar{q}-\bar{w}_1\right)}{R-(1-\delta)},\; \forall R \in \left(1-\delta, \bar{R}\right). \tag{53}$$

It is decreasing in R with $\lim_{R\searrow 1-\delta} a^p(R) = +\infty$.

Determination of the real interest rate The asset market clearing condition can be expressed as

$$\bar{\omega}(R) \geq a^p(R), \quad " = " \text{ if } R > \underline{R}. \tag{54}$$

If $\bar{\omega}$ is larger than the supply of private assets – a savings glut – then households do not save their full income, which requires $R = \underline{R}$.[26] An equilibrium is a list (n, θ, R) that solves (**??**), (**??**), and (**??**). The equilibrium condition (**??**) is represented graphically in Figure **??**. The following proposition characterizes equilibria in closed form.

Proposition 3 ***(Linear model.)*** *Suppose $U(c,e) = c$, $v(y) = \bar{v}y$ with $\bar{v} > 1$, and (**??**) holds. Assume $\bar{w}_1 > \alpha a^g$. There are two regimes with an active labor market ($\theta > 0$).*

(i) Savings glut. *If the following conditions hold,*

$$\bar{w}_1 - \alpha a^g \geq \frac{[\alpha\bar{v} - \rho(1-\alpha)]}{\rho + \delta - (1-\delta)\alpha(\bar{v}-1)}(\bar{q} - \bar{w}_1) \tag{55}$$

$$\rho + \delta > (1-\delta)\alpha(\bar{v}-1), \tag{56}$$

then r and θ are independent of a^g and solve:

$$r = \frac{\rho - \alpha(\bar{v}-1)}{1 + \alpha(\bar{v}-1)} \tag{57}$$

$$\frac{\theta k}{\lambda(\theta)} = \frac{[1 + \alpha(\bar{v}-1)](\bar{q} - \bar{w}_1)}{\rho + \delta - (1-\delta)\alpha(\bar{v}-1)}. \tag{58}$$

(ii) Abundant asset supply. *If (**??**)-(**??**) do not hold and*

$$a^g < \frac{\alpha\bar{w}_1 + (1-\alpha)\bar{q} - [\alpha + \delta(1-\alpha)]k}{\alpha}, \tag{59}$$

then r and θ solve:

$$r = \frac{\alpha(\bar{q} - \bar{w}_1) - \delta(\bar{w}_1 - \alpha a^g)}{(\bar{w}_1 - \alpha a^g) + (1-\alpha)(\bar{q} - \bar{w}_1)} \tag{60}$$

$$\frac{\theta k}{\lambda(\theta)} = \frac{\alpha(\bar{w}_1 - a^g) + (1-\alpha)\bar{q}}{\alpha + \delta(1-\alpha)}. \tag{61}$$

Moreover, $\partial r/\partial a^g > 0$, $\partial\theta/\partial a^g < 0$, $\partial r/\partial\bar{w}_1 < 0$, $\partial\theta/\partial\bar{w}_1 > 0$, and $\partial n/\partial\bar{w}_1 > 0$.

Proof. (i) The savings glut regime is defined by $R = \underline{R}$. From (**??**) and (**??**) r and θ solve (**??**) and (**??**). A necessary condition for (**??**) to hold at $R = \bar{R}$ is $\bar{R} > 1 - \delta$, i.e., $\rho + \delta > (1-\delta)\alpha(\bar{v}-1)$. A sufficient condition for (**??**) to hold at $R = \bar{R}$ is

$$\alpha\bar{v} - (1-\alpha)\rho \leq 0, \tag{62}$$

[26] By Walras's Law the clearing condition of the asset market, (**??**), and the clearing condition of the goods market are redundant. Hence, in the following we focus on the former.

in which case $\bar{\omega}(\bar{R}) = +\infty$. If $\rho < \alpha\bar{v}/(1-\alpha)$, then $\bar{\omega}(\bar{R}) \geq a^p(\bar{R})$ can be reexpressed as

$$\bar{w}_1 - \alpha a^g \geq \frac{[\alpha\bar{v} - \rho(1-\alpha)]}{\rho + \delta - (1-\delta)\alpha(\bar{v}-1)} (\bar{q} - \bar{w}_1). \tag{63}$$

Given $\bar{w}_1 - \alpha a^g > 0$, (**??**) implies (**??**). (ii) The second regime is such that $R \in (\underline{R}, \bar{R})$. The endogenous variables, r and θ, solve (**??**) and (**??**). It is easy to check that $R > \underline{R}$ is equivalent to (**??**) does not hold. Let's consider the comparative statics with respect to w_1. From (**??**),

$$\frac{\partial r}{\partial \bar{w}_1} = \frac{-(\alpha R + \delta)}{D}$$

where

$$D \equiv \bar{w}_1 - \alpha a^g + (1-\alpha)(\bar{q} - \bar{w}_1).$$

From (**??**), $\theta > 0$ implies $D > 0$. Hence, $\partial r/\partial \bar{w}_1 < 0$ since $R > 0$. The result $\partial\theta/\partial\bar{w}_1 > 0$ follows directly from (**??**). Let's consider next comparative statics with respect to a^g. From (**??**),

$$\frac{\partial r}{\partial a^g} = \frac{\alpha(\delta + r)}{D}.$$

From (**??**), $r > -\delta$. Hence, $\partial r/\partial a^g > 0$. The result $\partial\theta/\partial a^g < 0$ follows directly from (**??**). ■

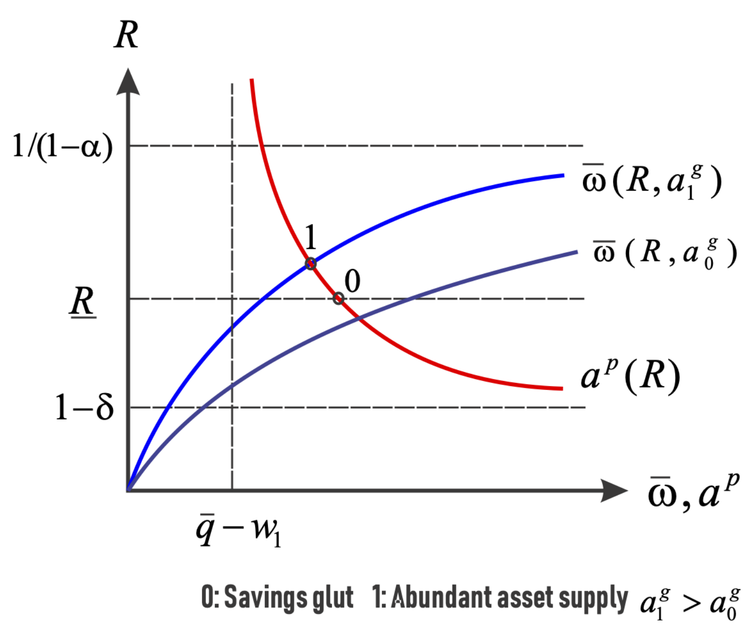

Figure 18: Equilibrium in simple linear model.

In the first regime, the supply of assets is scarce relative to the potential wealth that households can accumulate, which drives the (gross) real interest to its lower bounds, $\underline{R}$. In Figure ?? we indicate such an equilibrium where $\bar{\omega}(\underline{R}) > a^p(\underline{R})$ by the marker "0". The supply of public liquidity, a^g, has no effect on R, and θ. Indeed, if a^g increases, then households ramp up their asset holdings without asking for a higher interest rate. The fact that households raise their early consumption has not effect on firms' profits since early consumption and late consumption are sold at the same price. The condition for a savings glut, (**??**), holds if a^g is small, if $\bar{w}_1$ is large, or if α is small.

In the second regime the supply of assets is sufficiently abundant to drive the real interest rate above its lower bound. In Figure **??** we indicate such an equilibrium where $\bar{\omega}(R) = a^p(R)$ by the marker "1". Households save their full income in order to spend their wealth on early consumption opportunities. When a^g increases, the supply of assets becomes larger than the maximum wealth households can accumulate given their income. As a result, r increases, which reduces the supply of private assets, $\partial r/\partial a^g > 0$, $\partial\theta/\partial a^g < 0$, and $\partial n/\partial a^g < 0$. This effect is the interest channel of public liquidity.

Adding a markup

In order to allow the composition of sales to early and late consumers to matter for firms' profits, suppose now that early consumption is sold at a markup $\mu > 1$ over the opportunity cost of selling late. We treat this markup as exogenous in this simple version of the model but it arises endogenously in the general version when $\kappa'' > 0$. Analogous to the assumption of random matching in search models, the demand for early consumption is divided evenly among the n active firms in the market for early consumption.

Households' marginal value of assets solves (**??**) where $\bar{v}$ is replaced with $\bar{v}/\mu$. The lower bound for the real interest rate is $\underline{R} \equiv (1+\rho) \,/\, [\alpha(\bar{v}/\mu) + 1 - \alpha]$. The average sales of a firm in terms of the numeraire are now

$$q = \bar{q} + \alpha \frac{\mu - 1}{\mu}\left(a^g + \phi^f\right). \tag{64}$$

The second term on the right side of (**??**) corresponds to the additional profits received by a firm from selling to early consumers. Each unit of asset spent on early consumption generates a profit equal to $(1/\mu) - 1$ in terms of the numeraire, and the demand per firm is αa where, by market clearing, $a = a^g + \phi^f$. This second term creates a link between firms' average revenue and households' wealth. From (**??**) market tightness solves

$$\frac{\theta k}{\lambda(\theta)} = \frac{\mu\left(\bar{q} - \bar{w}_1\right) + \alpha\left(\mu - 1\right) a^g}{\delta\mu + [\alpha + (1-\alpha)\mu]\, r - \alpha\left(\mu - 1\right)}. \tag{65}$$

The provision of public liquidity has now a direct effect on market tightness. For given r, $\partial\theta/\partial a^g > 0$ if $\mu > 1$ because firms raise their profits by selling to early consumers. The upper bound for R above which the labor market shuts down is

$$\bar{R} \equiv \frac{\mu\left(\bar{q}-\bar{w}_1\right)+\alpha\left(\mu-1\right)a^g+(1-\delta)\mu k}{\left[\alpha+(1-\alpha)\mu\right]k}.$$

We impose $\underline{R} < \bar{R}$. The supply of private assets per employed worker as a function of the gross real interest rate as

$$a^p(R)=\frac{R\left[\bar{q}-\bar{w}_1+\alpha(1-\mu^{-1})a^g\right]}{R-(1-\delta)-\alpha\left(1-\mu^{-1}\right)},\ \forall R\in\left(1-\delta+\alpha\left(1-\mu^{-1}\right),\bar{R}\right). \tag{66}$$

The maximum wealth per employed worker, $\bar{\omega}(R)$, is still given by (**??**) and the market-clearing condition is given by (**??**). The outcome of the asset market is represented graphically in Figure **??**. A key difference is that the curve $a^p(R)$ is now parameterized by a^g.

Proposition 4 ***(Linear model with markup.)*** *Suppose $U(c,e)=c$ and $v(y)=\bar{v}y$ with $\bar{v}>1$. Moreover, early consumption is sold at a markup $\mu>1$. Assume $\bar{w}_1>\alpha a^g$. There are two regimes with an active labor market ($\theta>0$).*

(i) Savings glut. *If the following condition holds,*

$$\begin{aligned}\bar{w}_1-\alpha a^g &\geq \frac{\left[\alpha\bar{v}\mu^{-1}-\rho(1-\alpha)\right]\left[\bar{q}-\bar{w}_1+\alpha(1-\mu^{-1})a^g\right]}{1+\rho-\left[\alpha\left(\bar{v}\mu^{-1}-1\right)+1\right]\left[1-\delta+\alpha\left(1-\mu^{-1}\right)\right]} && (67)\\ 1+\rho &> \left[\alpha(\bar{v}\mu^{-1}-1)+1\right]\left[1-\delta+\alpha\left(1-\mu^{-1}\right)\right] && (68)\end{aligned}$$

then the real interest rate and market tightness are given by:

$$\begin{aligned} r &= \frac{\rho-\alpha\left(\bar{v}/\mu-1\right)}{\alpha\bar{v}/\mu+1-\alpha} && (69)\\ \frac{\theta k}{\lambda(\theta)} &= \frac{\left[1+\alpha\left(\frac{\bar{v}}{\mu}-1\right)\right]\left[\mu\left(\bar{q}-\bar{w}_1\right)+\alpha\left(\mu-1\right)a^g\right]}{\mu\left(\delta+\rho\right)-\alpha(1+\rho)\left(\mu-1\right)-(1-\delta)\mu\alpha\left(\frac{\bar{v}}{\mu}-1\right)}. && (70)\end{aligned}$$

Moreover, $\partial r/\partial a^g=0$ and $\partial\theta/\partial a^g>0$.

(ii) Abundant asset supply. *If (**??**) and (**??**) do not hold and*

$$a^g<\frac{\alpha(\bar{w}_1-k)+(1-\alpha)\mu\left(\bar{q}-\delta k\right)}{\alpha}, \tag{71}$$

then the real interest rate and market tightness are given by

$$\begin{aligned} r &= \frac{\alpha\left(\mu\bar{q}-\bar{w}_1\right)-\delta\mu\left(\bar{w}_1-\alpha a^g\right)}{\alpha(\bar{w}_1-a^g)+(1-\alpha)\mu\bar{q}} && (72)\\ \frac{\theta k}{\lambda(\theta)} &= \frac{\alpha(\bar{w}_1-a^g)+(1-\alpha)\mu\bar{q}}{\alpha+\mu(1-\alpha)\delta}. && (73)\end{aligned}$$

Moreover, $\partial r/\partial a^g>0$ and $\partial\theta/\partial a^g<0$.

In a savings glut, an increase in a^g does not affect the real interest rate but it raises firms' profits, market tightness, and employment. By raising the amount of wealth that households can accumulate, an increase in a^g raises the consumption of early consumers which is sold at a markup. This effect is the aggregate demand channel of public liquidity. Graphically, in Figure **??**, a small increase in a^g shifts the curve $\bar{\omega}$ upward but its intersection with the curve a^p, which also shifts upward, is still located below $\underline{R}$. In the case of abundant asset supply, the increase in a^g crowds out private assets by raising r – the interest rate channel of public liquidity. In that case, market tightness decreases and employment decreases.

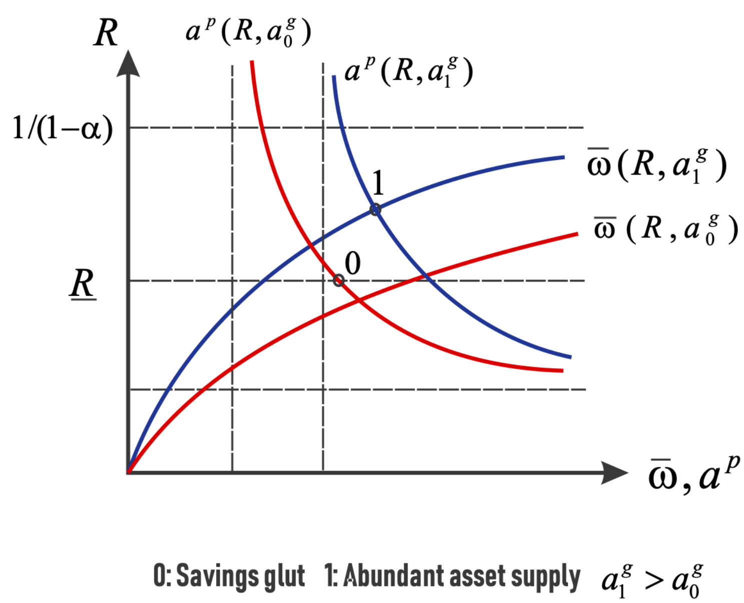

Figure 19: Equilibrium in simple linear model with a markup.

Appendix E. Additional numerical results

In this section, we report additional numerical results for i) a version of the when idiosyncratic labor productivity, z, is stochastic and ii) our baseline model under counter-factual changes in asset liquidity and money transfers schemes.

Lump-sum money creation with idiosyncratic labor-productivity risk

In our baseline environment, households face limited labor earnings risk through occasional unemployment spells. In this section, we introduce additional earnings risk through idiosyncratic shocks to labor productivity, z. We assume that households are ex-ante identical and face an labor productivity process $ln(z_{i,t+1}) = \rho_z ln(z_{i,t}) + \sigma_z \epsilon_{i,t}$, with $\epsilon_{i,t} \sim \mathcal{N}(0,1)$. We follow **?** and set ρ_z and σ_z such that the autocorrelation of earnings, $w_1(z)$, is 0.966 and the cross-sectional standard deviation of log earnings is 0.92. We discretize this process using the Rouwenhorst method using $\mathcal{Z} = \{z_\ell, z_m, z_h\}$. For simplicity, we assume that all households and firms interact in a common labor market with a common recruiting cost k. The free-entry condition (**??**) now becomes

$$-k + \frac{\lambda(\theta)}{\theta} \frac{\mathbb{E}_z \phi^f(z)}{R^\iota} \leq 0, \text{ "=" if } \theta > 0, \tag{74}$$

where the expectations operator is taken over the distribution of labor productivities of the unemployed (since there is no endogenous job destruction, this coincides with $\varpi(z)$). We also assume unemployed households also face risk in their non-employment income such that $w_0(z)/w_1(z)$ is fixed. Hence, a worker that loses their job with $z = z_h$ receives $w_0(z_h)$ but may get a negative productivity shock in unemployment such that $w_0(z) < w_0(z_h)$ for $z = \{z_\ell, z_m\}$. All other equilibrium conditions remain the same if expectations over employment states $\mathbb{E}_e$ are replaced with expectations over both employment and labor productivity $\mathbb{E}_{e,z}$.

To calibrate the model, we maintain the same strategy as outlined in Section **??**, however since our goal with this version of the model is to match cross-sectional features of both the liquid and total wealth distributions, we replace the target for the distribution of the share of liquid wealth to income with the wealth to income distribution. The parameters set independently, $(R^m, \bar{w}_0/\bar{w}_1, \mu, \delta, \nu)$, are identical to those in Table **??**. Table **??** reports the jointly calibrated parameters and how they compare to the model with ex-ante heterogeneous, constant z and lump-sum money transfers. The parameter with the largest difference to the environment with fixed z is supply of government bonds, A^g. The demand for precautionary savings in illiquid wealth is significantly larger when z is stochastic. In order to match returns while keeping average

Parameter	**Lump-sum** *(fixed z)*	**Lump-sum** *(stoch. z)*	**Moment**	**Data**	**Model**
Parameters Calibrated Jointly - outer loop					
bond supply, A^g	2.565	20.65	bond share of wealth	13%	63%
entry cost, k	6.573	7.576	monthly job finding rate	30%	30%
production cost curvature, a	0.780	0.783	average retail markup	30%	31%
acceptability of illiquid, α_1/α	0.360	0.422	liquidity premium	6.2%	6.2%
Parameters Calibrated Jointly - inner loop					
discount rate, β^{12}	0.952	0.954			
early consumption - curvature, ψ	0.200	0.204	see Figure **??**		
early consumption - level, Ψ	2.182	1.806			
preference shock, α	0.110	0.223			

Table 3: Lump-sum money creation with labor-productivity risk: jointly calibrated parameters

labor productivity set at 1, the bond supply must be larger.

Figure **??** illustrates the fit of the model with respect to the liquid and total wealth-to-income distributions. The model has a slightly more concentrated liquid wealth to income distribution compared to the data, however the fit of the total wealth distribution is good. Introducing idiosyncratic labor-productivity risk into the model allow it to match the wealth heterogeneity observed in the data. We now show that the main qualitative implications from our experiments remain unchanged.

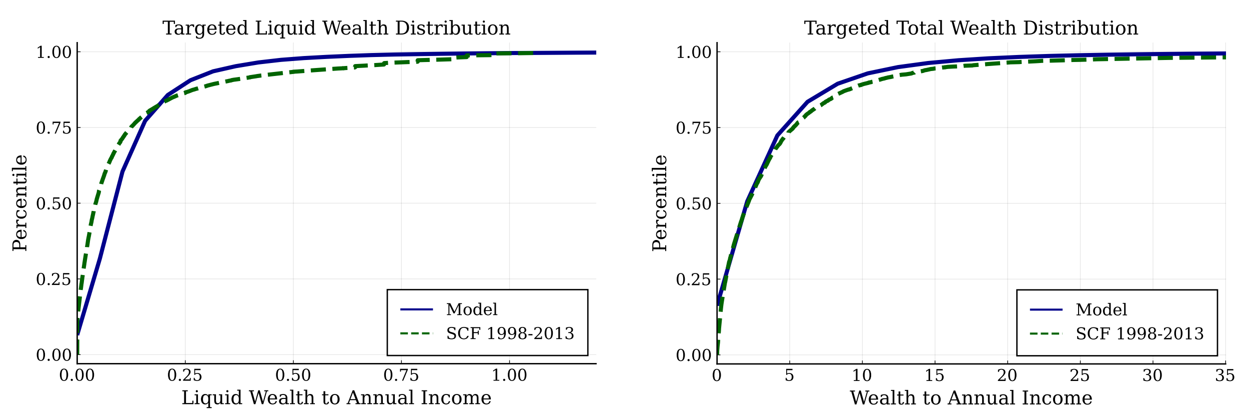

Figure 20: Lump-sum money creation with labor-productivity risk: the distribution of liquid wealth to income (left) and the distribution of the total wealth to income (right), in model versus data.

Figure ?? illustrates the long-run Phillips curve and the interest-rate and aggregate-demand channels, as discussed in Section ??. The Phillips curve is still upward-sloping. The aggregate-demand and interest-rate channels are weaker compared to the version of the model with fixed permanent heterogeneity in labor productivity (and lump-sum transfers of money creation). Inflation reduces the financial discount rate, increases asset prices, and reduces firms' expected revenue.

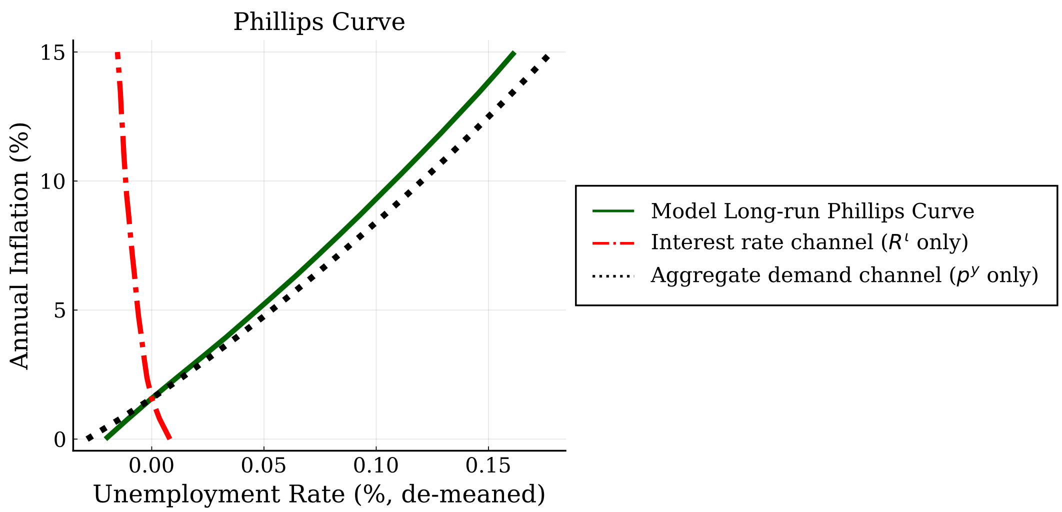

Figure 21: Lump-sum money creation with labor-productivity risk: aggregate demand and interest channels of the long-run Phillips curve.

More on the slope of the long-run Phillips curve

For our calibration, the long-run Phillips curve is almost vertical, i.e., the unemployment rate is largely unresponsive to a change in anticipated inflation. We now show that changes in fundamentals or policy can alter the strengths of the aggregate demand and interest rates channels, with quantitative implications for the long-run trade-off between inflation and unemployment.

Liquidity of financial assets and the long-run Phillips curve In the benchmark calibration, conditional on a preference shock, financial wealth can be liquidated 6% of the time. Suppose now that innovations in the finance and banking industry makes it easier to liquidate and transfer financial wealth in order to allow households to finance unexpected expenditures. We capture this idea by assuming that financial wealth is more liquid than in the baseline, while keeping the same rate of expenditure shocks $\alpha = 0.075$. We set $\alpha_1/\alpha = 0.5$. Figure ?? illustrates how the long-run Phillips curve, and the strength of the aggregate demand and interest rate channels, change under these assumptions.

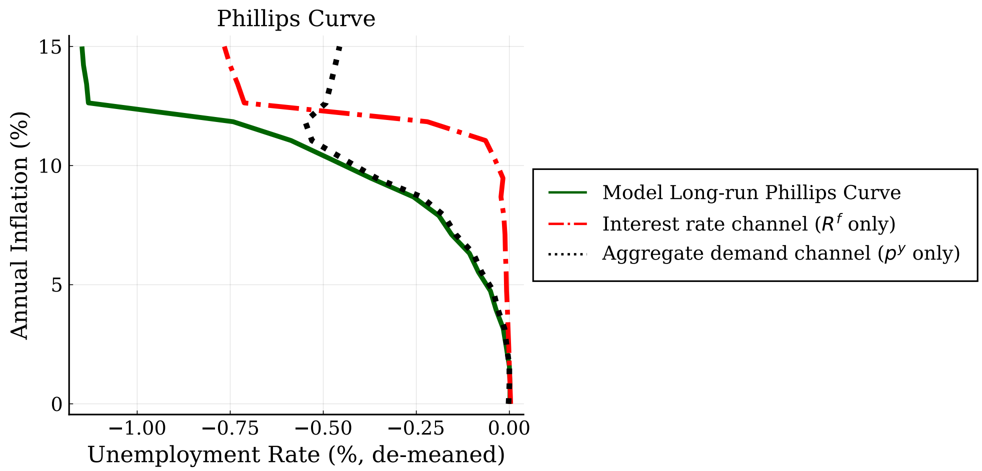

Figure 22: Long-run Phillips curve when asset liquidity is higher, $\alpha_1/\alpha = 0.5$.

Increasing the liquidity of wealth leads to a negatively-sloped long-run Phillips curve, illustrated with the solid-green line in Figure **??**. Quantitatively, an increase in the inflation rate from 0 to 10% reduces unemployment by about 0.5 percent. When the liquidity of financial wealth increases, stocks and bonds become more substitutable with money. As a result, the portfolio substitution effect strengthens and inflation reduces the real return on financial wealth. This effect reduces unemployment as firms' values are boosted, illustrated with a dashed-red line. Further, the increase in the liquidity of financial wealth implies that inflation up to 10% has minimal effects on aggregate demand and the price of early consumption, shown as the dash-dotted red line. For inflation rates larger than 10%, we find these effects reverse; asset prices fall and the price of early consumption rises.

Targeted 'helicopter drops' We now consider a change in policy according to which 'helicopter' drops of money target the unemployed, i.e., money creation is distributed lump sum to unemployed households only. Formally, the transfers conditional on employment status are equal to $\tau_0 = \pi\phi_t M_t/(1-n) + (1/R^g - 1)A^g$ and $\tau_{1=}(1/R^g - 1)A^g$. It means that taxes required to service government debt affect all households, but the revenue generated from money creation is only distributed to the unemployed. This transfer scheme captures the possibility of income-progressivity in monetary transfers.

When money creation is used to finance unemployment benefits, the slope of the long-run Phillips curve flattens relative to the baseline, as illustrated in the solid-green line in Figure **??**. An increase of the inflation rate from 0 to 10 percent raises equilibrium unemployment by about 0.36 percent. The insurance provided by targeted transfers reduces households' precautionary demand for higher-return, less-liquid wealth. Inflation

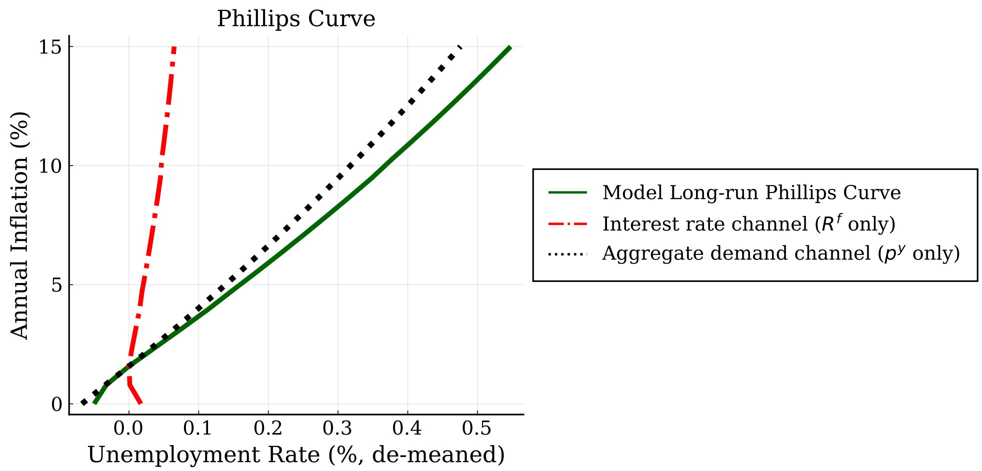

Figure 23: Long-run Phillips curve when money creation is distributed lump-sum to unemployed households.

has a stronger, positive, effect on the return on illiquid wealth, relative to the baseline.